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Optimization of Pin-Fin Heat Sinks Using Entropy Generation Minimization

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Outline

- ✓ Introduction
- ✓ Literature Review
- ✓ Objectives and Criterion
- ✓ Optimization
- ✓ Results
- ✓ Conclusions
- ✓ Acknowledgments

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Introduction

Constraints: L, W, l, w, Q or T_c , T_a and Overall height

Design Variables: D, H, N, U_{app} In-Line/Staggered

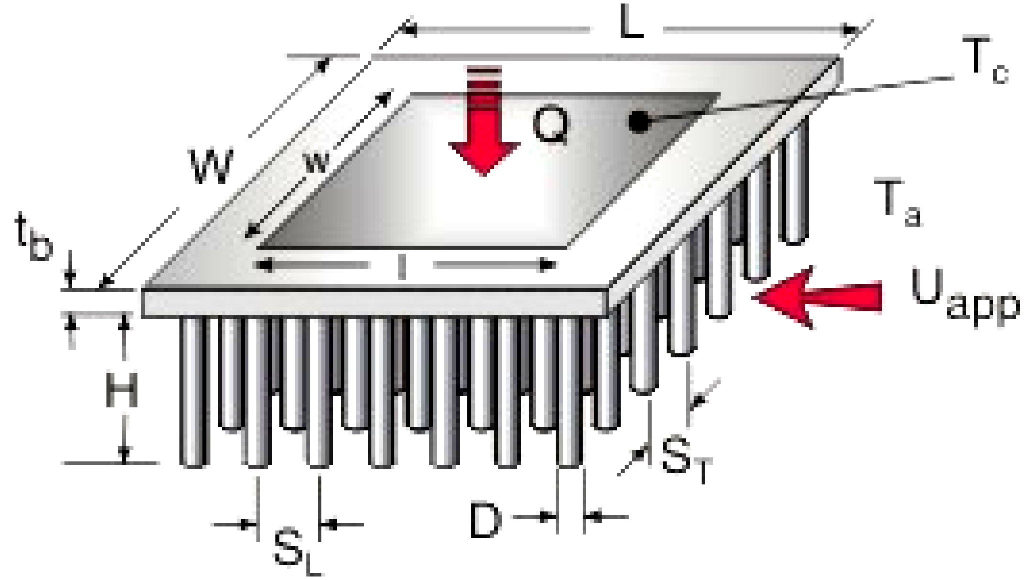
$$40 \le Re_D \le 1000$$

$$3 \le H / D \le 8$$

$$1.25 \le S_T / D \le 3$$

$$1.25 \le S_L / D \le 3$$

$$H + t_b \le 12$$



Literature Review

- Theoretical
- Experimental
- No analytical work available to optimize all design variables

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Objectives

Possible Objective Functions:

- T_c
- ΔP
- Cost
- Weight

Goal: To Transfer maximum heat, keep chip temperature at minimum

Selected Objective Function:

Minimize T_c

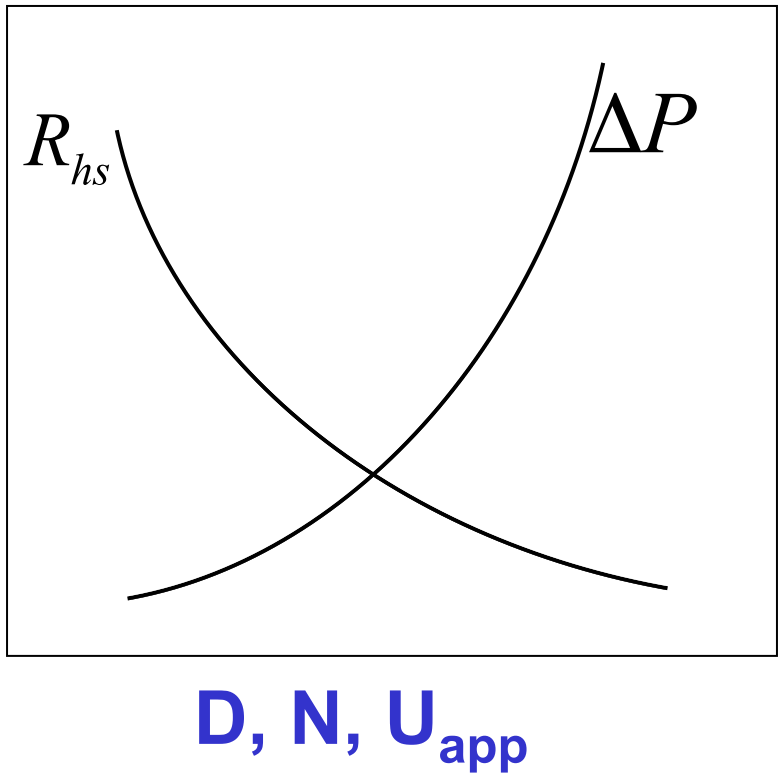
Criterion

$$\downarrow T_c = QR_{hs} \downarrow + T_a$$

$\downarrow \mathrm{R_{hs}}$

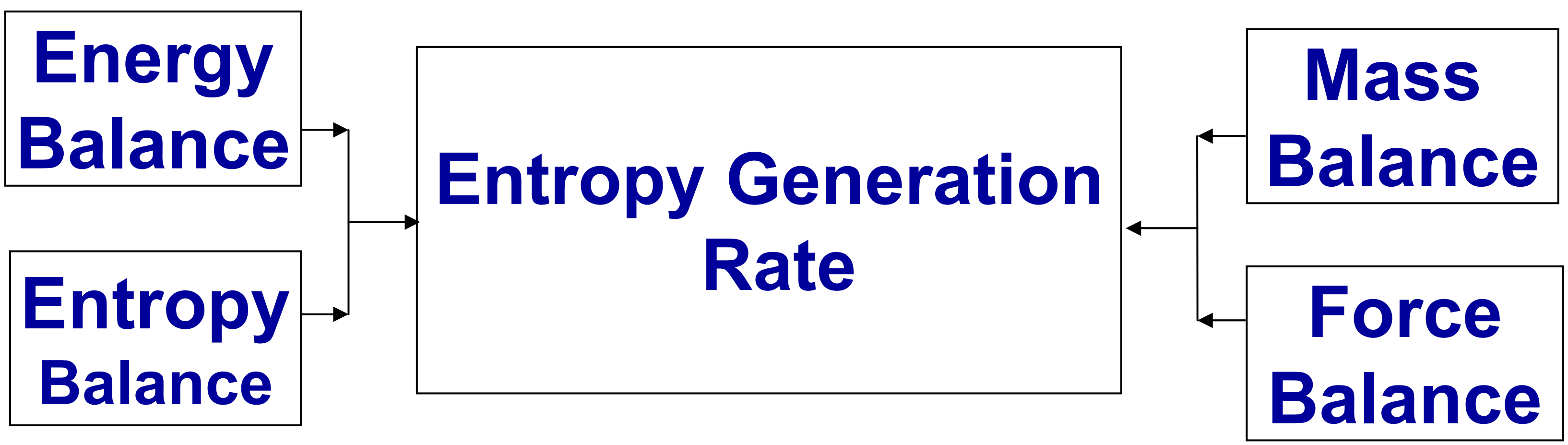
- $h \uparrow$
  - **Fluid properties**
  - **Fluid velocity**
  - **Geometry**
- $A \uparrow \rightarrow \Delta P \uparrow \rightarrow R_{fin} \uparrow$

Criterion



Criterion (EGR)

- Criterion for rating the performance of heat sinks
- Requires thermodynamics, fluid mechanics, heat transfer, material and geometry



Optimization

- **Model Development**
- **Problem Formulation**
- **Implementation**

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EGM Model

Specify System Parameters

$L, W, t_b, N, D, H, l, w, k, Q, k_f, \rho, c_p, \mu$

Heat Sink Resistance Model

Pressure Drop Model

EGM Model

$$\dot{S}_{gen} = \left(\frac{Q^2}{T_a T_b}\right) R_{hs} + \left(\frac{\dot{m}}{\rho T_a}\right) \Delta P$$

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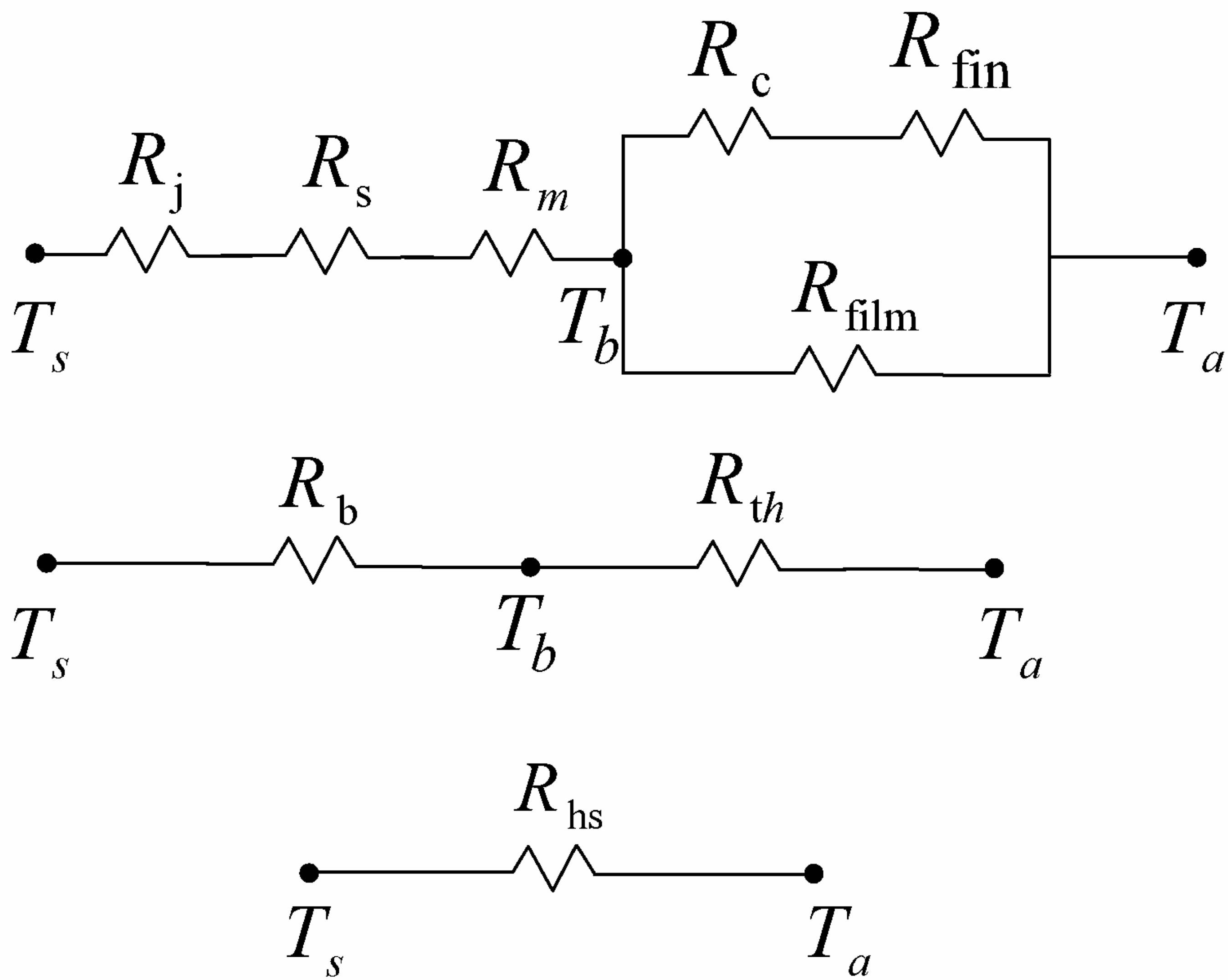
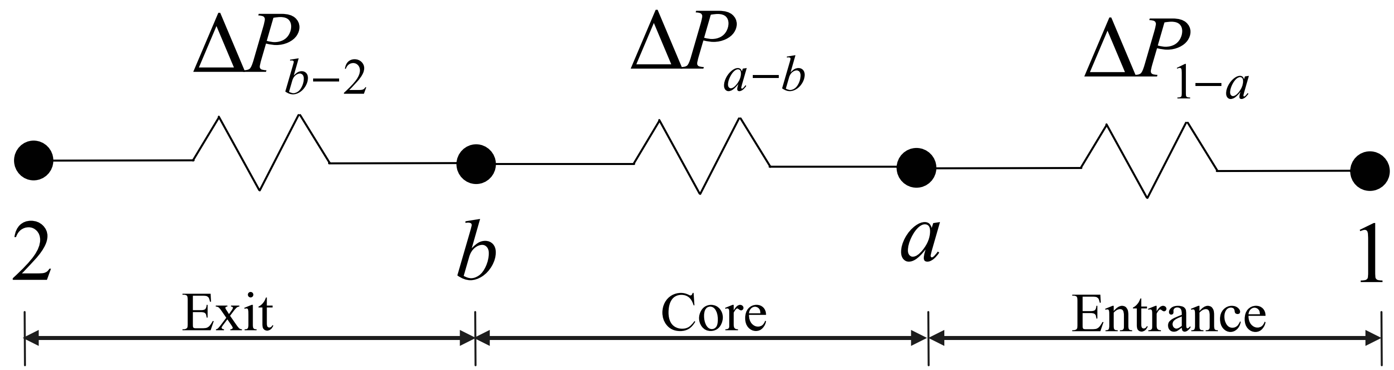
Heat Sink Resistance Model



Pressure Drop Model



$$\Delta P_{1-a} = k_c \cdot \frac{\rho U_{max}^2}{2}$$

$$\Delta P_{b-2} = k_e \cdot \frac{\rho U_{max}^2}{2}$$

$$\Delta P_{a-b} = f N_L \cdot \frac{\rho U_{max}^2}{2}$$

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Problem Formulation

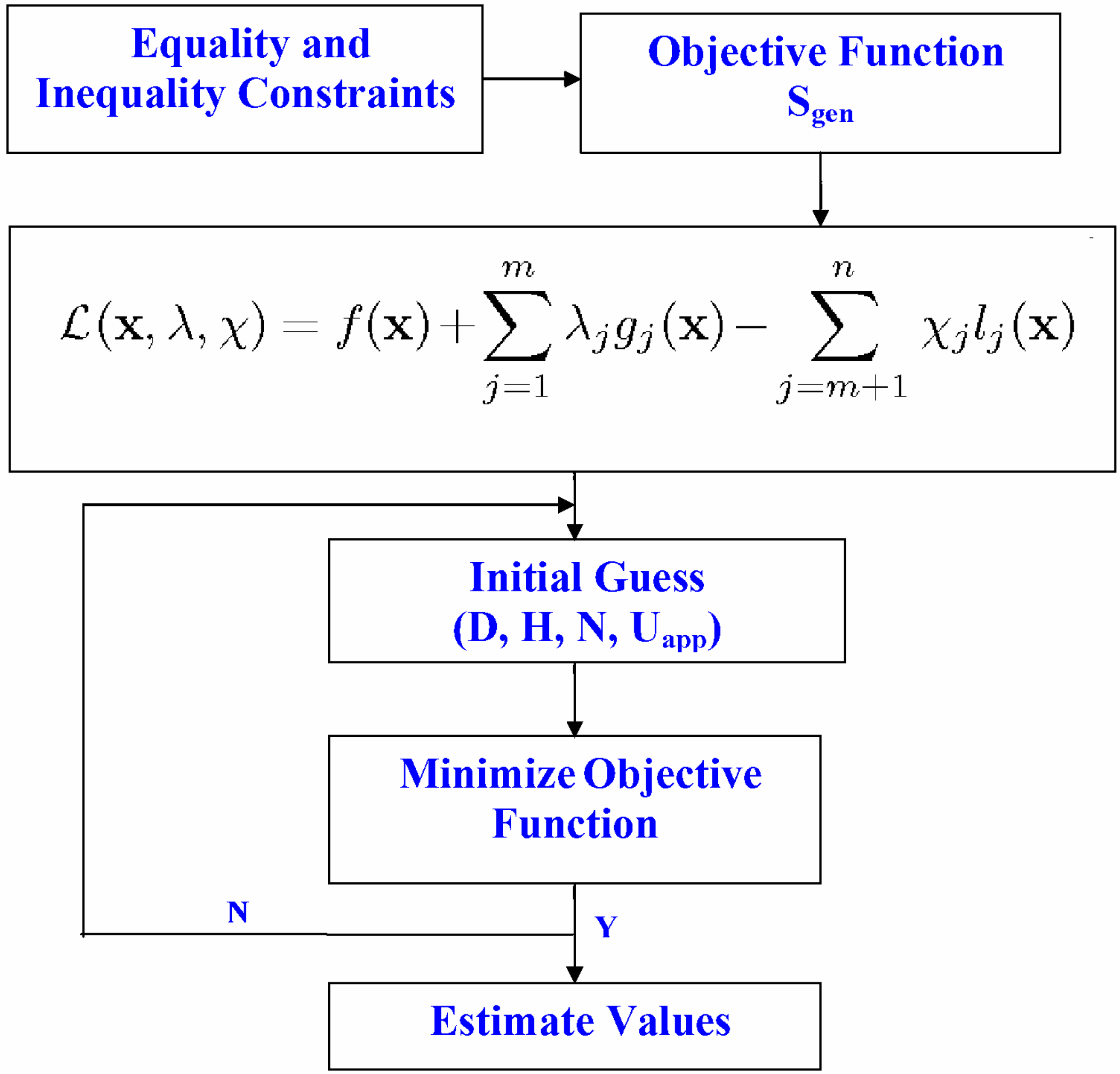
$$minimize \;\; f(\mathrm{x}) = \dot{S}_{gen}(\mathrm{x})$$

$$g_j(\mathrm{x}) = 0, \quad j = 1, 2, ..., m$$

$$l_j(\mathrm{x}) \geq 0, \quad j = m+1, ..., n$$

$$\mathcal{L}(\mathrm{x}, \lambda, \chi) = f(\mathrm{x}) + \sum_{j=1}^{m} \lambda_j g_j(\mathrm{x}) - \sum_{j=m+1}^{n} \chi_j l_j(\mathrm{x})$$

Implementation



Results

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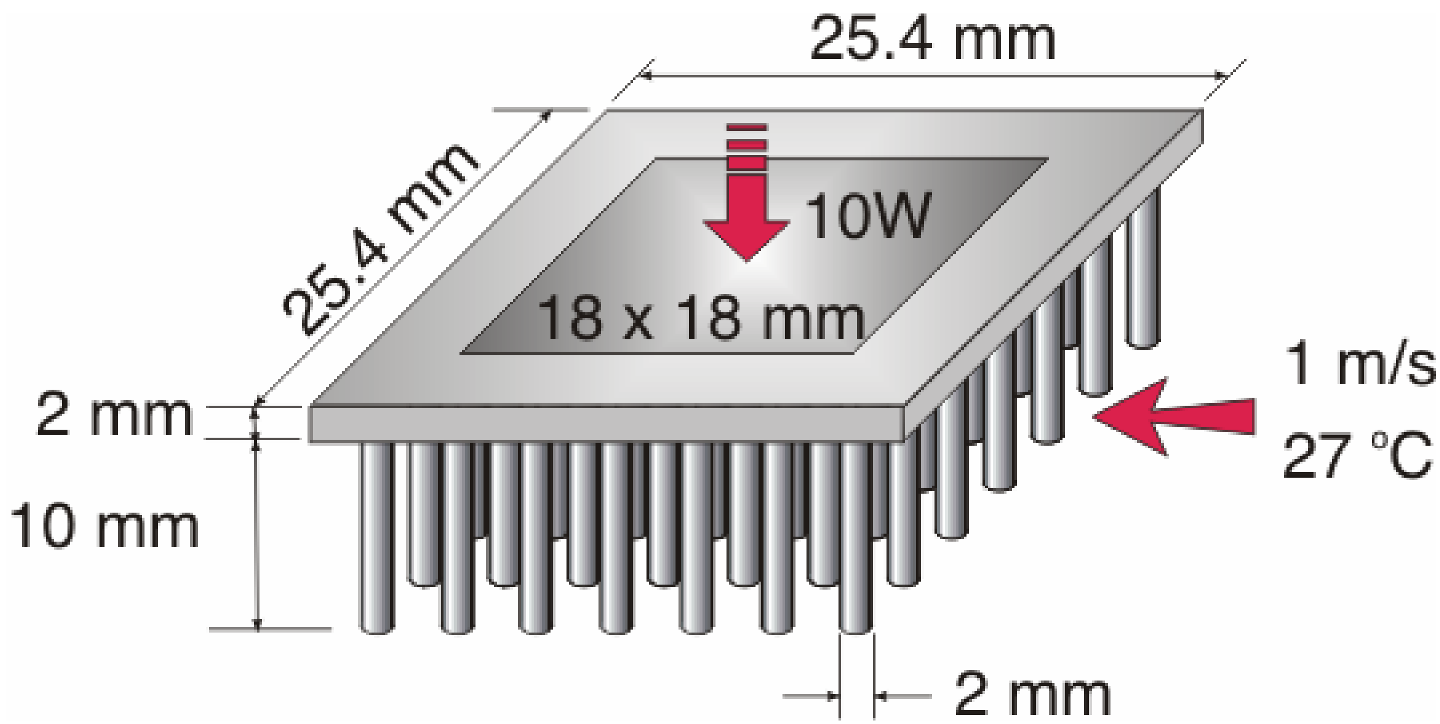
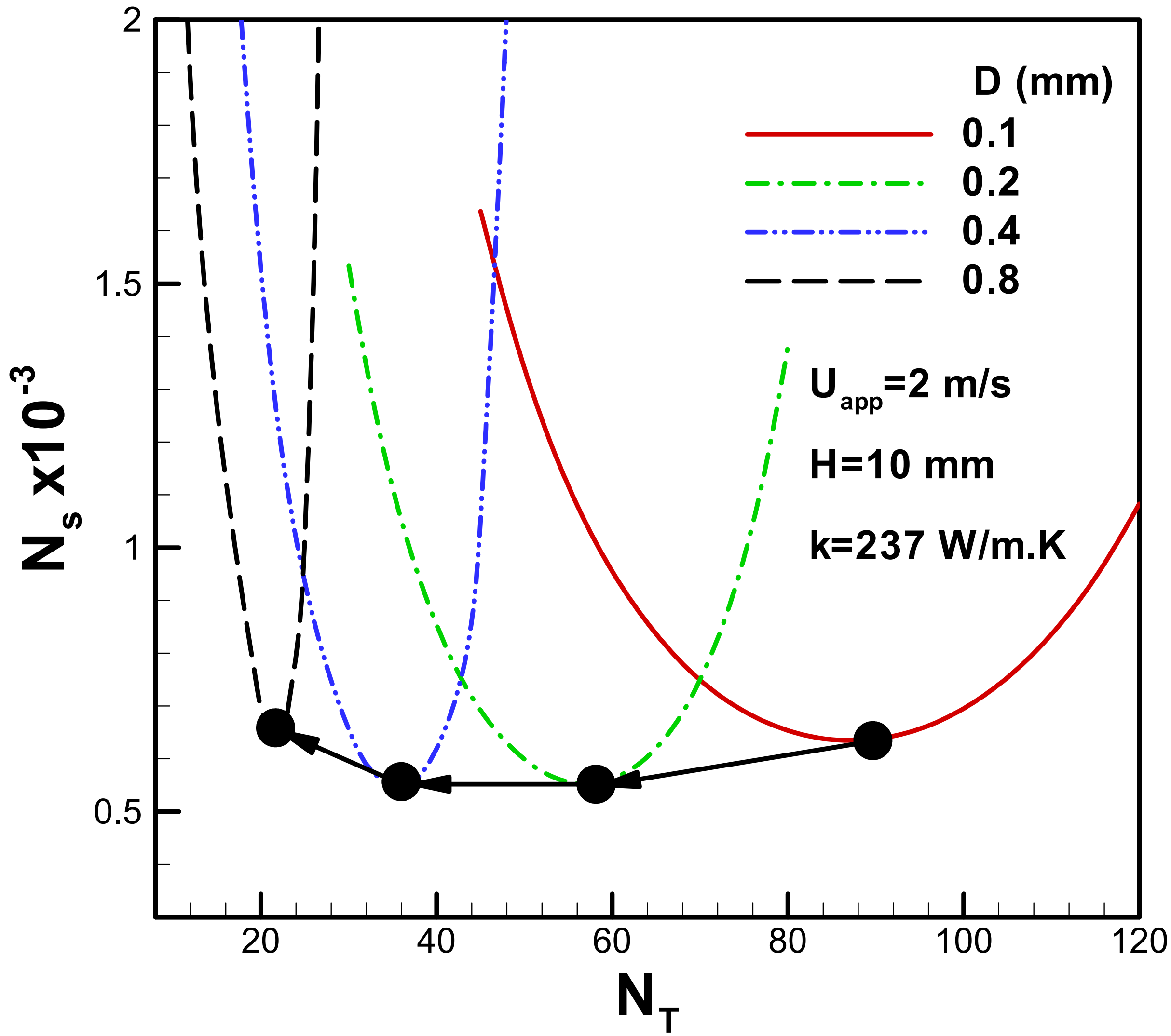
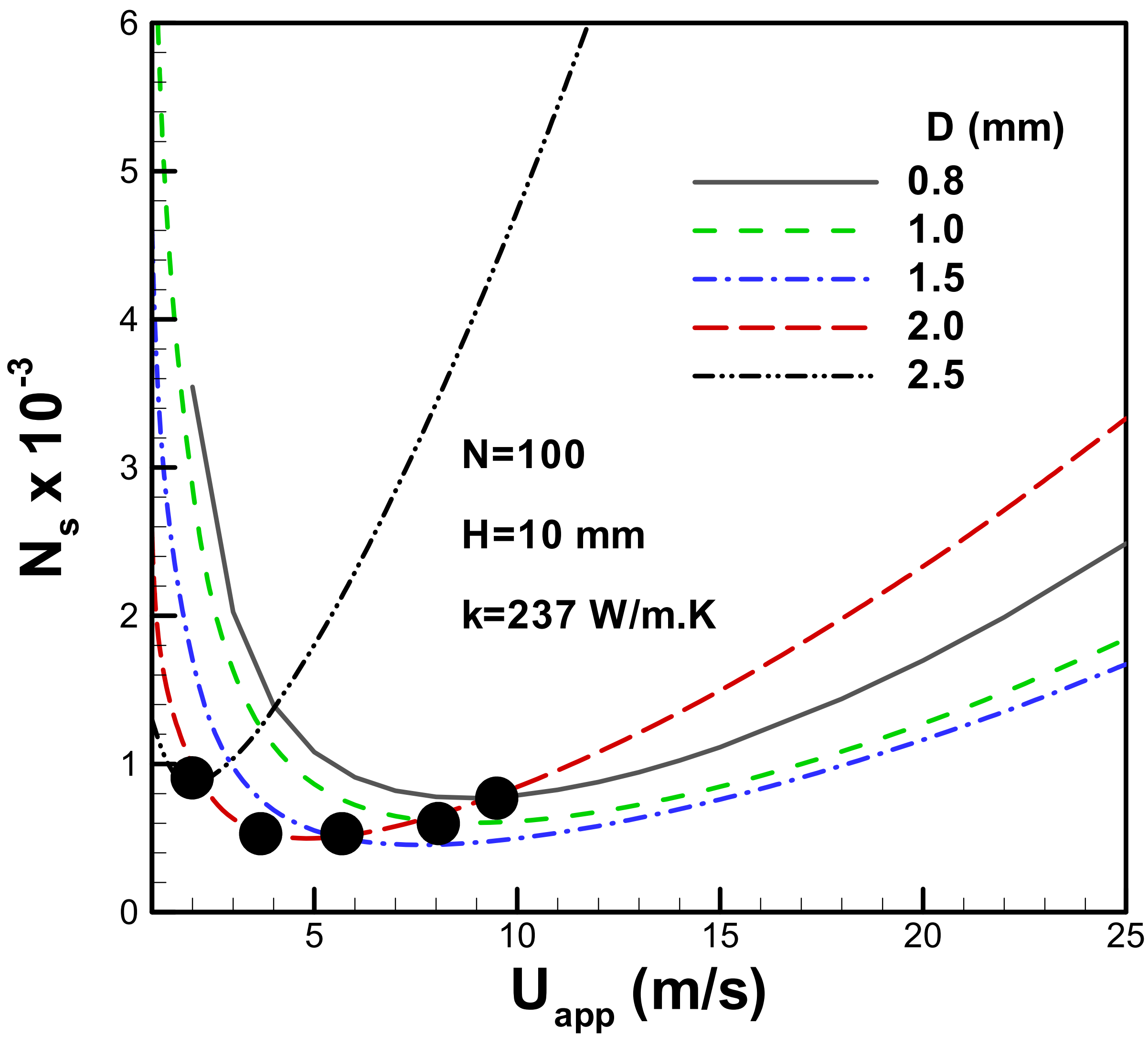
In-Line PFHS



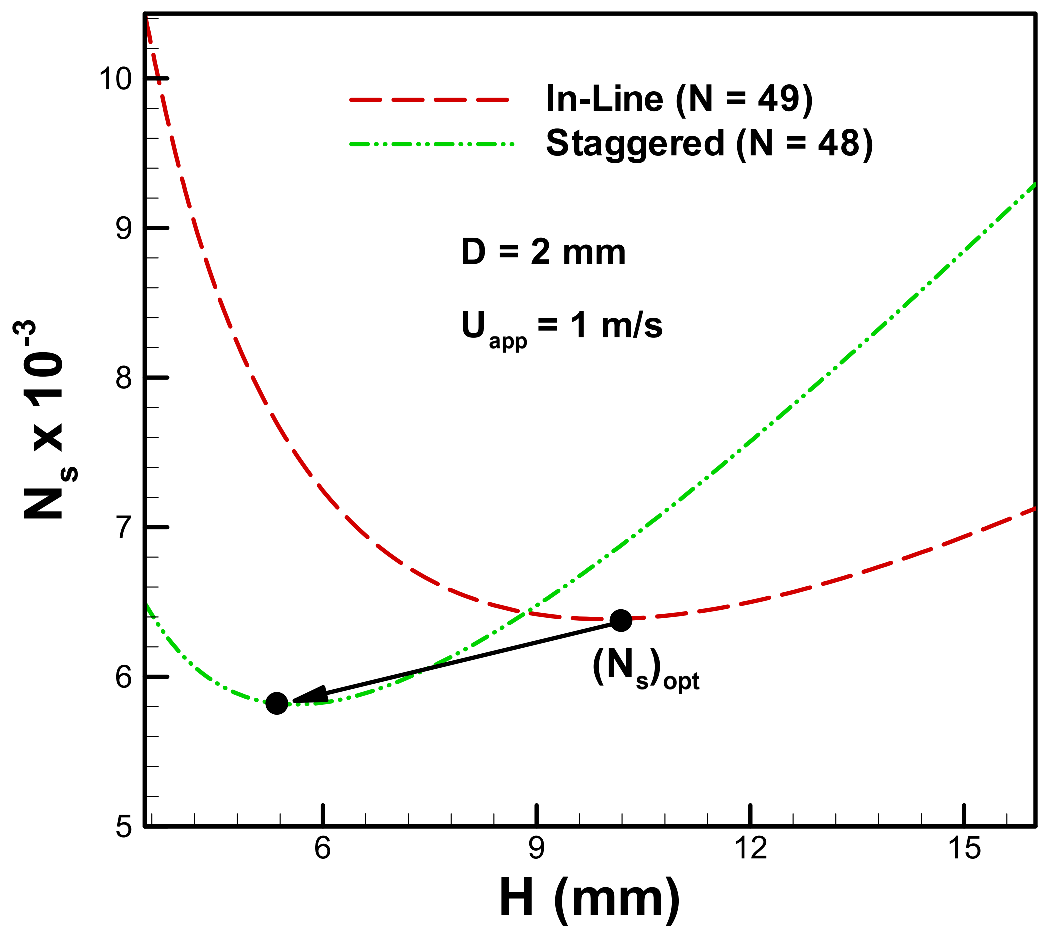
Optimization (In-Line PFHS)



Optimization (In-Line PFHS)



Optimization (Comparison)



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Optimization (Four Parameters)

In-Line Arrangemment

k	Optimized Design Variables				R_{hs}	ΔP	$\dot{S}_{gen}$
W/mK	D (mm)	H (mm)	U (m/s)	$N_T \times N_L$	$^oC/W$	N/m^2	W/K
25	0.92	4	4.58	11×11	2.12	107.4	0.002
237	0.50	4	3.35	18×18	1.17	95.7	0.001

Staggered Arrangemment

k	Optimized Design Variables				R_{hs}	ΔP	$\dot{S}_{gen}$
W/mK	D (mm)	H (mm)	U (m/s)	$N_T \times N_L$	$^oC/W$	N/m^2	W/K
25	0.86	4	2.29	14×14	1.71	175.1	0.002
237	0.62	4	2.00	18×18	1.06	168.7	0.001

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Conclusions

- ✓ EGM is a powerful technique that can
  - optimize several parameters
  - trade-off between R_{hs} and ΔP
  - minimize irreversibility penalty costs
- ✓ Possible to design an optimum PFHS

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Optimization Problem

$$minimize \;\; f(\mathbf{x}) = \dot{S}_{gen}(\mathbf{x})$$

subject to equality constraints

L-25.4=0

W-25.4=0

l-25.4=0

w-25.4=0

T_a-27=0

Q-10=0

and inequality constraints

$H_{overall} - 12 \leq 0$

$S_T/D - 3 \leq 0$

$S_L/D - 3 \leq 0$

$Re_D \leq 1000$

$H/D \leq 8$

$$\mathcal{L}(\mathbf{x}, \lambda, \chi) = f(\mathbf{x}) + \sum_{j=1}^{m} \lambda_j g_j(\mathbf{x}) - \sum_{j=m+1}^{n} \chi_j l_j(\mathbf{x})$$

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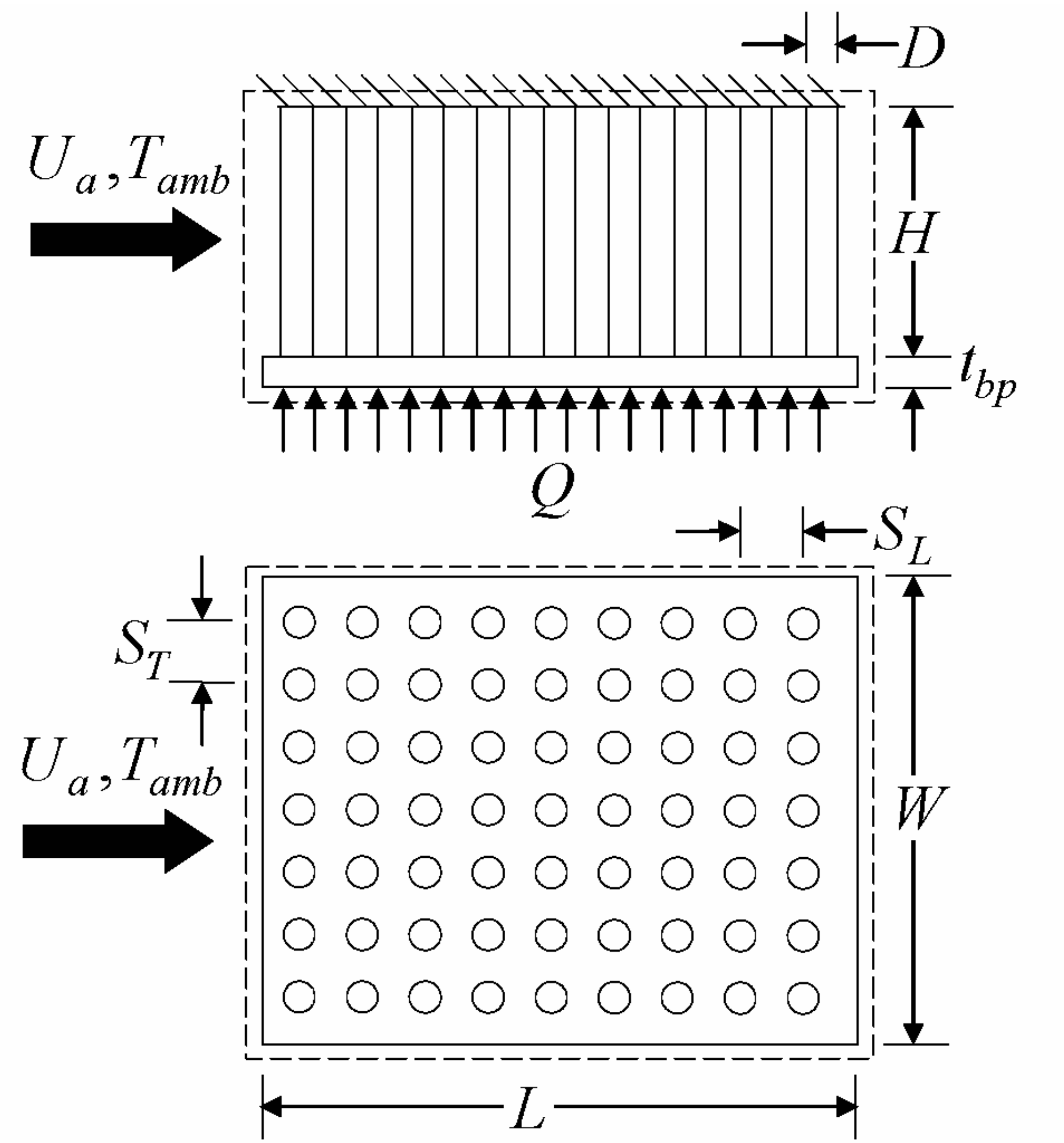
Control Volume



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Heat Transfer

Pin-Fin Arrays

$$\frac{Nu_D}{Re_D^{1/2} Pr^{1/3}} = \begin{cases} \dfrac{[0.2 + \exp(-0.55\mathcal{S}_T)]\mathcal{S}_T^{\,0.785}\mathcal{S}_L^{\,0.212}}{(\mathcal{S}_T - 1)^{0.5}} & \text{for In-Line} \\[2ex] \dfrac{0.61\,\mathcal{S}_T^{\,0.591}\mathcal{S}_L^{\,0.053}}{(\mathcal{S}_T - 1)^{0.5}\,[1 - 2\exp(-1.09\,\mathcal{S}_T)]} & \text{for Staggered} \end{cases}$$

Base Plate

$$\frac{Nu_L}{Re_L^{1/2} Pr^{1/3}} = \begin{cases} 0.750 & \text{for UWT} \\ 0.912 & \text{for UWF} \end{cases}$$

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Fluid Flow

Friction factors

$$f = \begin{cases} K_1 \left[0.233 + 45.78/(\mathcal{S}_T - 1)^{1.1} Re_D\right] & \text{for In-Line arrays} \\ K_1 \left[378.6/\mathcal{S}_T^{13.1/\mathcal{S}_T}\right] / Re_D^{0.68/\mathcal{S}_T^{1.29}} & \text{for Staggered arrays} \end{cases}$$

Correction factors when $\mathcal{S}_T \neq \mathcal{S}_L$

$$K_1 = \begin{cases} 1.009\left(\frac{\mathcal{S}_T - 1}{\mathcal{S}_L - 1}\right)^{1.09/Re_D^{0.0553}} & \text{for In-Line arrays} \\ 1.175(\mathcal{S}_L/\mathcal{S}_T Re_D^{0.3124}) + 0.5 Re_D^{0.0807} & \text{for Staggered arrays} \end{cases}$$

Contraction coefficient

$$k_c = -0.0311\sigma^2 - .3722\sigma + 1.0676$$

Expansion coefficient

$$k_e = 0.9301\sigma^2 - 2.5746\sigma + 0.973$$

where

$$\sigma = \frac{\mathcal{S}_T - 1}{\mathcal{S}_T}$$

Newton – Raphson Method

- Newton – Raphson Method
  - Suitable for multiple non-linear equations
  - More rapidly convergent
  - Sequence of equations listed is immaterial
  - Easily adapted in mathematical software packages

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Reference Velocity

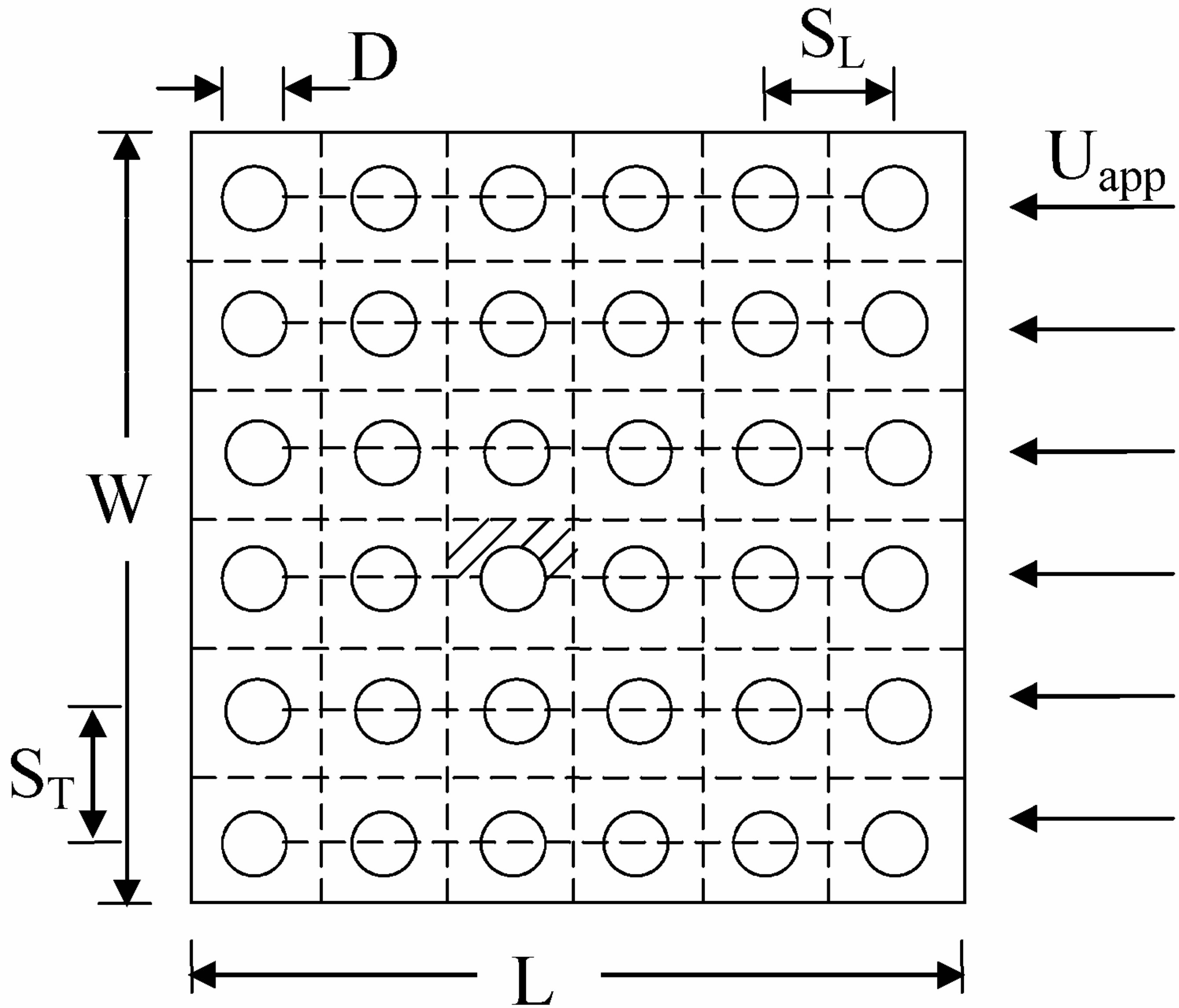
- For In-Line Pins

$$\frac{U_{\max}}{U_1} = \frac{S_T}{S_T - D}$$

- For Staggered Pins

$$\frac{U_{\max}}{U_1} = \max\left\{ \frac{S_T}{S_T - D}, \frac{S_T}{\sqrt{4S_L^2 + S_T^2} - 2D} \right\}$$

In-Line Arrangement



Control Volume