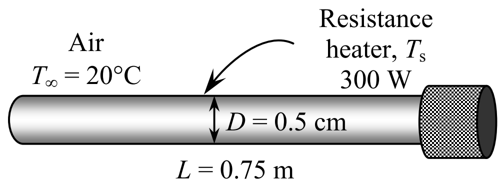
14-24 A cylindrical resistance heater is placed horizontally in a fluid. The outer surface temperature of the resistance wire is to be determined for two different fluids.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** The local atmospheric pressure is 1 atm. **4** Any heat transfer by radiation is ignored. **5** Properties are evaluated at 500°C for air and 40°C for water.

Properties The properties of air at 1 atm and 500°C are (Table A-22)

$$k = 0.05572 \text{ W/m.°C}$$
$$\nu = 7.804\times10^{-5} \text{ m}^2\text{/s}$$
$$\text{Pr} = 0.6986,$$
$$\beta = \frac{1}{T_f} = \frac{1}{(500+273)\text{K}} = 0.001294 \text{ K}^{-1}$$



The properties of water at 40°C are (Table A-15)

$$k = 0.631 \text{ W/m.°C}, \quad \nu = \mu/\rho = 0.6582\times10^{-6} \text{ m}^2\text{/s}$$
$$\text{Pr} = 4.32, \quad \beta = 0.000377 \text{ K}^{-1}$$

Analysis (*a*) The solution of this problem requires a trial-and-error approach since the determination of the Rayleigh number and thus the Nusselt number depends on the surface temperature which is unknown. We start the solution process by "guessing" the surface temperature to be 1200°C for the calculation of *h*. We will check the accuracy of this guess later and repeat the calculations if necessary. The characteristic length in this case is the outer diameter of the wire, $L_c = D = 0.005$ m. Then,

$$Ra = \frac{g\beta(T_s - T_\infty)D^3}{\nu^2}\text{Pr} = \frac{(9.81 \text{ m/s}^2)(0.001294 \text{ K}^{-1})(1200-20)\text{°C}(0.005 \text{ m})^3}{(7.804\times10^{-5} \text{ m}^2\text{/s})^2}(0.6986) = 214.7$$

$$Nu = \left\{0.6 + \frac{0.387Ra^{1/6}}{\left[1+\left(0.559/\text{Pr}\right)^{9/16}\right]^{8/27}}\right\}^2 = \left\{0.6 + \frac{0.387(214.7)^{1/6}}{\left[1+\left(0.559/0.6986\right)^{9/16}\right]^{8/27}}\right\}^2 = 1.919$$

$$h = \frac{k}{D}Nu = \frac{0.05572 \text{ W/m.°C}}{0.005 \text{ m}}(1.919) = 21.38 \text{ W/m}^2\text{.°C}$$

$$A_s = \pi DL = \pi(0.005 \text{ m})(0.75 \text{ m}) = 0.01178 \text{ m}^2$$

and

$$\dot{Q} = hA_s(T_s - T_\infty) \rightarrow 300 \text{ W} = (21.38 \text{ W/m}^2\text{.°C})(0.01178 \text{ m}^2)(T_s - 20)\text{°C} \rightarrow T_s = \mathbf{1211°C}$$

which is sufficiently close to the assumed value of 1200°C used in the evaluation of *h*, and thus it is not necessary to repeat calculations.

(*b*) For the case of water, we "guess" the surface temperature to be 40°C. The characteristic length in this case is the outer diameter of the wire, $L_c = D = 0.005$ m. Then,

$$Ra = \frac{g\beta(T_s - T_\infty)D^3}{\nu^2}\text{Pr} = \frac{(9.81 \text{ m/s}^2)(0.000377 \text{ K}^{-1})(40-20 \text{ K})(0.005 \text{ m})^3}{(0.6582\times10^{-6} \text{ m}^2\text{/s})^2}(4.32) = 92{,}197$$

$$Nu = \left\{0.6 + \frac{0.387Ra^{1/6}}{\left[1+\left(0.559/\text{Pr}\right)^{9/16}\right]^{8/27}}\right\}^2 = \left\{0.6 + \frac{0.387(92{,}197)^{1/6}}{\left[1+\left(0.559/4.32\right)^{9/16}\right]^{8/27}}\right\}^2 = 8.986$$

$$h = \frac{k}{D}Nu = \frac{0.631 \text{ W/m.°C}}{0.005 \text{ m}}(8.986) = 1134 \text{ W/m}^2\text{.°C}$$

and $\dot{Q} = hA_s(T_s - T_\infty) \longrightarrow 300 \text{ W} = (1134 \text{ W/m}^2\text{.°C})(0.01178 \text{ m}^2)(T_s - 20)\text{°C} \longrightarrow T_s = \mathbf{42.5°C}$

which is sufficiently close to the assumed value of 40°C in the evaluation of the properties and *h*. The film temperature in this case is $(T_s+T_\infty)/2$ = (42.5+20)/2 =31.3°C, which is close to the value of 40°C used in the evaluation of the properties.