10-69 Chilled water is flowing inside a pipe. The thickness of the insulation needed to reduce the temperature rise of water to one-fourth of the original value is to be determined.

Assumptions 1 Heat transfer is steady since there is no indication of any change with time. 2 Heat transfer is one-dimensional since there is thermal symmetry about the centerline and no variation in the axial direction, 3 Thermal conductivities are constant. 4 The thermal contact resistance at the interface is negligible.

Properties The thermal conductivity is given to be $k = 0.05 \text{ W/m} \cdot ^{\circ}\text{C}$ for insulation.

Analysis The rate of heat transfer without the insulation is

$$\dot{Q}_{\text{old}} = \dot{m}c_n \Delta T = (0.98 \text{ kg/s})(4180 \text{ J/kg} \cdot ^{\circ}\text{C})(8-7)^{\circ}\text{C} = 4096 \text{ W}$$

The total resistance in this case is

$$\dot{Q}_{\text{old}} = \frac{T_{\infty} - T_{w}}{R_{\text{total}}}$$

$$4096 \text{ W} = \frac{(30 - 7.5)^{\circ}\text{C}}{R_{\text{total}}} \longrightarrow R_{\text{total}} = 0.005493^{\circ}\text{C/W}$$

$$The convection resistance on the outer surface is$$

$$Water$$

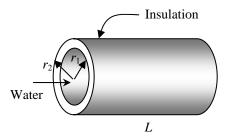
$$R_{1}$$

$$R_{0}$$

$$R_{\text{ins}}$$

$$T_{\infty 1}$$

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$$R_o = \frac{1}{h_o A_o} = \frac{1}{(9 \text{ W/m}^2 \cdot ^{\circ}\text{C})\pi (0.05 \text{ m})(150 \text{ m})} = 0.004716 \, ^{\circ}\text{C/W}$$

The rest of thermal resistances are due to convection resistance on the inner surface and the resistance of the pipe and it is determined from

$$R_1 = R_{\text{total}} - R_0 = 0.005493 - 0.004716 = 0.0007769 \,^{\circ}\text{C/W}$$

The rate of heat transfer with the insulation is

$$\dot{Q}_{\text{new}} = \dot{m}c_p \Delta T = (0.98 \text{ kg/s})(4180 \text{ J/kg} \cdot ^{\circ}\text{C})(0.25 ^{\circ}\text{C}) = 1024 \text{ W}$$

The total thermal resistance with the insulation is

$$\dot{Q}_{\text{new}} = \frac{T_{\infty} - T_{w}}{R_{\text{total,new}}} \longrightarrow 1024 \text{ W} = \frac{[30 - (7 + 7.25) / 2)]^{\circ}\text{C}}{R_{\text{total,new}}} \longrightarrow R_{\text{total,new}} = 0.02234^{\circ}\text{C/W}$$

It is expressed by

$$R_{\text{total,new}} = R_1 + R_{\text{o,new}} + R_{\text{ins}} = R_1 + \frac{1}{h_o A_o} + \frac{\ln(D_2 / D_1)}{2\pi k_{\text{ins}} L}$$

$$0.02234^{\circ}\text{C/W} = 0.0007769 + \frac{1}{(9 \text{ W/m}^2 \cdot {}^{\circ}\text{C})\pi D_2 (150 \text{ m})} + \frac{\ln(D_2 / 0.05)}{2\pi (0.05 \text{ W/m} \cdot {}^{\circ}\text{C})(150 \text{ m})}$$

Solving this equation by trial-error or by using an equation solver such as EES, we obtain

$$D_2 = 0.1265 \,\mathrm{m}$$

Then the required thickness of the insulation becomes

$$t_{\text{ins}} = (D_2 - D_1) / 2 = (0.05 - 0.1265) / 2 = 0.0382 \text{ m} = 3.8 \text{ cm}$$