**10-54** A wall consists of horizontal bricks separated by plaster layers. There are also plaster layers on each side of the wall, and a rigid foam on the inner side of the wall. The rate of heat transfer through the wall is to be determined.

Assumptions **1** Heat transfer is steady since there is no indication of change with time. **2** Heat transfer through the wall is one-dimensional. **3** Thermal conductivities are constant. **4** Heat transfer by radiation is disregarded.

Properties The thermal conductivities are given to be $k = 0.72$ W/m·°C for bricks, $k = 0.22$ W/m·°C for plaster layers, and $k = 0.026$ W/m·°C for the rigid foam.

Analysis We consider 1 m deep and 0.33 m high portion of wall which is representative of the entire wall. The thermal resistance network and individual resistances are
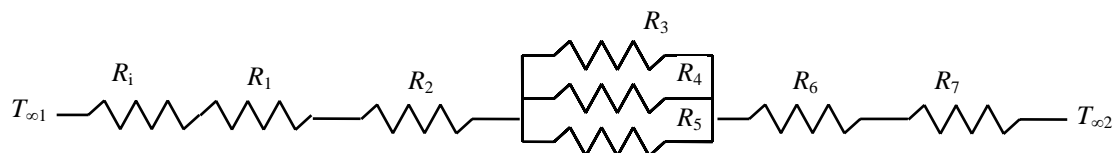
$$R_i = R_{conv,1} = \frac{1}{h_1 A} = \frac{1}{(10\ \text{W/m}^2 \cdot °\text{C})(0.33 \times 1\,\text{m}^2)} = 0.303\ °\text{C/W}$$

$$R_1 = R_{foam} = \frac{L}{kA} = \frac{0.02\ \text{m}}{(0.026\ \text{W/m} \cdot °\text{C})(0.33 \times 1\,\text{m}^2)} = 2.33\ °\text{C/W}$$

$$R_2 = R_6 = R_{plaster \atop side} = \frac{L}{kA} = \frac{0.02\ \text{m}}{(0.22\ \text{W/m} \cdot °\text{C})(0.33 \times 1\,\text{m}^2)} = 0.275\ °\text{C/W}$$

$$R_3 = R_5 = R_{plaster \atop center} = \frac{L}{h_o A} = \frac{0.18\ \text{m}}{(0.22\ \text{W/m} \cdot °\text{C})(0.015 \times 1\,\text{m}^2)} = 54.55\ °\text{C/W}$$

$$R_4 = R_{brick} = \frac{L}{kA} = \frac{0.18\ \text{m}}{(0.72\ \text{W/m} \cdot °\text{C})(0.30 \times 1\,\text{m}^2)} = 0.833\ °\text{C/W}$$

$$R_o = R_{conv,2} = \frac{1}{h_2 A} = \frac{1}{(20\ \text{W/m} \cdot °\text{C})(0.33 \times 1\,\text{m}^2)} = 0.152\ °\text{C/W}$$

$$\frac{1}{R_{mid}} = \frac{1}{R_3} + \frac{1}{R_4} + \frac{1}{R_5} = \frac{1}{54.55} + \frac{1}{0.833} + \frac{1}{54.55} \longrightarrow R_{mid} = 0.81\ °\text{C/W}$$

$$R_{total} = R_i + R_1 + 2R_2 + R_{mid} + R_o = 0.303 + 2.33 + 2(0.275) + 0.81 + 0.152 = 4.145\ °\text{C/W}$$

The steady rate of heat transfer through the wall per 0.33 m^2 is

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{total}} = \frac{[(22 - (-4)]°\text{C}}{4.145°\text{C/W}} = 6.27\ \text{W}$$

Then steady rate of heat transfer through the entire wall becomes

$$\dot{Q}_{total} = (6.27\ \text{W}) \frac{(4 \times 6)\text{m}^2}{0.33\ \text{m}^2} = \textbf{456 W}$$