**11-97 CD EES** The trunks of some dry oak trees are exposed to hot gases. The time for the ignition of the trunks is to be determined.

*Assumptions* **1** Heat conduction in the trunks is one-dimensional since it is long and it has thermal symmetry about the center line. **2** The thermal properties of the trunks are constant. **3** The heat transfer coefficient is constant and uniform over the entire surface. **4** The Fourier number is $\tau > 0.2$ so that the one-term approximate solutions (or the transient temperature charts) are applicable (this assumption will be verified).

*Properties* The properties of the trunks are given to be $k = 0.17$ W/m.°C and $\alpha = 1.28 \times 10^{-7}$ m$^2$/s.

*Analysis* We treat the trunks of the trees as an infinite cylinder since heat transfer is primarily in the radial direction. Then the Biot number becomes
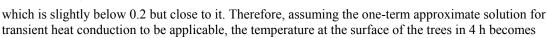
$$Bi = \frac{hr_o}{k} = \frac{(65 \text{ W/m}^2.°\text{C})(0.1\,\text{m})}{(0.17 \text{ W/m.}°\text{C})} = 38.24$$

The constants $\lambda_1$ and $A_1$ corresponding to this Biot number are, from Table 11-2,

$$\lambda_1 = 2.3420 \quad \text{and} \quad A_1 = 1.5989$$

The Fourier number is

$$\tau = \frac{\alpha t}{r_o^2} = \frac{(1.28 \times 10^{-7} \text{ m}^2/\text{s})(4\,\text{h} \times 3600 \text{ s/h})}{(0.1\,\text{m})^2} = 0.184$$



Tree
$T_i = 30°C$

Hot gases
$T_\infty = 520°C$

$D = 0.2$ m

which is slightly below 0.2 but close to it. Therefore, assuming the one-term approximate solution for transient heat conduction to be applicable, the temperature at the surface of the trees in 4 h becomes

$$\theta(r_o,t)_{cyl} = \frac{T(r_o,t) - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} J_0(\lambda_1 r / r_o)$$

$$\frac{T(r_o,t) - 520}{30 - 520} = (1.5989)e^{-(2.3420)^2(0.184)}(0.0332) = 0.01935 \longrightarrow T(r_o,t) = \mathbf{511\,°C} \; > 410°C$$

Therefore, the trees will ignite. (Note: $J_0$ is read from Table 11-3).