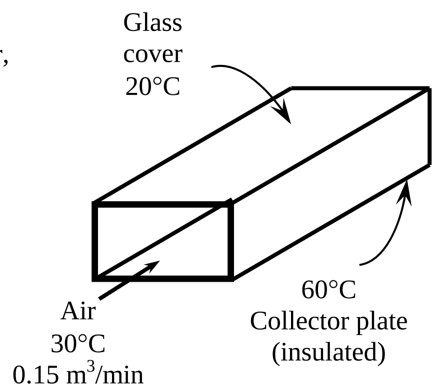
13-47 Air enters the constant spacing between the glass cover and the plate of a solar collector. The net rate of heat transfer and the temperature rise of air are to be determined.

Assumptions **1** Steady operating conditions exist. **2** The inner surfaces of the spacing are smooth. **3** Air is an ideal gas with constant properties. **4** The local atmospheric pressure is 1 atm.

Properties The properties of air at 1 atm and estimated average temperature of 35°C are (Table A-22)

$$\rho = 1.145 \text{ kg/m}^3, \quad k = 0.02625 \text{ W/m.°C}$$

$$\nu = 1.655\times10^{-5} \text{ m}^2\text{/s}, \quad c_p = 1007 \text{ J/kg.°C}, \quad \text{Pr} = 0.7268$$



Analysis Mass flow rate, cross sectional area, hydraulic diameter, mean velocity of air and the Reynolds number are

$$\dot{m} = \rho\dot{V} = (1.145 \text{ kg/m}^3)(0.15 \text{ m}^3\text{/s}) = 0.1718 \text{ kg/s}$$

$$A_c = (1 \text{ m})(0.03 \text{ m}) = 0.03 \text{ m}^2$$

$$D_h = \frac{4A_c}{P} = \frac{4(0.03 \text{ m}^2)}{2(1 \text{ m} + 0.03 \text{ m})} = 0.05825 \text{ m}$$

$$V_{\text{avg}} = \frac{\dot{V}}{A_c} = \frac{0.15 \text{ m}^3\text{/s}}{0.03 \text{ m}^2} = 5 \text{ m/s}$$

$$\text{Re} = \frac{V_{\text{avg}} D_h}{\nu} = \frac{(5 \text{ m/s})(0.05825 \text{ m})}{1.655\times10^{-5} \text{ m}^2\text{/s}} = 17{,}600$$

which is greater than 10,000. Therefore, the flow is turbulent and the entry lengths in this case are roughly

$$L_h \approx L_t \approx 10D_h = 10(0.05825 \text{ m}) = 0.5825 \text{ m}$$

which are much shorter than the total length of the collector. Therefore, we can assume fully developed turbulent flow in the entire collector, and determine the Nusselt number from

$$Nu = \frac{hD_h}{k} = 0.023 \text{ Re}^{0.8} \text{ Pr}^{0.4} = 0.023(17{,}600)^{0.8}(0.7268)^{0.4} = 50.43$$

and $$h = \frac{k}{D_h} Nu = \frac{0.02625 \text{ W/m.°C}}{0.05825 \text{ m}}(50.43) = 22.73 \text{ W/m}^2\text{.°C}$$

The exit temperature of air can be calculated using the "average" surface temperature as

$$A_s = 2(5 \text{ m})(1 \text{ m}) = 10 \text{ m}^2, \qquad T_{s,avg} = \frac{60+20}{2} = 40\text{°C}$$

$$T_e = T_{s,avg} - (T_{s,avg} - T_i)\exp\left(-\frac{hA_s}{\dot{m}c_p}\right) = 40 - (40-30)\exp\left(-\frac{22.73\times10}{0.1718\times1007}\right) = 37.31\text{°C}$$

The temperature rise of air is

$$\Delta T = 37.3\text{°C} - 30\text{°C} = \mathbf{7.3°C}$$

The logarithmic mean temperature difference and the heat loss from the glass are

$$\Delta T_{\ln,glass} = \frac{T_e - T_i}{\ln\frac{T_s - T_e}{T_s - T_i}} = \frac{37.31-30}{\ln\frac{20-37.31}{20-30}} = 13.32\text{°C}$$

$$\dot{Q}_{glass} = hA_s\Delta T_{\ln} = (22.73 \text{ W/m}^2\text{.°C})(5 \text{ m}^2)(13.32\text{°C}) = 1514 \text{ W}$$

The logarithmic mean temperature difference and the heat gain of the absorber are

$$\Delta T_{\ln,absorber} = \frac{T_e - T_i}{\ln\frac{T_s - T_e}{T_s - T_i}} = \frac{37.31-30}{\ln\frac{60-37.31}{60-30}} = 26.17\text{°C}$$

$$\dot{Q}_{absorber} = hA\Delta T_{\ln} = (22.73 \text{ W/m}^2\text{.°C})(5 \text{ m}^2)(26.17\text{°C}) = 2975 \text{ W}$$

Then the net rate of heat transfer becomes

$$\dot{Q}_{net} = 2975 - 1514 = \mathbf{1461 \text{ W}}$$