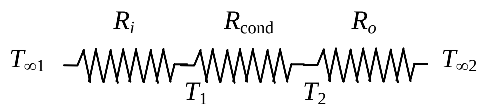
10-126 Two cast iron steam pipes are connected to each other through two 1-cm thick flanges exposed to cold ambient air. The average outer surface temperature of the pipe, the fin efficiency, the rate of heat transfer from the flanges, and the equivalent pipe length of the flange for heat transfer are to be determined.

Assumptions **1** Steady operating conditions exist. **2** The temperature along the flanges (fins) varies in one direction only (normal to the pipe). **3** The heat transfer coefficient is constant and uniform over the entire fin surface. **4** The thermal properties of the fins are constant. **5** The heat transfer coefficient accounts for the effect of radiation from the fins.

Properties The thermal conductivity of the cast iron is given to be k = 52 W/m·°C.

Analysis (a) We treat the flanges as fins. The individual thermal resistances are



$$A_i = \pi D_i L = \pi(0.092\text{ m})(6\text{ m}) = 1.73\text{ m}^2$$

$$A_o = \pi D_o L = \pi(0.1\text{ m})(6\text{ m}) = 1.88\text{ m}^2$$

$$R_{\text{i}} = \frac{1}{h_i A_i} = \frac{1}{(180\text{ W/m}^2.°\text{C})(1.73\text{ m}^2)} = 0.0032\text{ °C/W}$$

$$R_{\text{cond}} = \frac{\ln(r_2/r_1)}{2\pi kL} = \frac{\ln(5/4.6)}{2\pi(52\text{ W/m.°C})(6\text{ m})} = 0.00004\text{ °C/W}$$

$$R_{\text{o}} = \frac{1}{h_o A_o} = \frac{1}{(25\text{ W/m}^2.°\text{C})(1.88\text{ m}^2)} = 0.0213\text{ °C/W}$$

$$R_{\text{total}} = R_{\text{i}} + R_{\text{cond}} + R_{\text{o}} = 0.0032 + 0.00004 + 0.0213 = 0.0245\text{ °C/W}$$

The rate of heat transfer and average outer surface temperature of the pipe are

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{\text{total}}} = \frac{(200-12)°\text{C}}{0.0245\text{ °C}} = 7673\text{ W}$$

$$\dot{Q} = \frac{T_2 - T_{\infty 2}}{R_{\text{o}}} \longrightarrow T_2 = T_{\infty 2} + \dot{Q}R_{\text{o}} = 12\text{ °C} + (7673\text{ W})(0.0213\text{ °C/W}) = \mathbf{175.4°C}$$

(b) The fin efficiency can be determined from (Fig. 10-43)

$$\left.\begin{aligned} &\frac{r_2 + \frac{t}{2}}{r_1} = \frac{0.1 + \frac{0.02}{2}}{0.05} = 2.2 \\ &\xi = L_c^{3/2}\left(\frac{h}{kA_p}\right)^{1/2} = \left(L + \frac{t}{2}\right)\sqrt{\frac{h}{kt}} = \left(0.05\text{ m} + \frac{0.02}{2}\text{ m}\right)\sqrt{\frac{25\text{ W/m}^2\,{}^\circ\text{C}}{(52\text{ W/m}^\circ\text{C})(0.02\text{ m})}} = 0.29 \end{aligned}\right\} \eta_{\text{fin}} = 0.88$$

$$A_{\text{fin}} = 2\pi(r_2^2 - r_1^2) + 2\pi r_2 t = 2\pi[(0.1\text{ m})^2 - (0.05\text{ m})^2] + 2\pi(0.1\text{ m})(0.02\text{ m}) = 0.0597\text{ m}^2$$

The heat transfer rate from the flanges is

$$\begin{aligned} \dot{Q}_{\text{finned}} &= \eta_{\text{fin}}\dot{Q}_{\text{fin,max}} = \eta_{\text{fin}} h A_{\text{fin}}(T_b - T_\infty) \\ &= 0.88(25\text{ W/m}^2.°\text{C})(0.0597\text{ m}^2)(175.4-12)°\text{C} = \mathbf{215\text{ W}} \end{aligned}$$

(c) A 6-m long section of the steam pipe is losing heat at a rate of 7673 W or 7673/6 = 1279 W per m length. Then for heat transfer purposes the flange section is equivalent to

$$\text{Equivalent length} = \frac{215\text{ W}}{1279\text{ W/m}} = 0.168\text{ m} = \mathbf{16.8\text{ cm}}$$

Therefore, the flange acts like a fin and increases the heat transfer by 16.8/2 = 8.4 times.