Modeling of Natural Convection in Electronic Enclosures

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Outline

• Introduction and problem description
• Model development
• Numerical simulations
• Validation
• Summary
Introduction

• Current design practice for sealed electronic enclosures
  ▪ Numerical CFD simulations
  ▪ Experimental prototype testing
  ▪ Time consuming, expensive

• Analytically-based modeling
  ▪ Quick, easy to implement
  ▪ Ideal for preliminary design, parametric studies

• Objective: to develop and validate a natural convection model for simple, sealed enclosures
  ▪ Vertical rectangular flat plate at center of a cuboid shaped enclosure
  ▪ Full range of Rayleigh number from laminar natural convection to conduction
Problem Description

- Enclosure dimensions
  \[
  \frac{L_o}{L_i}, \quad \frac{L_i}{W_i}, \quad \frac{L_o}{W_o}, \quad \frac{b}{L_o}
  \]

- Isothermal boundary conditions
  \[T_i > T_o\]

- Total heat transfer rate
  \[Nu \sqrt{A_i} = \frac{Q}{k \sqrt{A_i} (T_i - T_o)} = S^*_{\sqrt{A_i}} \text{ for } Ra \rightarrow 0\]

- Rayleigh number
  \[Ra \sqrt{A_i} = \frac{g \beta (T_i - T_o) (\sqrt{A_i})^3}{\nu \alpha}\]
• Combination of three asymptotic solutions (Teertstra, 2003)

\[ \text{Nu}_{\sqrt{A_i}} = S_{\sqrt{A_i}}^* + \left[ \left( \frac{1}{\text{Nu}_{tr}} \right)^2 + \left( \frac{1}{\text{Nu}_{bl}} \right)^2 \right]^{-1/2} \]

\[ S^*_{\sqrt{A_i}} = \text{conduction shape factor} \]

\[ \text{Nu}_{tr} = \text{transition flow convection} \]

\[ \text{Nu}_{bl} = \text{laminar boundary layer flow convection} \]
Conduction Shape Factor

- Composite model (Churchill and Usagi, 1972)

\[
S_{\sqrt{A_i}}^* = \left[ \left( S_{b/L \to 0}^* \right)^{3/2} + \left( S_{b/L \to \infty}^* \right)^{3/2} \right]^{2/3}
\]

- \(b/L \to 0\) one dimensional conduction in gap

\[
S = \frac{A_i}{b}, \quad S_{b/L \to 0}^* = \frac{S}{\sqrt{A_i}} = \frac{\sqrt{A_i}}{b}
\]

- \(b/L \to \infty\) shape factor independent of \(b/L\)

\[
S_{b/L \to \infty}^* = \frac{1}{k \sqrt{A_i R}}, \quad R = R_{\text{plate}} - R_{\text{sphere}}
\]

\(R_{\text{plate}}\) = isothermal flat plate in full space region

\(R_{\text{sphere}}\) = equivalent sphere in full space region

\[d_{\text{eff}} = \frac{(L_o + W_o)}{2}\]
Laminar Boundary Layer

- Assumptions
  - $T_b$ uniform
  - Non-intersecting boundary layers
- Series combination of resistances

$$ R_{conv} = R_i + R_o $$

$$ Nu_{bl} = \frac{1}{k \sqrt{A_i}} \frac{1}{R_{conv}} = \frac{1}{k \sqrt{A_i}} \left( \frac{1}{1 + R_o/R_i} \right) $$

$$ R_i = \frac{T_i - T_b}{Q} = \frac{1}{k \sqrt{A_i}} \frac{1}{Nu_i} $$

$$ R_o = \frac{T_b - T_o}{Q} = \frac{1}{k \sqrt{A_o}} \frac{1}{Nu_o} $$

- Convection at boundaries modeled using Lee, Yovanovich and Jafarpur (1991)

$$ Nu_{\sqrt{A}} = F(Pr) G_{\sqrt{A}} Ra_{\sqrt{A}}^{1/4} $$
Laminar Boundary Layer

\[ \text{Nu}_{bl} = \frac{F(\text{Pr}) G \sqrt{A_i} \text{Ra}^{1/4}}{1 + \left( \frac{A_i}{A_o} \right)^{7/10} \left( \frac{G \sqrt{A_i}}{G \sqrt{A_o}} \right)^{4/5}} \]

- Prandtl number function \( F(\text{Pr}) = 0.513 \) for air at STP
- Body gravity functions

\[ G \sqrt{A_i} = 2^{1/8} \left( \frac{W_i}{L_i} \right)^{1/8} \quad \text{(Lee et al., 1991)} \]

\[ G \sqrt{A_o} = 2^{1/8} \left[ \frac{0.625(2b)^{4/3} W_o + L_o (2b + W_o)^{4/3}}{(L_o W_o + 2b (L_o + W_o))^{7/6}} \right]^{3/4} \quad \text{(Jafarpur and Yovanovich, 1993)} \]
Transition Flow

- Boundary layers merge at low Rayleigh numbers
- Linear temperature distribution in core
- Convective heat transfer in top and bottom recirculation regions
- Enthalpy balance in end regions

\[
Nu_{tr} = \frac{\sqrt{2}}{360} \frac{\sqrt{W_i/L_i}}{(1 + L_o/L_i)} \left( \frac{\delta_{\text{eff}}}{\sqrt{A_i}} \right)^3 Ra \frac{1}{\sqrt{A_i}}
\]

\( \delta_{\text{eff}} = \text{gap spacing of equivalent spherical cavity} \)

\( L_o, L_i = \text{effective flow length on outer, inner wall} \)
Numerical Simulations

- Conduction shape factor
  - Flotherm simulations of 56 enclosure geometries
    \[ T_i = 40^\circ C \quad T_o = 20^\circ C \]
    \[ \frac{L_o}{L_i} = 1.05, 1.2, 1.6, 2 \]
    \[ \frac{L_i}{W_i} = \frac{L_o}{W_o} = 0.5, 1, 2 \]
    \[ b/L_o = 1 \rightarrow 0.05 \]

- Natural convection
  - Icepak simulations of 42 enclosure geometries
    \[ \frac{L_o}{L_i} = 1.05, 1.2, 1.6, 2 \]
    \[ \frac{L_i}{W_i} = \frac{L_o}{W_o} = 1, 2 \]
    \[ b/L_o = 1 \rightarrow 0.05 \]
Conduction Model Validation

\[ S^* \sqrt{\frac{A_i}{L_i}} \]

\[ L_i / W_i = 1 \]

\[ L_o / W_o = 1 \]
Convection Model Validation

\[ \frac{L_o}{L_i} = 1.05 \]
\[ \frac{L_o}{W_o} = 1 \]
\[ \frac{L_i}{W_i} = 1 \]
Convection Model Validation

\[
\begin{align*}
N_v \sqrt{A_i} & \approx 25 \\
Ra \sqrt{A_i} & \approx 10^2 \quad 10^3 \quad 10^4 \quad 10^5 \quad 10^6 \quad 10^7
\end{align*}
\]

\[
\begin{align*}
L_o / L_i & = 1.2 \\
L_o / W_o & = 0.5 \\
L_i / W_i & = 0.5
\end{align*}
\]

\[
\begin{align*}
\text{model} & \{ \\
\frac{b}{L_o} & \{ \\
1 & \text{black, solid} \\
0.5 & \text{red, solid} \\
0.2 & \text{green, solid} \\
0.1 & \text{blue, solid} \\
0.05 & \text{violet, solid}
\}
\end{align*}
\]

\[
\begin{align*}
\text{data} & \{ \\
\text{square} & \text{black, solid} \\
\text{triangle} & \text{red, solid} \\
\text{down triangle} & \text{green, solid} \\
\text{diamond} & \text{blue, solid} \\
\text{circle} & \text{violet, solid}
\}
\end{align*}
\]
Convection Model Validation

\[ \text{model} \{ \begin{align*}
L_o / L_i &= 1.6 \\
L_o / W_o &= 2 \\
L_i / W_i &= 2
\end{align*} \}

\[ \text{data} \{ \begin{align*}
b / L_o &= 1 \\
&= 0.5 \\
&= 0.2 \\
&= 0.1 \\
&= 0.05
\end{align*} \}

\[ \text{model} \{ \begin{align*}
b / L_o &= 1 \\
&= 0.5 \\
&= 0.2 \\
&= 0.1 \\
&= 0.05
\end{align*} \}

\[ \text{data} \{ \begin{align*}
b / L_o &= 1 \\
&= 0.5 \\
&= 0.2 \\
&= 0.1 \\
&= 0.05
\end{align*} \}

\[ \begin{align*}
10^2 & \quad 10^3 & \quad 10^4 & \quad 10^5 & \quad 10^6 & \quad 10^7
\end{align*} \]

\[ \begin{align*}
13 & \quad 25 & \quad 75 & \quad 125 & \quad 175
\end{align*} \]
Convection Model Validation

\[ \text{model} \begin{cases} \frac{b}{L_o} = 1 \\ \frac{b}{L_o} = 0.5 \\ \frac{b}{L_o} = 0.2 \\ \frac{b}{L_o} = 0.1 \\ \frac{b}{L_o} = 0.05 \\ \end{cases} \]

\[ L_o / L_i = 2 \]
\[ L_o / W_o = 1 \]
\[ L_i / W_i = 1 \]

\[ \text{data} \begin{cases} 
\begin{align*} 
\frac{b}{L_o} = 1 & : \square \\
\frac{b}{L_o} = 0.5 & : \triangle \\
\frac{b}{L_o} = 0.2 & : \downarrow \\
\frac{b}{L_o} = 0.1 & : \diamond \\
\frac{b}{L_o} = 0.05 & : \circ 
\end{align*}
\end{cases} \]

\[ \text{Nu}_{\sqrt{Ai}} \]

\[ Ra_{\sqrt{Ai}} \]

\[ 14 \quad 10^2 \quad 10^3 \quad 10^4 \quad 10^5 \quad 10^6 \quad 10^7 \]
Summary

• Analytical model developed for natural convection for a vertical plate in a sealed, cuboid shaped enclosure

• Validated with data from CFD simulations
  ▪ 10 % average difference with numerical data

• Demonstrates trends in data as function of geometry and Rayleigh number

• Future work
  ▪ Isoflux inner boundary condition
  ▪ Array of vertical plates
  ▪ Experimental validation of analytical models
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