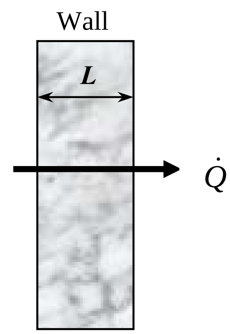
10-35 Two of the walls of a house have no windows while the other two walls have single- or double-pane windows. The average rate of heat transfer through each wall, and the amount of money this household will save per heating season by converting the single pane windows to double pane windows are to be determined.

Assumptions **1** Heat transfer through the window is steady since the indoor and outdoor temperatures remain constant at the specified values. **2** Heat transfer is one-dimensional since any significant temperature gradients will exist in the direction from the indoors to the outdoors. **3** Thermal conductivities of the glass and air are constant. **4** Heat transfer by radiation is disregarded.

Properties The thermal conductivities are given to be k = 0.026 W/m·°C for air, and 0.78 W/m·°C for glass.

Analysis The rate of heat transfer through each wall can be determined by applying thermal resistance network. The convection resistances at the inner and outer surfaces are common in all cases.

Walls without windows:



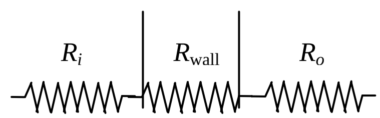
$$R_i = \frac{1}{h_i A} = \frac{1}{(7\ \text{W/m}^2\cdot°\text{C})(10\times 4\ \text{m}^2)} = 0.003571\ °\text{C/W}$$

$$R_{\text{wall}} = \frac{L_{\text{wall}}}{kA} = \frac{R-value}{A} = \frac{2.31\ \text{m}^2\cdot°\text{C/W}}{(10\times 4\ \text{m}^2)} = 0.05775\ °\text{C/W}$$

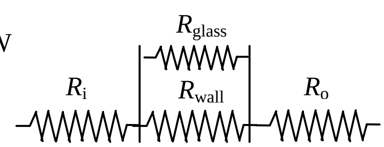
$$R_o = \frac{1}{h_o A} = \frac{1}{(18\ \text{W/m}^2\cdot°\text{C})(10\times 4\ \text{m}^2)} = 0.001389°\text{C/W}$$

$$R_{\text{total}} = R_i + R_{\text{wall}} + R_o = 0.003571 + 0.05775 + 0.001389 = 0.06271\ °\text{C/W}$$

Then $\quad \dot{Q} = \dfrac{T_{\infty 1} - T_{\infty 2}}{R_{total}} = \dfrac{(24-8)°\text{C}}{0.06271°\text{C/W}} = \mathbf{255.1\ W}$



Wall with single pane windows:



$$R_i = \frac{1}{h_i A} = \frac{1}{(7\ \text{W/m}^2\cdot°\text{C})(20\times 4\ \text{m}^2)} = 0.001786\ °\text{C/W}$$

$$R_{\text{wall}} = \frac{L_{\text{wall}}}{kA} = \frac{R-value}{A} = \frac{2.31\ \text{m}^2\cdot°\text{C/W}}{(20\times 4) - 5(1.2\times 1.8)\ \text{m}^2} = 0.033382\ °\text{C/W}$$

$$R_{\text{glass}} = \frac{L_{\text{glass}}}{kA} = \frac{0.005\ \text{m}}{(0.78\ \text{W/m}^2\cdot{}^{\text{o}}\text{C})(1.2\times 1.8)\text{m}^2} = 0.002968\ °\text{C/W}$$

$$\frac{1}{R_{\text{eqv}}} = \frac{1}{R_{\text{wall}}} + 5\frac{1}{R_{\text{glass}}} = \frac{1}{0.033382} + 5\frac{1}{0.002968} \rightarrow R_{eqv} = 0.000583\ {}^{\text{o}}\text{C/W}$$

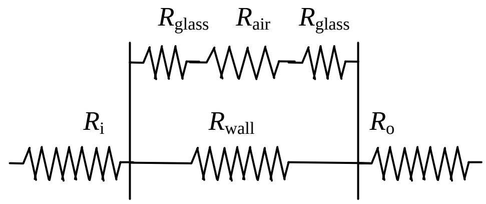
$$R_o = \frac{1}{h_o A} = \frac{1}{(18\ \text{W/m}^2\cdot°\text{C})(20\times 4\ \text{m}^2)} = 0.000694\ °\text{C/W}$$

$$R_{\text{total}} = R_i + R_{\text{eqv}} + R_o = 0.001786 + 0.000583 + 0.000694 = 0.003063\ °\text{C/W}$$

Then

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{\text{total}}} = \frac{(24-8)°\text{C}}{0.003063°\text{C/W}} = \mathbf{5224\ W}$$

4th wall with double pane windows:



$$R_{\text{wall}} = \frac{L_{\text{wall}}}{kA} = \frac{R-value}{A} = \frac{2.31\,\text{m}^2\cdot{}^\circ\text{C/W}}{(20\times 4)-5(1.2\times 1.8)\text{m}^2} = 0.033382\,^\circ\text{C/W}$$

$$R_{\text{glass}} = \frac{L_{\text{glass}}}{kA} = \frac{0.005\,\text{m}}{(0.78\,\text{W/m}^2\cdot{}^\circ\text{C})(1.2\times 1.8)\text{m}^2} = 0.002968\,^\circ\text{C/W}$$

$$R_{\text{air}} = \frac{L_{\text{air}}}{kA} = \frac{0.015\,\text{m}}{(0.026\,\text{W/m}^2\cdot{}^\text{o}\,\text{C})(1.2\times 1.8)\text{m}^2} = 0.267094\,^\circ\text{C/W}$$

$$R_{\text{window}} = 2R_{\text{glass}} + R_{\text{air}} = 2\times 0.002968 + 0.267094 = 0.27303\,^\circ\text{C/W}$$

$$\frac{1}{R_{\text{eqv}}} = \frac{1}{R_{\text{wall}}} + 5\frac{1}{R_{\text{window}}} = \frac{1}{0.033382} + 5\frac{1}{0.27303} \longrightarrow R_{\text{eqv}} = 0.020717\,^\circ\text{C/W}$$

$$R_{\text{total}} = R_i + R_{\text{eqv}} + R_o = 0.001786 + 0.020717 + 0.000694 = 0.023197\,^\circ\text{C/W}$$

Then $\dot{Q} = \dfrac{T_{\infty 1} - T_{\infty 2}}{R_{\text{total}}} = \dfrac{(24-8)^\circ\text{C}}{0.023197^\circ\text{C/W}} = \mathbf{690\ W}$

The rate of heat transfer which will be saved if the single pane windows are converted to double pane windows is

$$\dot{Q}_{\text{save}} = \dot{Q}_{\substack{\text{single}\\\text{pane}}} - \dot{Q}_{\substack{\text{double}\\\text{pane}}} = 5224 - 690 = 4534\ \text{W}$$

The amount of energy and money saved during a 7-month long heating season by switching from single pane to double pane windows become

$$Q_{save} = \dot{Q}_{save}\Delta t = (4.534\,\text{kW})(7\times 30\times 24\,\text{h}) = 22{,}851\,\text{kWh}$$

Money savings = (Energy saved)(Unit cost of energy) = (22,851 kWh)($0.08/kWh) = **$1828**