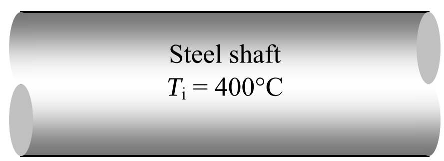
11-41 A long cylindrical shaft at 400°C is allowed to cool slowly. The center temperature and the heat transfer per unit length of the cylinder are to be determined.

Assumptions **1** Heat conduction in the shaft is one-dimensional since it is long and it has thermal symmetry about the center line. **2** The thermal properties of the shaft are constant. **3** The heat transfer coefficient is constant and uniform over the entire surface. **4** The Fourier number is $\tau > 0.2$ so that the one-term approximate solutions (or the transient temperature charts) are applicable (this assumption will be verified).

Properties The properties of stainless steel 304 at room temperature are given to be k = 14.9 W/m.°C, ρ = 7900 kg/m^3, c_p = 477 J/kg.°C, $\alpha = 3.95\times10^{-6}$ m^2/s

Analysis First the Biot number is calculated to be

$$Bi = \frac{hr_o}{k} = \frac{(60\ \text{W/m}^2.\text{°C})(0.175\ \text{m})}{(14.9\ \text{W/m.°C})} = 0.705$$



The constants λ_1 and A_1 corresponding to this Biot number are, from Table 11-2,

$$\lambda_1 = 1.0904 \quad \text{and} \quad A_1 = 1.1548$$

The Fourier number is

$$\tau = \frac{\alpha t}{L^2} = \frac{(3.95\times10^{-6}\ \text{m}^2/\text{s})(20\times60\ \text{s})}{(0.175\ \text{m})^2} = 0.1548$$

which is very close to the value of 0.2. Therefore, the one-term approximate solution (or the transient temperature charts) can still be used, with the understanding that the error involved will be a little more than 2 percent. Then the temperature at the center of the shaft becomes

$$\theta_{0,cyl} = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} = (1.1548)e^{-(1.0904)^2(0.1548)} = 0.9607$$

$$\frac{T_0 - 150}{400 - 150} = 0.9607 \longrightarrow T_0 = \mathbf{390\ °C}$$

The maximum heat can be transferred from the cylinder per meter of its length is

$$m = \rho V = \rho \pi r_o^2 L = (7900\ \text{kg/m}^3)[\pi(0.175\ \text{m})^2(1\ \text{m})] = 760.1\ \text{kg}$$

$$Q_{\max} = mc_p[T_\infty - T_i] = (760.1\ \text{kg})(0.477\ \text{kJ/kg.°C})(400-150)\text{°C} = 90{,}640\ \text{kJ}$$

Once the constant J_1= 0.4679 is determined from Table 11-3 corresponding to the constant λ_1=1.0904, the actual heat transfer becomes

$$\left(\frac{Q}{Q_{\max}}\right)_{cyl} = 1 - 2\left(\frac{T_o - T_\infty}{T_i - T_\infty}\right)\frac{J_1(\lambda_1)}{\lambda_1} = 1 - 2\left(\frac{390-150}{400-150}\right)\frac{0.4679}{1.0904} = 0.1761$$

$$Q = 0.1761(90{,}640\ \text{kJ}) = \mathbf{15{,}960\ kJ}$$