Optimization of Microchannel Heat Sinks Using Entropy Generation Minimization Method

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SEMI-THERM 22, March 14-16, 2006, Dallas, TX, USA

Agenda

- Introduction
- Literature Review
- Objectives
- Assumptions
- Modeling
- Results
- Summary and Conclusions
- Acknowledgments

Introduction (Microchannel Heat Sinks)

- Dissipate high heat fluxes with small ΔT
- High heat transfer coefficients and lower friction factors due to slip
- Used in microelectronics, aviation and aerospace, medical treatment, biological engineering, material sciences, cooling of high temperature superconductors, and thermal control of film deposition

Literature Review

Tuckerman and Pease [1]

Theoretical

- Knight et al. [4,5]
- Perret et al. [6,7]
- Kim [8]
- Upadhye and Kandilkar [9]
- Kandilkar and Grande [3]
- Liu and Gariella [10]
- Singhal et al. [11]

Experimental

- Kleiner et al. [13]
- Aranyosi et al. [14]
- Harris et al. [15]
- Garimella and Singhal [16]
- Jang and Kim [17]

Numerical

- Choquette et al. [18]
- Zhimin and Fah [19]
- Meysenc et al. [20]
- Chong et al. [21]
- Liao et al. [22]
- Ryu et al. [23]
- Wei and Joshi [24]

Objectives

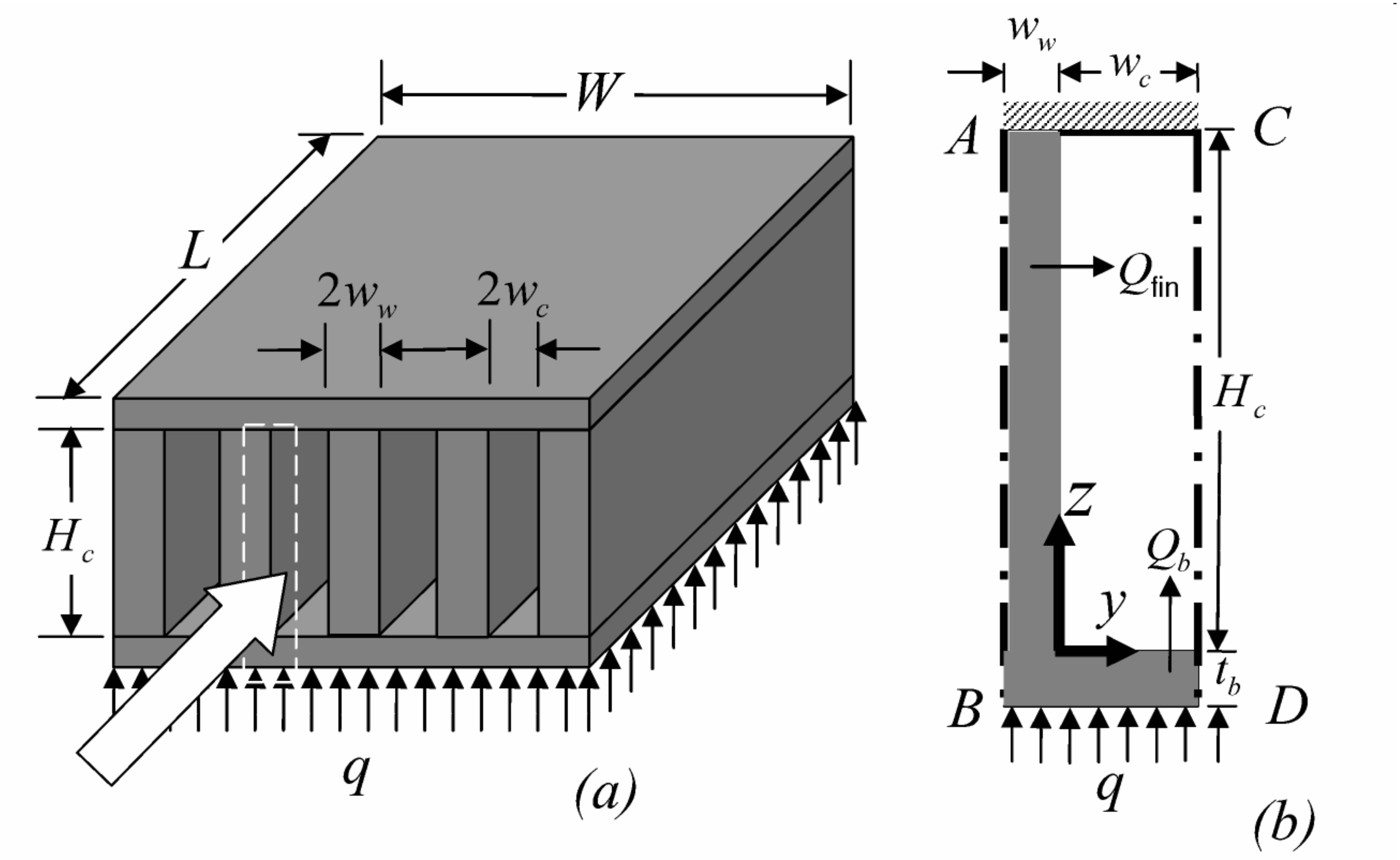
Previous Optimization Objectives:

- **Minimize thermal resistance R_{th} for given ΔP**
- **Minimize pumping power for specified R_{th}**

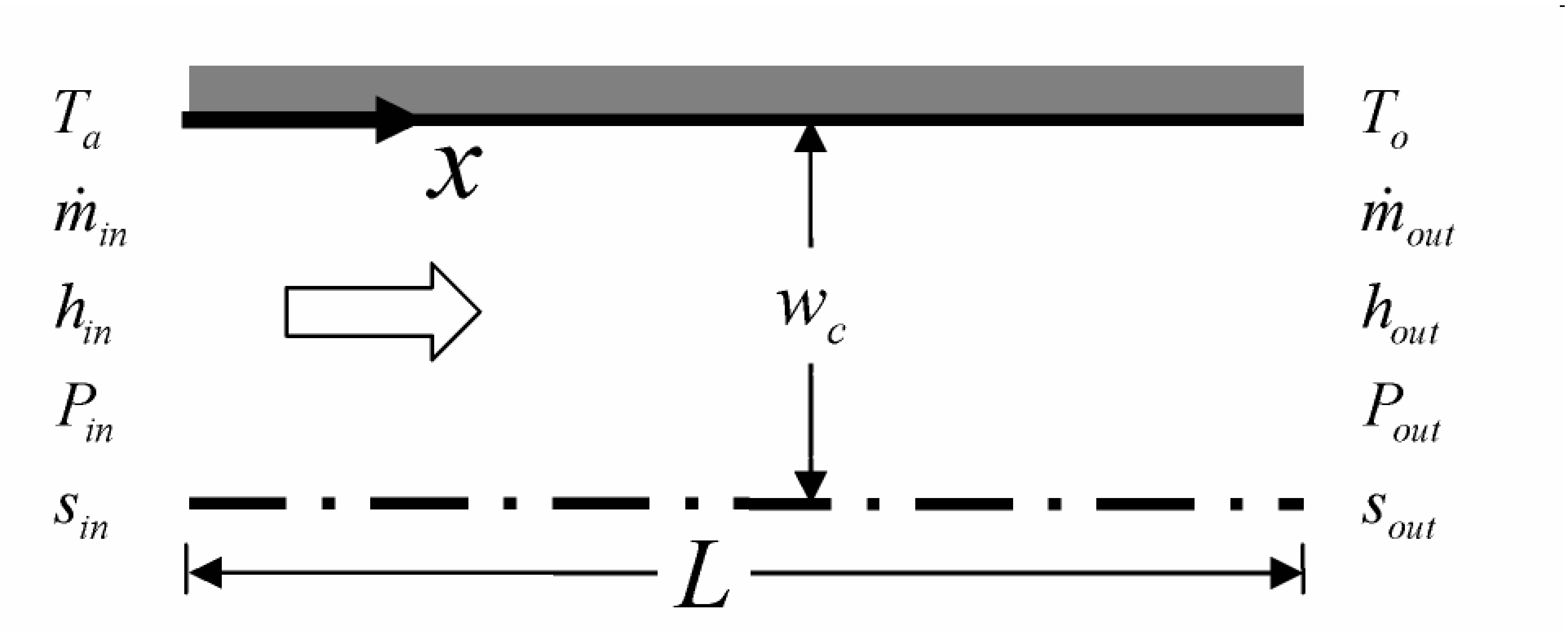
Current Objectives:

- **Minimize both thermal resistance and pressure drop simultaneously using EGM method**

Geometry (MCHS)



Geometry (MCHS)



Assumptions

1. Uniform heat flux on bottom surface
2. Steady, laminar, fully developed and 2-D flow
3. Incompressible fluid
4. Constant thermophysical properties
5. Slip flow ($0.001 \leq Kn \leq 0.1$) with negligible creep effects ($Kn = \lambda / D_h$)
6. Microchannel with smooth surfaces

Entropy Generation Rate

Mass balance:
$$\dot{m}_{in} = \dot{m}_{out} = \dot{m}$$

Energy balance:
$$Q = \dot{m}(h_{out} - h_{in})$$

Entropy balance:
$$S_{gen} = \dot{m}(s_{out} - s_{in}) - \frac{Q}{T_b}$$

Entropy generation rate:
$$S_{gen} = \frac{Q^2 R_{th}}{T_a T_b} + \frac{\dot{m} \Delta P}{\rho T_a}$$

Entropy Generation Rate (Contd.)

$$R_{th} = \frac{1}{h_{av} A} + \frac{1}{\dot{m} C_p}$$

$$\Delta P = \frac{\rho U_{av}^2}{2} \left[k_{ce} + \left(f \frac{L}{D_h} \right) \right]$$

with

$$U_{av} = \frac{\dot{m}}{N \rho (2 w_c) H_c}$$

$$N = \frac{W - w_w}{w_c + w_w}$$

$$Nu_{D_h} = \frac{h_{av} D_h}{k_f} = \frac{140}{17(1+\alpha_c) K_s}$$

Entropy Generation Rate (Contd.)

$$K_s = 1 - \frac{6}{17}\left(\frac{U_s}{U_{av}}\right) + \frac{2}{51}\left(\frac{U_s}{U_{av}}\right)^2 - \frac{140}{17}\zeta_t$$

$$\alpha_c = \frac{2w_c}{H_c} < 1 \qquad \frac{U_s}{U_{av}} = \frac{6\alpha}{1+6\alpha}$$

$$\alpha = \frac{2\zeta_u}{1+\alpha_c} \qquad \zeta_u = \left(\frac{2-\sigma}{\sigma}\right) Kn$$

$$\zeta_t = \left(\frac{2-\sigma_t}{\sigma_t}\right) \cdot \frac{2\gamma}{\gamma+1} \cdot \frac{Kn}{Pr}$$

Entropy Generation Rate (Contd.)

$$f = \frac{24}{Re_{D_h}}\left(\frac{1}{1+6\alpha}\right)\left(\frac{1}{1+\alpha_c}\right)$$

$$k_{ce} = 1.79 - 2.32\left(\frac{w_c}{w_c+w_w}\right) + 0.53\left(\frac{w_c}{w_c+w_w}\right)^2$$

with

$$Re_{D_h} = \frac{U_{av}D_h}{\nu}$$

$$D_h = \frac{4w_c}{1+\alpha_c}$$

Optimization Procedure

$$minimize \;\; f(\mathbf{x}) = S_{gen}(\mathbf{x})$$

subject to the equality constraints:

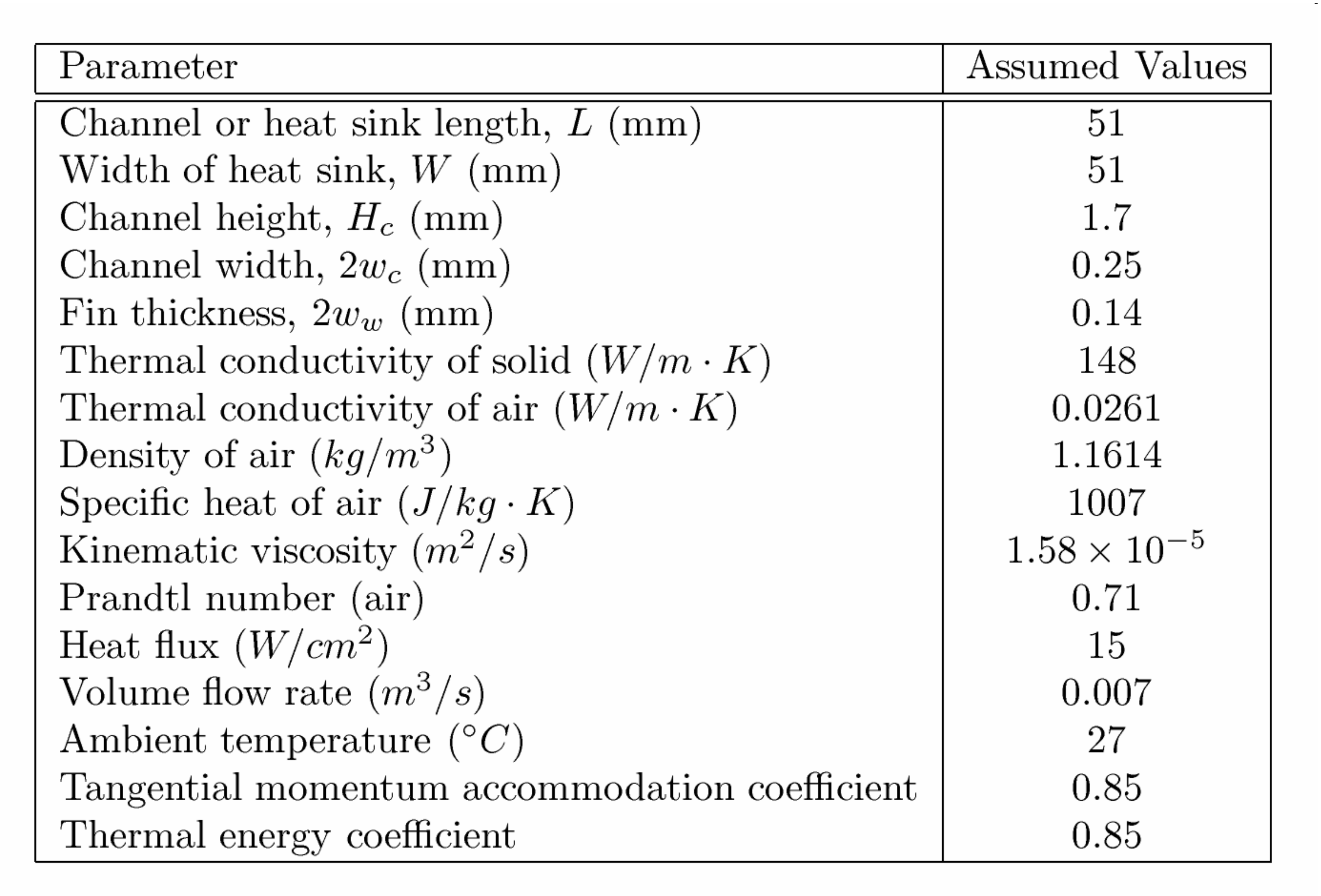
$$g_j(\mathbf{x}) = 0, \qquad j = 1, 2, ..., m$$

and inequality constraints

$$l_j(\mathbf{x}) \geq 0, \qquad j = m + 1, ..., n$$

$$\mathcal{L}(\mathbf{x}, \lambda, \chi) = f(\mathbf{x}) + \sum_{j=1}^{m} \lambda_j g_j(\mathbf{x}) - \sum_{j=m+1}^{n} \chi_j l_j(\mathbf{x})$$

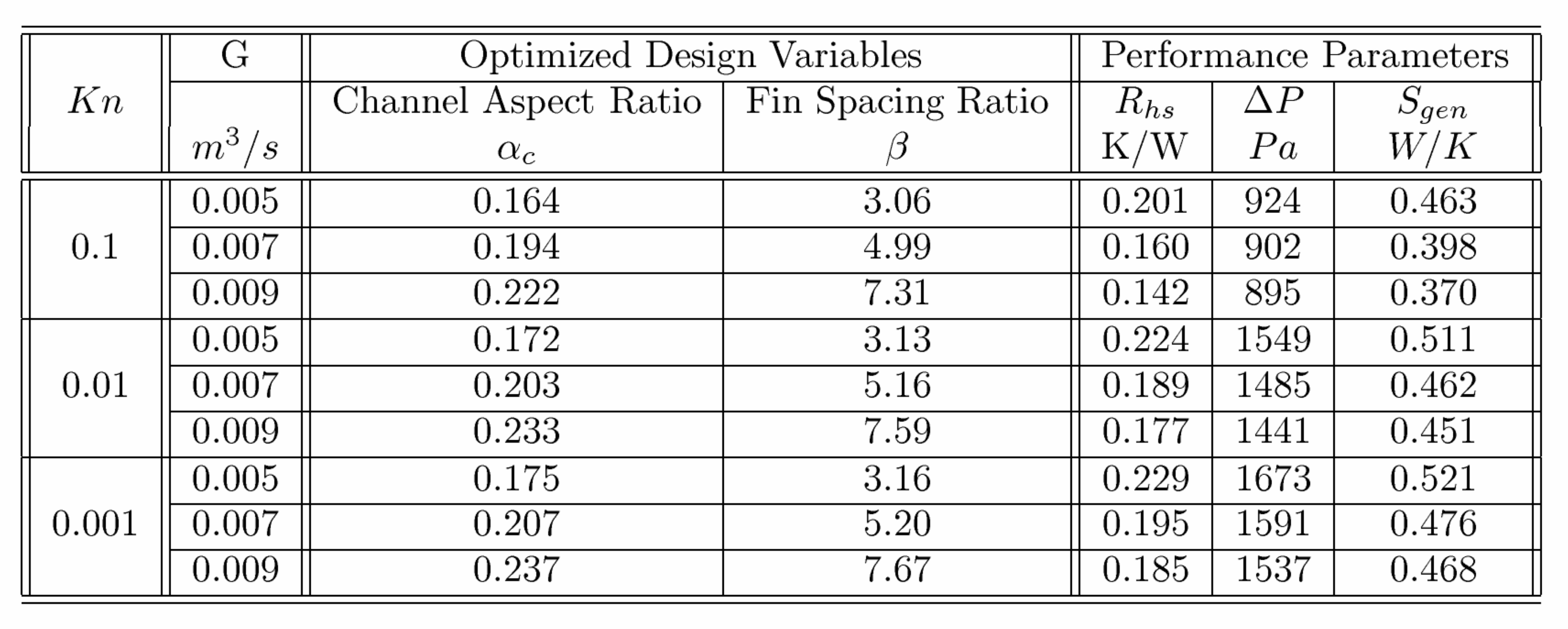
Assumed Parameter Values



Parameter	Assumed Values
Channel or heat sink length, L (mm)	51
Width of heat sink, W (mm)	51
Channel height, H_c (mm)	1.7
Channel width, $2w_c$ (mm)	0.25
Fin thickness, $2w_w$ (mm)	0.14
Thermal conductivity of solid $(W/m \cdot K)$	148
Thermal conductivity of air $(W/m \cdot K)$	0.0261
Density of air (kg/m^3)	1.1614
Specific heat of air $(J/kg \cdot K)$	1007
Kinematic viscosity (m^2/s)	1.58×10^{-5}
Prandtl number (air)	0.71
Heat flux (W/cm^2)	15
Volume flow rate (m^3/s)	0.007
Ambient temperature $(°C)$	27
Tangential momentum accommodation coefficient	0.85
Thermal energy coefficient	0.85

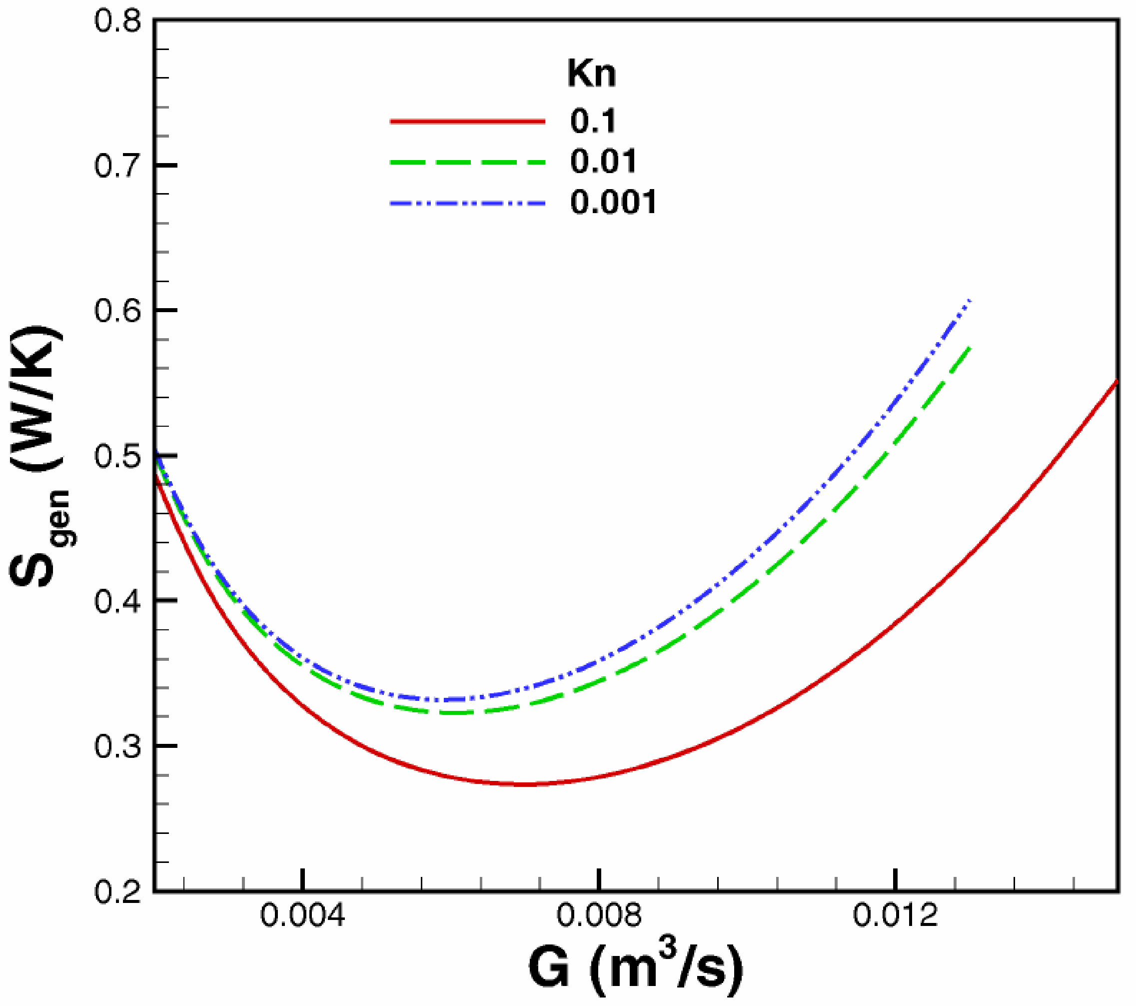
Reference: Kleiner et al. [13]

Results of Optimization

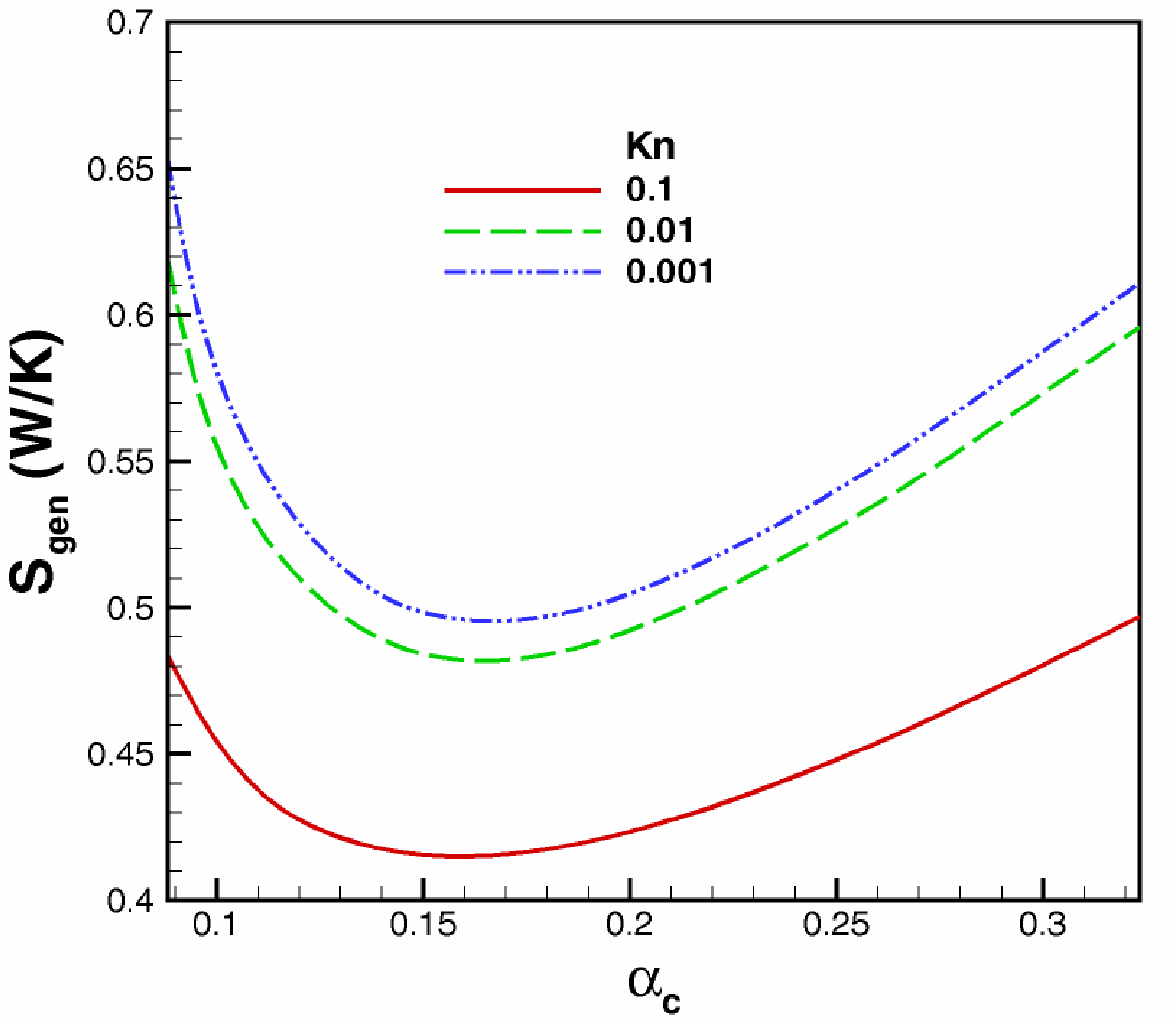


Kn	G	Optimized Design Variables		Performance Parameters		
	m^3/s	Channel Aspect Ratio α_c	Fin Spacing Ratio β	R_{hs} K/W	ΔP Pa	S_{gen} W/K
0.1	0.005	0.164	3.06	0.201	924	0.463
	0.007	0.194	4.99	0.160	902	0.398
	0.009	0.222	7.31	0.142	895	0.370
0.01	0.005	0.172	3.13	0.224	1549	0.511
	0.007	0.203	5.16	0.189	1485	0.462
	0.009	0.233	7.59	0.177	1441	0.451
0.001	0.005	0.175	3.16	0.229	1673	0.521
	0.007	0.207	5.20	0.195	1591	0.476
	0.009	0.237	7.67	0.185	1537	0.468

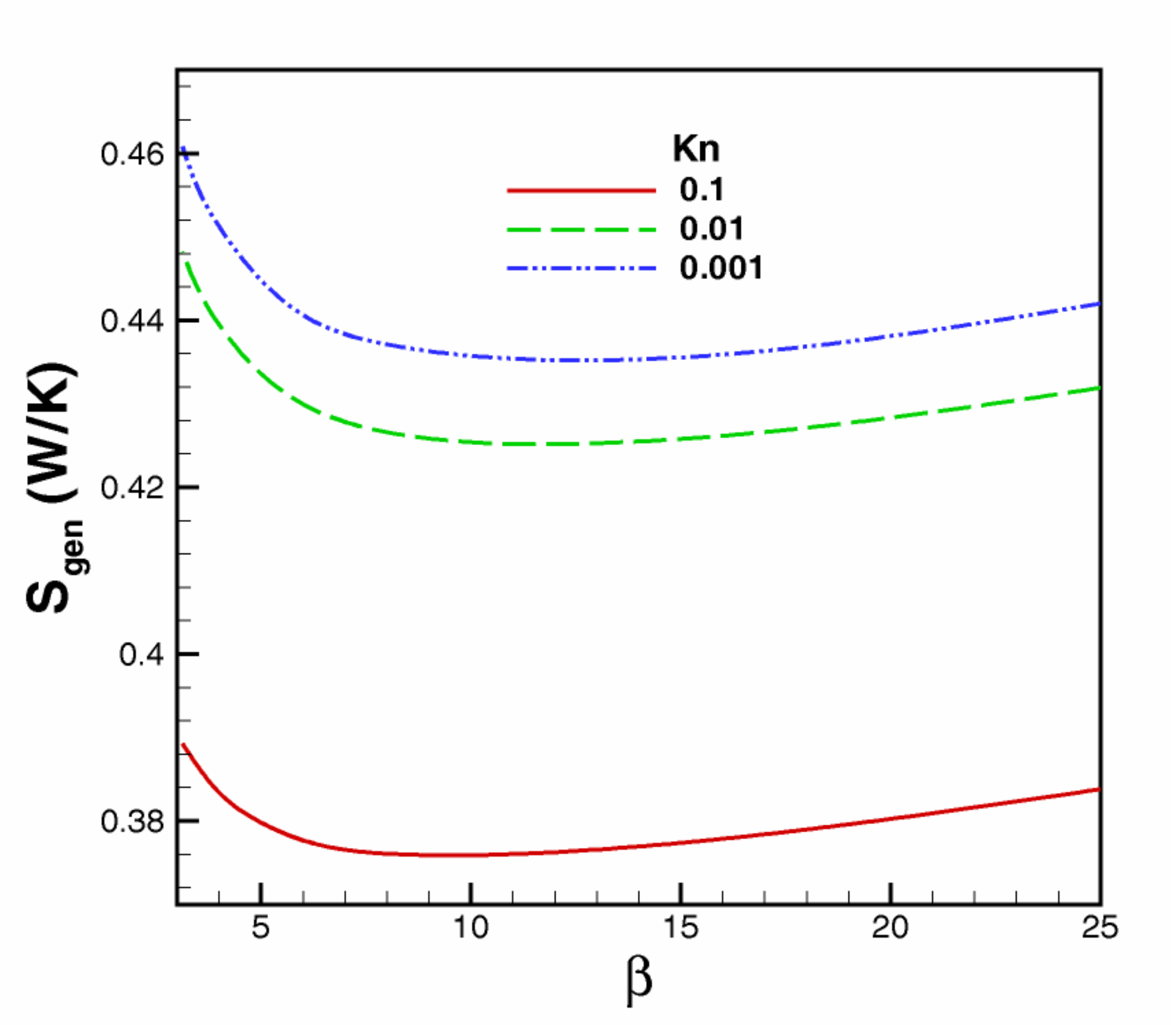
Effect of Volume Flow Rate on S_{gen}



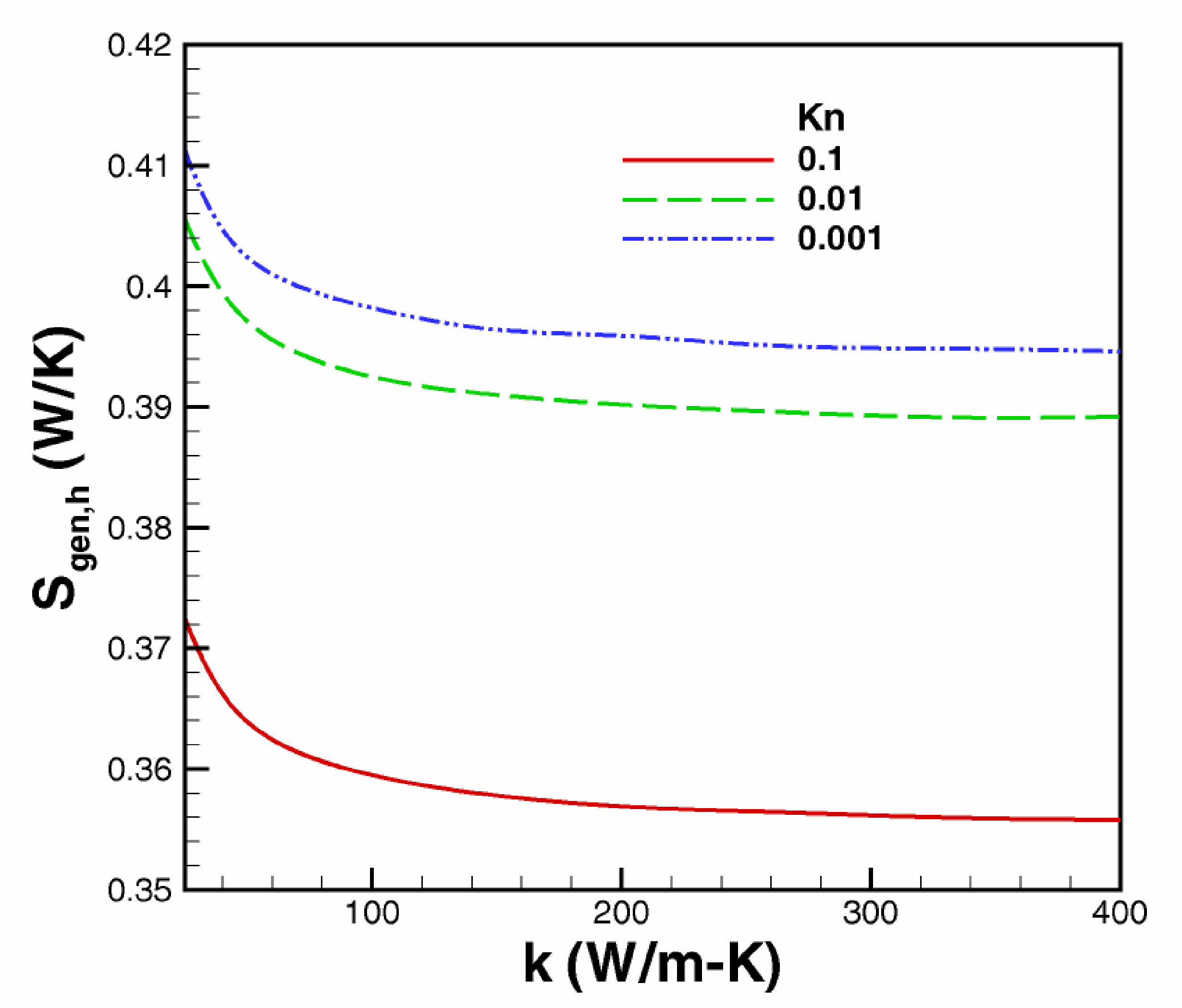
Effect of Channel Aspect Ratio on S_{gen}



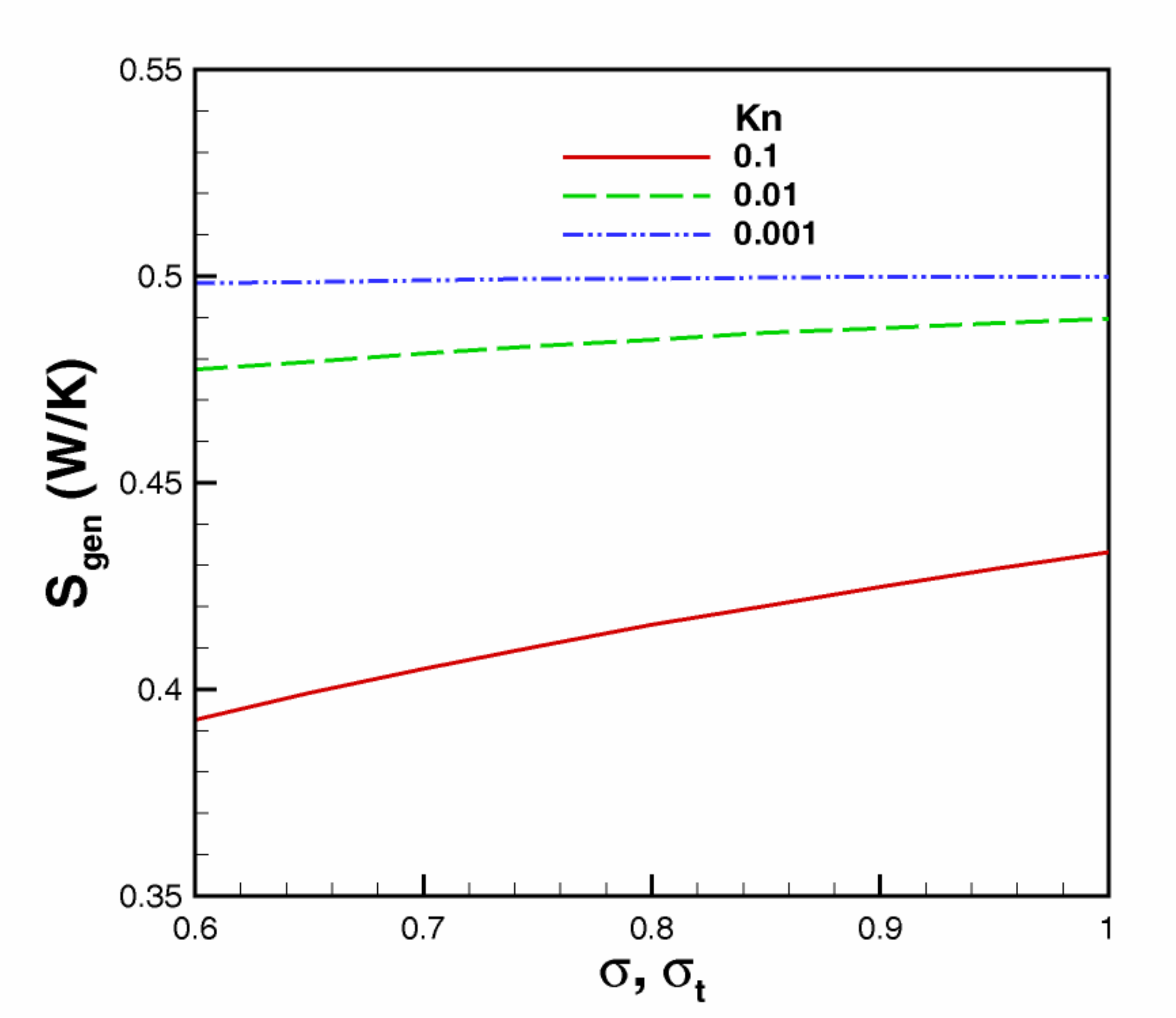
Effect of Fin Spacing Ratio on S_{gen}



Effect of Heat Sink Material on S_{gen}



Effect of Accomodation Coefficients on S_{gen}



Summary and Conclusions

- Due to slip flow and temperature jump, fluid friction decreases and heat transfer increases
- Thermal resistance and pressure drop decrease with increase in volume flow rate and increase with decrease in Knudsen number
- Optimum channel aspect ratio and fin spacing ratio increase with volume flow rate

Summary and Conclusions

- **Optimum entropy generation rate decreases with increase in Knudsen number**
- **Low thermal conductivity heat sinks with large number of microchannels gives acceptable performance in terms of entropy generation rate**
- **For fixed Knudsen numbers, lower tangential momentum and energy accomodation coefficients result in higher entropy generation rates**

Acknowledgements

The authors gratefully acknowledge the financial support of

- Natural Sciences and Engineering Research Council of Canada
- Centre for Microelectronics Assembly and Packaging