

10-69 Chilled water is flowing inside a pipe. The thickness of the insulation needed to reduce the temperature rise of water to one-fourth of the original value is to be determined.

Assumptions 1 Heat transfer is steady since there is no indication of any change with time. 2 Heat transfer is one-dimensional since there is thermal symmetry about the centerline and no variation in the axial direction. 3 Thermal conductivities are constant. 4 The thermal contact resistance at the interface is negligible.

Properties The thermal conductivity is given to be $k = 0.05 \text{ W/m} \cdot ^\circ\text{C}$ for insulation.

Analysis The rate of heat transfer without the insulation is

$$\dot{Q}_{\text{old}} = \dot{m} c_p \Delta T = (0.98 \text{ kg/s})(4180 \text{ J/kg} \cdot ^\circ\text{C})(8 - 7)^\circ\text{C} = 4096 \text{ W}$$

The total resistance in this case is

$$\dot{Q}_{\text{old}} = \frac{T_\infty - T_w}{R_{\text{total}}} \\ 4096 \text{ W} = \frac{(30 - 7.5)^\circ\text{C}}{R_{\text{total}}} \longrightarrow R_{\text{total}} = 0.005493^\circ\text{C/W}$$

The convection resistance on the outer surface is

$$R_o = \frac{1}{h_o A_o} = \frac{1}{(9 \text{ W/m}^2 \cdot ^\circ\text{C})\pi(0.05 \text{ m})(150 \text{ m})} = 0.004716^\circ\text{C/W}$$

The rest of thermal resistances are due to convection resistance on the inner surface and the resistance of the pipe and it is determined from

$$R_1 = R_{\text{total}} - R_o = 0.005493 - 0.004716 = 0.0007769^\circ\text{C/W}$$

The rate of heat transfer with the insulation is

$$\dot{Q}_{\text{new}} = \dot{m} c_p \Delta T = (0.98 \text{ kg/s})(4180 \text{ J/kg} \cdot ^\circ\text{C})(0.25^\circ\text{C}) = 1024 \text{ W}$$

The total thermal resistance with the insulation is

$$\dot{Q}_{\text{new}} = \frac{T_\infty - T_w}{R_{\text{total,new}}} \longrightarrow 1024 \text{ W} = \frac{[30 - (7 + 7.25)/2]^\circ\text{C}}{R_{\text{total,new}}} \longrightarrow R_{\text{total,new}} = 0.02234^\circ\text{C/W}$$

It is expressed by

$$R_{\text{total,new}} = R_1 + R_{o,\text{new}} + R_{\text{ins}} = R_1 + \frac{1}{h_o A_o} + \frac{\ln(D_2 / D_1)}{2\pi k_{\text{ins}} L} \\ 0.02234^\circ\text{C/W} = 0.0007769 + \frac{1}{(9 \text{ W/m}^2 \cdot ^\circ\text{C})\pi D_2 (150 \text{ m})} + \frac{\ln(D_2 / 0.05)}{2\pi(0.05 \text{ W/m} \cdot ^\circ\text{C})(150 \text{ m})}$$

Solving this equation by trial-error or by using an equation solver such as EES, we obtain

$$D_2 = 0.1265 \text{ m}$$

Then the required thickness of the insulation becomes

$$t_{\text{ins}} = (D_2 - D_1) / 2 = (0.05 - 0.1265) / 2 = 0.0382 \text{ m} = \mathbf{3.8 \text{ cm}}$$

