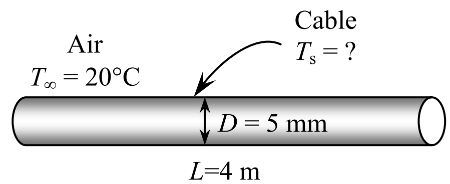
14-18 Heat generated by the electrical resistance of a bare cable is dissipated to the surrounding air. The surface temperature of the cable is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** The local atmospheric pressure is 1 atm. **4** The temperature of the surface of the cable is constant.

Properties We assume the surface temperature to be 100°C. Then the properties of air at 1 atm and the film temperature of $(T_s+T_\infty)/2 = (100+20)/2 = 60°C$ are (Table A-22)

$$k = 0.02808 \text{ W/m.°C}$$
$$\nu = 1.896\times10^{-5} \text{ m}^2\text{/s}$$
$$\Pr = 0.7202$$
$$\beta = \frac{1}{T_f} = \frac{1}{(60+273)\text{K}} = 0.003003 \text{ K}^{-1}$$



Analysis The characteristic length in this case is the outer diameter of the pipe, $L_c = D = 0.005$ m. Then,

$$Ra = \frac{g\beta(T_s - T_\infty)D^3}{\nu^2}\Pr = \frac{(9.81 \text{ m/s}^2)(0.003003 \text{ K}^{-1})(100-20 \text{ K})(0.005 \text{ m})^3}{(1.896\times10^{-5} \text{ m}^2\text{/s})^2}(0.7202) = 590.2$$

$$Nu = \left\{0.6 + \frac{0.387Ra^{1/6}}{\left[1+(0.559/\Pr)^{9/16}\right]^{8/27}}\right\}^2 = \left\{0.6 + \frac{0.387(590.2)^{1/6}}{\left[1+(0.559/0.7202)^{9/16}\right]^{8/27}}\right\}^2 = 2.346$$

$$h = \frac{k}{D}Nu = \frac{0.02808 \text{ W/m.°C}}{0.005 \text{ m}}(2.346) = 13.17 \text{ W/m}^2\text{.°C}$$

$$A_s = \pi DL = \pi(0.005 \text{ m})(4 \text{ m}) = 0.06283 \text{ m}^2$$

$$\dot{Q} = hA_s(T_s - T_\infty)$$
$$(60 \text{ V})(1.5 \text{ A}) = (13.17 \text{ W/m}^2\text{.°C})(0.06283 \text{ m}^2)(T_s - 20)\text{°C}$$
$$T_s = 128.8\text{°C}$$

which is not close to the assumed value of 100°C. Repeating calculations for an assumed surface temperature of 120°C, $[T_f = (T_s+T_\infty)/2 = (120+20)/2 = 70°C]$

$$k = 0.02881 \text{ W/m.°C}$$
$$\nu = 1.995\times10^{-5} \text{ m}^2\text{/s}$$
$$\Pr = 0.7177$$
$$\beta = \frac{1}{T_f} = \frac{1}{(70+273)\text{K}} = 0.002915 \text{ K}^{-1}$$

$$Ra = \frac{g\beta(T_s - T_\infty)D^3}{\nu^2}\Pr = \frac{(9.81 \text{ m/s}^2)(0.002915 \text{ K}^{-1})(120-20 \text{ K})(0.005 \text{ m})^3}{(1.995\times10^{-5} \text{ m}^2\text{/s})^2}(0.7177) = 644.6$$

$$Nu = \left\{0.6 + \frac{0.387Ra^{1/6}}{\left[1+(0.559/\Pr)^{9/16}\right]^{8/27}}\right\}^2 = \left\{0.6 + \frac{0.387(644.6)^{1/6}}{\left[1+(0.559/0.7177)^{9/16}\right]^{8/27}}\right\}^2 = 2.387$$

$$h = \frac{k}{D}Nu = \frac{0.02881 \text{ W/m.°C}}{0.005 \text{ m}}(2.387) = 13.76 \text{ W/m}^2\text{.°C}$$

$$\dot{Q} = hA_s(T_s - T_\infty)$$
$$(60 \text{ V})(1.5A) = (13.76 \text{ W/m}^2\text{.°C})(0.06283 \text{ m}^2)(T_s - 20)\text{°C}$$
$$T_s = \mathbf{124.1°C}$$

which is sufficiently close to the assumed value of 120°C.