Modeling of Cylindrical Pin-Fin Heat Sinks for Electronic Packaging

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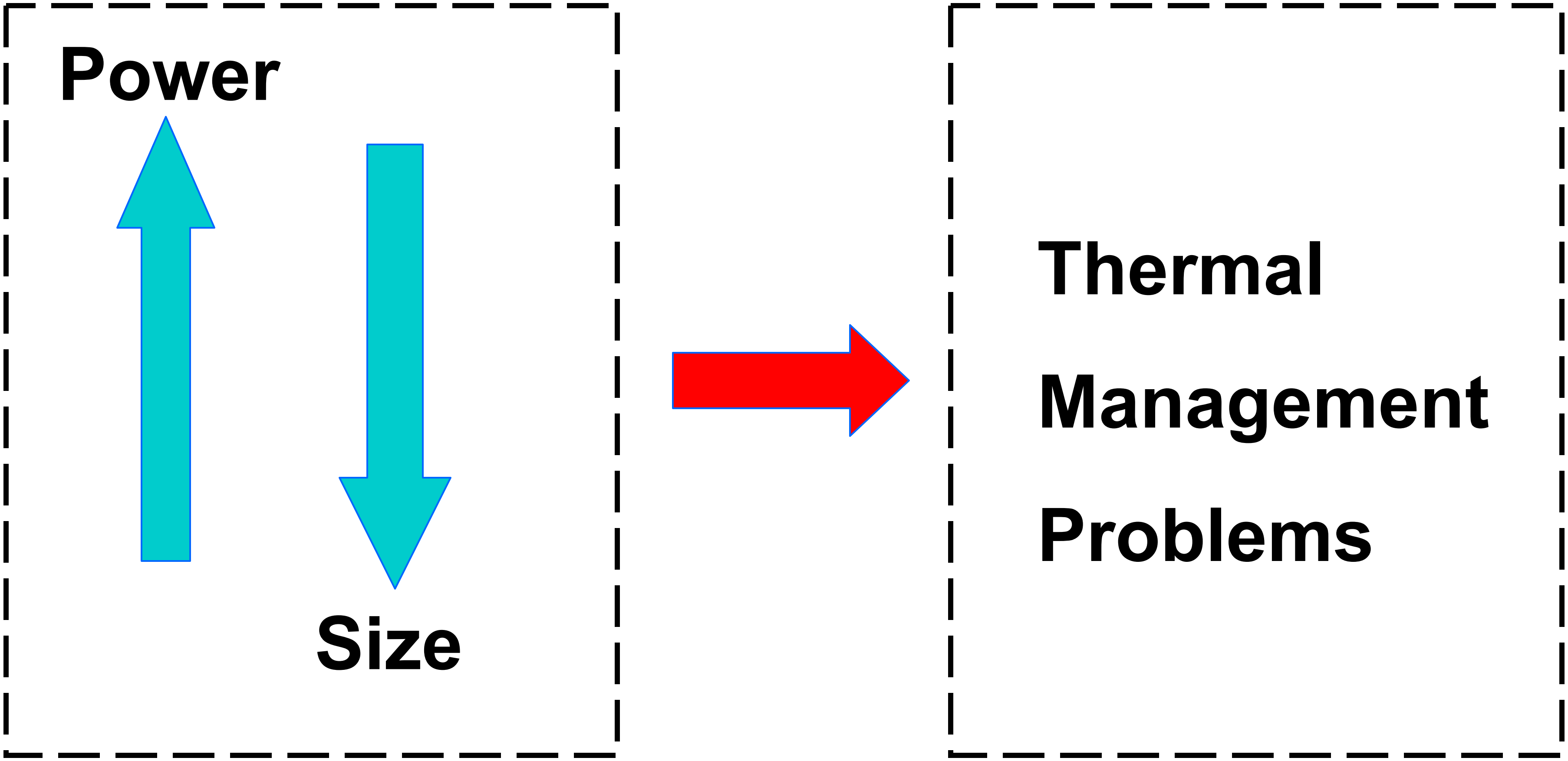
Agenda

- Introduction
- Literature Review
- Objectives
- Assumptions
- Modeling
- Results and Comparisons
- Conclusions
- Acknowledgments

Introduction



Power

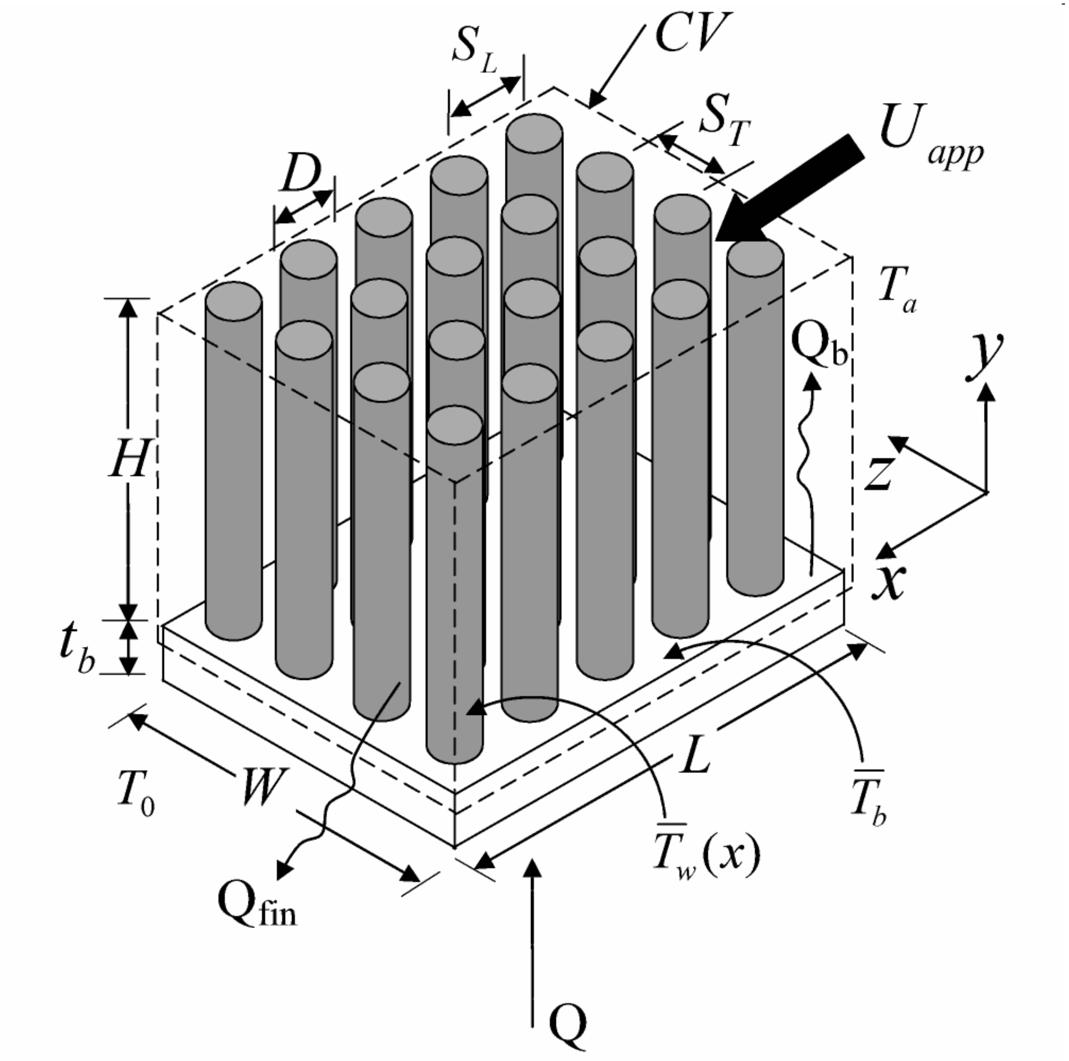
Size

Thermal

Management

Problems

Introduction



Literature Review

Tube Banks (Experimental)

1910-1933 ---- Chilton and Genereaux (1933)

---- Colburn (1933)

1933-1945 ---- Grimison (1937)

---- Jakob (1938)

Zukauskas (1972)

Zukauskas and Ulinskas (1988)

Available Correlations (Tube Banks)

- **Grimison**, 1937: $\mathrm{Nu_D = FCRe_D^n Pr^{1/3}}$

F = Correction factor for $N_L \leq 10$

$S_T/D \rightarrow$ $S_L/D \downarrow$	1.25		1.5		2		3	
	C	**n**	**C**	**n**	**C**	**n**	**C**	**n**
In-Line								
1.250	0.348	0.592	0.275	0.608	0.100	0.704	0.0633	0.752
1.500	0.367	0.586	0.250	0.620	0.101	0.762	0.0678	0.744
2.000	0.418	0.570	0.299	0.602	0.229	0.632	0.198	0.648
3.000	0.290	0.601	0.357	0.584	0.374	0.581	0.286	0.608

Available Correlations (Tube Banks)

- **Zukauskas, 1972:** $\mathrm{Nu_D} = \mathrm{FCRe_D^n Pr^m}$

F = Correction factor for $N_L \leq 16$

Geometry	C	n	m	Conditions
In-Line				
	0.9	0.4	0.36	$10 \leq \mathrm{Re_{Dmax}} \leq 100$
	0.52	0.5	0.36	$100 \leq \mathrm{Re_{Dmax}} \leq 10^3$
	0.27	0.63	0.36	$10^3 \leq \mathrm{Re_{Dmax}} \leq 2 \times 10^5$
	0.21	0.84	0.4	$\mathrm{Re_{Dmax}} > 2 \times 10^5$

Literature Review

Pin-Fin Heat sinks (Numerical)

Hamilton (2002)-------Staggered PFHS

$$\mathrm{Nu_D} = \mathrm{C Re_D^n} \qquad \text{where} \qquad 3500 < \mathrm{Re_D} < 14000$$

$S_T/D \rightarrow$	1.25		1.50		2.00		3.00	
$S_L/D \downarrow$	**C**	**n**	**C**	**n**	**C**	**n**	**C**	**n**
1.25	0.0905	0.7140	0.1024	0.7099	0.0860	0.7355	0.1377	0.6832
1.50	0.1881	0.6277	0.1247	0.6850	0.1465	0.6696	0.1826	0.6476
2.00	0.1406	0.6542	0.0813	0.7297	0.1203	0.6935	0.1619	0.6621
3.00	-	-	0.0750	0.7309	0.1504	0.6692	0.1094	0.7019

Literature Review

Pin-Fin Heat sinks (Experimental)

Tahat et. al

(1994, 2000)

Correlations/Models	Conditions
$Nu_D = 0.355 \left(\frac{S_T}{W}\right)^{0.0446} \left(\frac{S_L}{H}\right)^{0.048} Re_D^{0.585}$	$2\times 10^3 \leq Re_D \leq 9\times 10^3$ $0.019 \leq \frac{S_T}{W} \leq 0.409$ $0.003 \leq \frac{S_L}{H} \leq 0.272$
$Nu_D = 0.00902 \left(\frac{S_T}{W}\right)^{0.285} \left(\frac{S_L}{H}\right)^{0.212} Re_D^{1.011}$	In-Line Arrangement $3341.4 \leq Re_D \leq 6683$ $0.004 \leq \frac{S_T}{W} \leq 0.332$ $0.033 \leq \frac{S_L}{H} \leq 0.212$
$Nu_D = 0.00704 \left(\frac{S_T}{W}\right)^{0.091} \left(\frac{S_L}{H}\right)^{0.053} Re_D^{0.953}$	Staggered Arrangement $3138 \leq Re_D \leq 4980$ $0.004 \leq \frac{S_T}{W} \leq 0.332$ $0.033 \leq \frac{S_L}{H} \leq 0.1522$

Objectives

- ❖ Develop models to determine heat transfer and friction coefficients for :
  - ▪ In-line and
  - ▪ Staggered arrangements

that can be used for a wide range of geometric parameters

- ❖ Validate above models with experimental data

Assumptions

1. Forced Convection
2. Steady, laminar, fully developed and 2-D flow
3. Incompressible fluid
4. Constant properties
5. Inviscid flow outside the boundary layer
6. Fully shrouded heat sink

Boundary Conditions

- Velocity is zero at all boundaries of pins
- Adiabatic conditions are applied at all boundaries

At Inlet:

$$u = U_{app}, \quad v = 0, \quad w = 0\Big|_{x=0,\ 0 \le y \le H,\ 0 \le z \le W}$$

$$T = T_a\Big|_{x=0,\ 0 \le y \le H,\ 0 \le z \le W}$$

At Exit:

$$T = T_0\Big|_{x=L,\ 0 \le y \le H,\ 0 \le z \le W}$$

Reference Velocity

$$U_{max} = max\left(\frac{\mathcal{S}_T}{\mathcal{S}_T - 1}U_{app}, \frac{\mathcal{S}_T}{\mathcal{S}_D - 1}U_{app}\right)$$

Reynolds Number:

$$Re_D = \frac{DU_{max}}{\nu}$$

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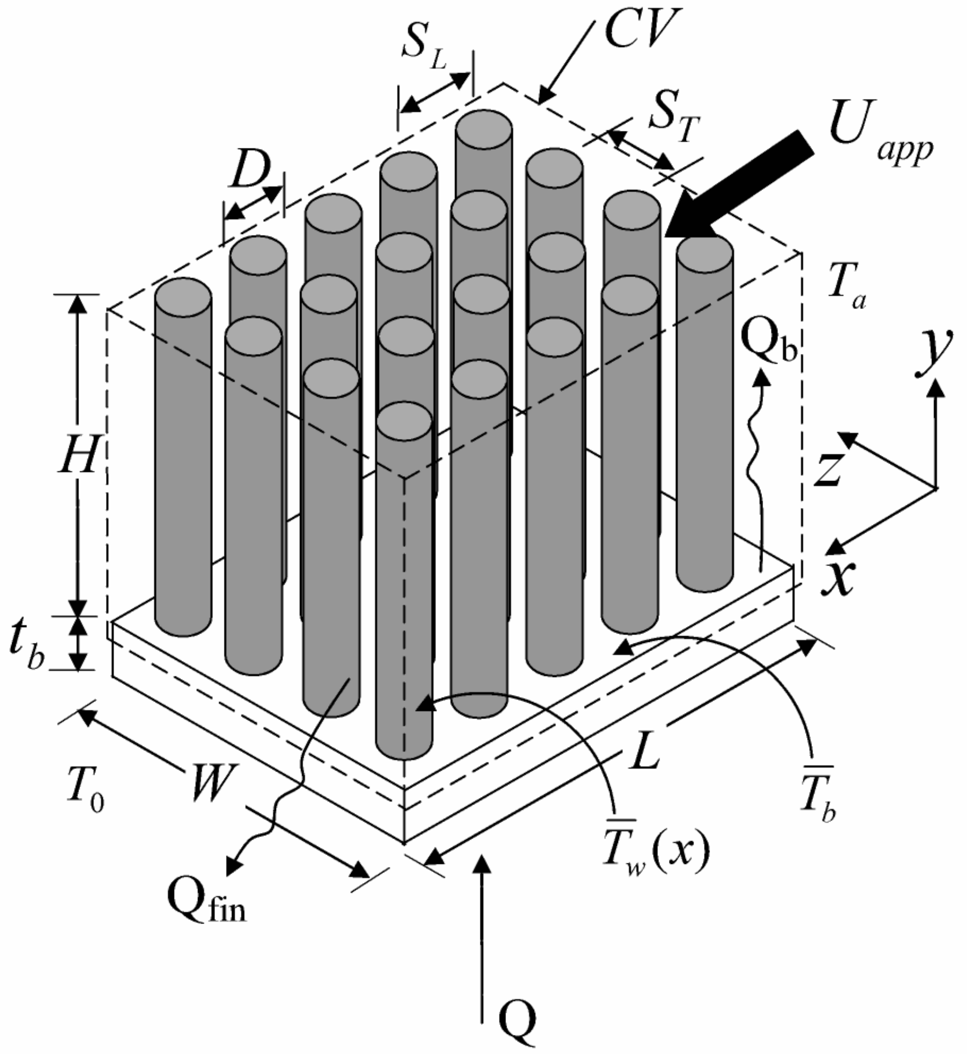
Analysis (Energy Balance)

$$Q = NQ_{fin} + Q_b$$

where

$$\begin{aligned} Q &= (hA)_{hs}\,\theta_b \\ Q_{fin} &= (Ah\eta)_{fin}\,\theta_b \\ Q_b &= (hA)_b\,\theta_b \end{aligned}$$



Models

$$
\begin{aligned}
h_b &= \frac{0.75 k_f}{D} \sqrt{\frac{\mathcal{S}_T - 1}{N_L \mathcal{S}_L \mathcal{S}_T}} Re_D^{1/2} Pr^{1/3} \\
h_{fin} &= \frac{C_1 k_f}{D} Re_D^{1/2} Pr^{1/3}
\end{aligned}
$$

where

$$
C_1 = \begin{cases}
[0.2 + \exp(-0.55 \mathcal{S}_T)] \mathcal{S}_T{}^{0.285} \mathcal{S}_L{}^{0.212} & \text{In-Line arrangement} \\
\dfrac{0.61\, \mathcal{S}_T{}^{0.091} \mathcal{S}_L{}^{0.053}}{[1 - 2 \exp(-1.09\, \mathcal{S}_T)]} & \text{Staggered arrangement}
\end{cases}
$$

Models

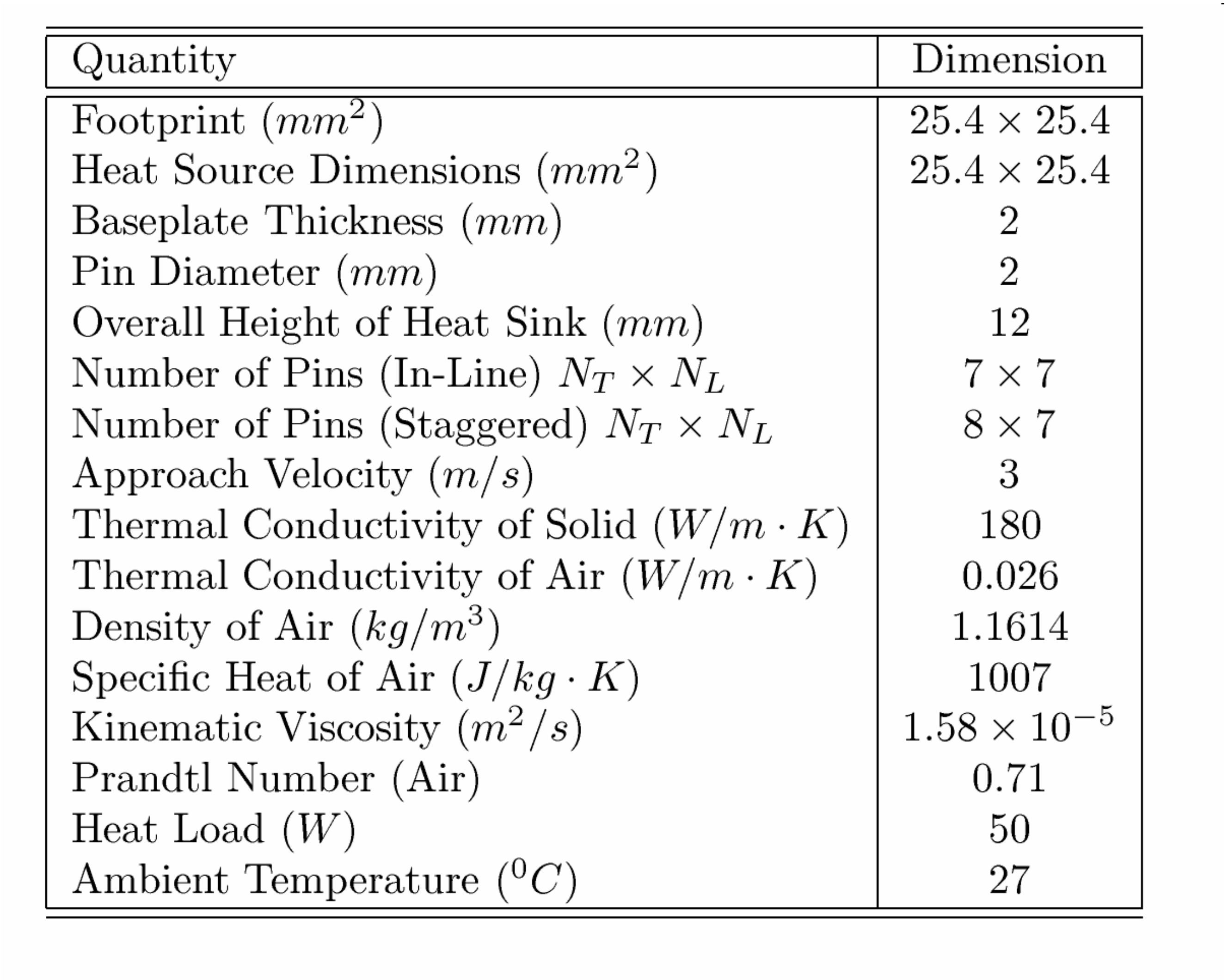
$$Nu_{D_{hs}} = \frac{h_{hs} D}{k_f} = C_2 Re_D^{1/2} Pr^{1/3}$$

$$C_2 = \frac{C_1 \pi \gamma \eta_{fin} + 0.75 \sqrt{\frac{\mathcal{S}_T - 1}{N_L \mathcal{S}_L \mathcal{S}_T}} \left(\mathcal{S}_T \mathcal{S}_L - \frac{\pi}{4} \right)}{\pi(\gamma - 1/4) + \mathcal{S}_T \mathcal{S}_L}$$

$$\overline{T}_f \quad = \quad \overline{T}_b - (\overline{T}_b - T_a) \left[\frac{\dot{m} c_p}{(hA)_{hs}} \right] \cdot \left[1 - \exp\left(-\frac{(hA)_{hs}}{\dot{m} c_p} \right) \right]$$

$$T_o \quad = \quad \overline{T}_b - (\overline{T}_b - T_a) \cdot \exp\left[-\frac{(hA)_{hs}}{\dot{m} c_p} \right]$$

Dimensions Used for Modeling of PFHS

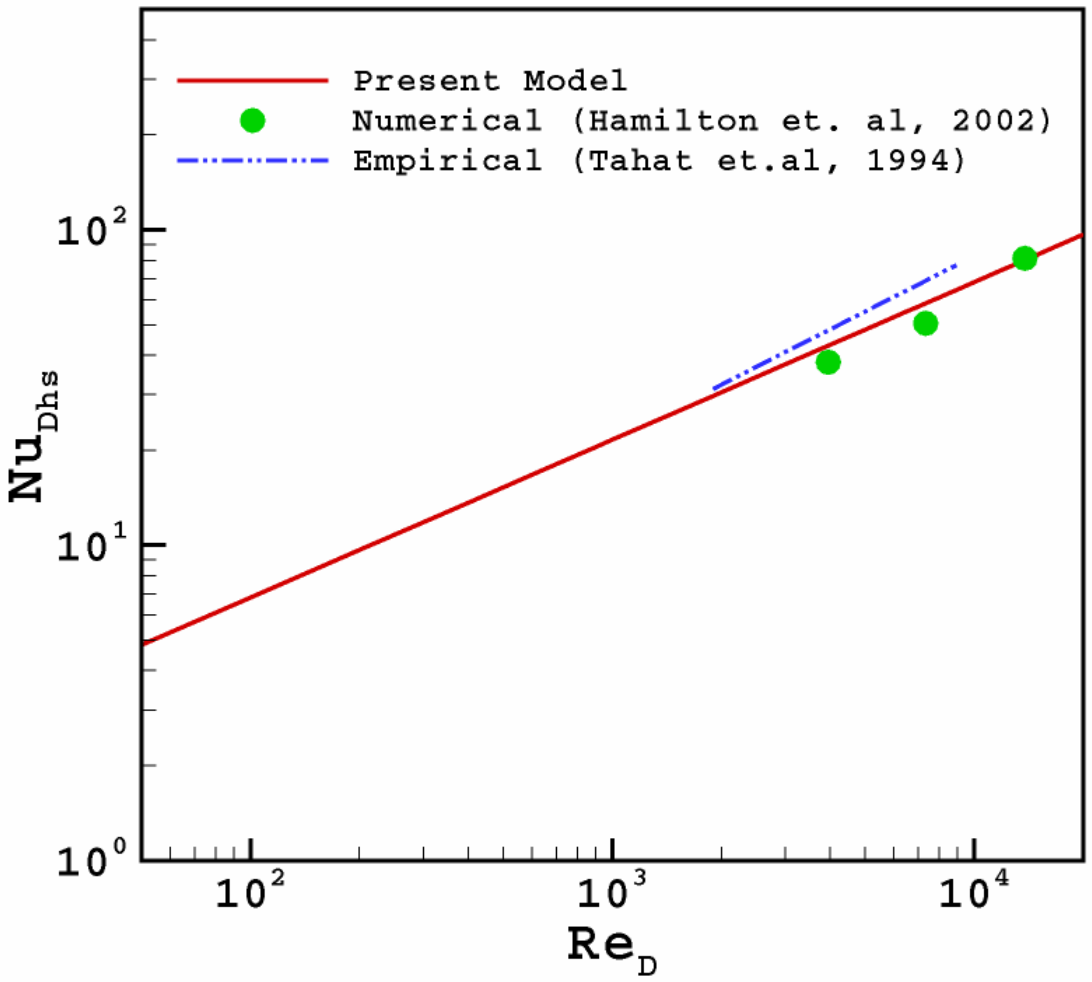


Quantity	Dimension
Footprint (mm^2)	25.4×25.4
Heat Source Dimensions (mm^2)	25.4×25.4
Baseplate Thickness (mm)	2
Pin Diameter (mm)	2
Overall Height of Heat Sink (mm)	12
Number of Pins (In-Line) $N_T \times N_L$	7×7
Number of Pins (Staggered) $N_T \times N_L$	8×7
Approach Velocity (m/s)	3
Thermal Conductivity of Solid $(W/m \cdot K)$	180
Thermal Conductivity of Air $(W/m \cdot K)$	0.026
Density of Air (kg/m^3)	1.1614
Specific Heat of Air $(J/kg \cdot K)$	1007
Kinematic Viscosity (m^2/s)	1.58×10^{-5}
Prandtl Number (Air)	0.71
Heat Load (W)	50
Ambient Temperature (^0C)	27

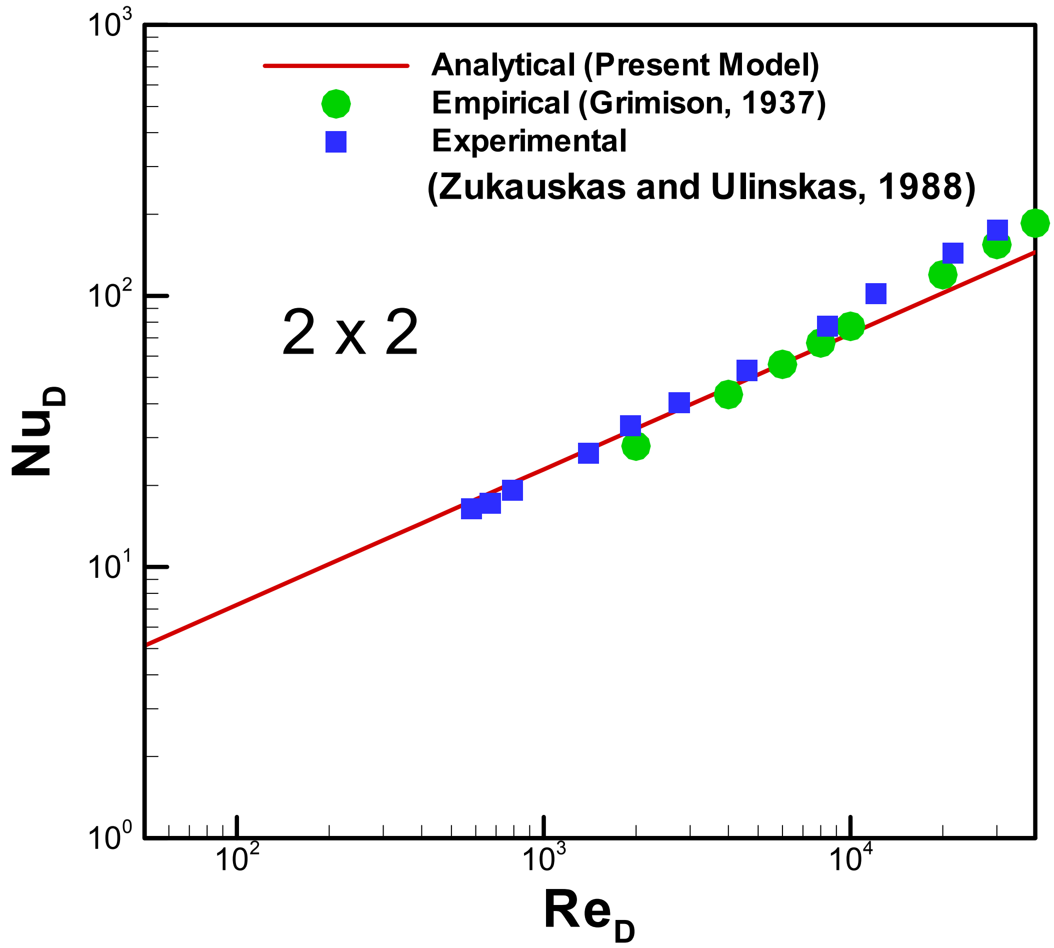
Results

Quantity	In-Line	Staggered
Thermal Resistance ($^0C/W$)	1.35	0.94
Average Heat Transfer Coefficient ($W/m^2 \cdot K$)	210.7	271.8
Pressure Drop (Pa)	78.5	211.9
Average Fluid Temperature (^0C)	48.9	46.8
Average Baseplate Temperature (^0C)	94.3	74.0
Air Temperature Leaving Heat Sink (^0C)	65.4	60.1

Staggered Arrangement (2.5 x 1.5)



In-Line Tube Bank



Conclusions

- ➢ Heat transfer from and pressure drop across the heat sink increases with the increase in approach velocity, pin diameter, pin height, and pin density
- ➢ Heat transfer also increases with k
- ➢ Both In-Line and Staggered models are:
  - ✓ applicable over a wide range of parameters
  - ✓ suitable for use in modeling PFHS

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