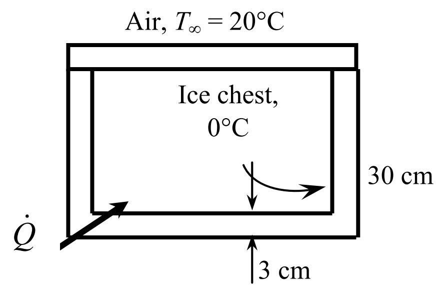
14-82 An ice chest filled with ice at 0°C is exposed to ambient air. The time it will take for the ice in the chest to melt completely is to be determined for natural and forced convection cases.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** Heat transfer from the base of the ice chest is disregarded. **4** Radiation effects are negligible. **5** Heat transfer coefficient is the same for all surfaces considered. **6** The local atmospheric pressure is 1 atm.

Properties The properties of air at 1 atm and the anticipated film temperature of $(T_s+T_\infty)/2 = (15+20)/2 = 17.5$°C are (Table A-22)



$$k = 0.02495 \text{ W/m.°C}$$
$$\nu = 1.493\times10^{-5} \text{ m}^2/\text{s}$$
$$\text{Pr} = 0.7316$$
$$\beta = \frac{1}{T_f} = \frac{1}{(17.5+273)\text{K}} = 0.003442 \text{ K}^{-1}$$

Analysis The solution of this problem requires a trial-and-error approach since the determination of the Rayleigh number and thus the Nusselt number depends on the surface temperature which is unknown. We start the solution process by "guessing" the surface temperature to be 15°C for the evaluation of the properties and h. We will check the accuracy of this guess later and repeat the calculations if necessary. The characteristic length for the side surfaces is the height of the chest, $L_c = L = 0.3$ m Then,

$$Ra = \frac{g\beta(T_\infty - T_s)L^3}{\nu^2}\text{Pr} = \frac{(9.81\text{ m/s}^2)(0.003442\text{ K}^{-1})(20-15\text{ K})(0.3\text{ m})^3}{(1.493\times10^{-5}\text{ m}^2/\text{s})^2}(0.7316) = 1.495\times10^7$$

$$Nu = \left\{0.825 + \frac{0.387\text{Ra}^{1/6}}{\left[1+\left(\frac{0.492}{\text{Pr}}\right)^{9/16}\right]^{8/27}}\right\}^2 = \left\{0.825 + \frac{0.387(1.495\times10^7)^{1/6}}{\left[1+\left(\frac{0.492}{0.7316}\right)^{9/16}\right]^{8/27}}\right\}^2 = 35.15$$

$$h = \frac{k}{L}Nu = \frac{0.02495\text{ W/m.°C}}{0.3\text{ m}}(35.15) = 2.923\text{ W/m}^2.\text{°C}$$

The heat transfer coefficient at the top surface can be determined similarly. However, the top surface constitutes only about one-fourth of the heat transfer area, and thus we can use the heat transfer coefficient for the side surfaces for the top surface also for simplicity. The heat transfer surface area is

$$A_s = 4(0.3\text{ m})(0.4\text{ m}) + (0.4\text{ m})(0.4\text{ m}) = 0.64\text{ m}^2$$

Then the rate of heat transfer becomes

$$\dot{Q} = \frac{T_\infty - T_{s,i}}{R_{wall} + R_{conv,o}} = \frac{T_\infty - T_{s,i}}{\frac{L}{kA_s} + \frac{1}{hA_s}} = \frac{(20-0)\text{°C}}{\frac{0.03\text{ m}}{(0.033\text{ W/m.°C})(0.64\text{ m}^2)} + \frac{1}{(2.923\text{ W/m}^2.\text{°C})(0.64\text{ m}^2)}} = 10.23\text{ W}$$

The outer surface temperature of the ice chest is determined from Newton's law of cooling to be

$$\dot{Q} = hA_s(T_\infty - T_s) \rightarrow T_s = T_\infty - \frac{\dot{Q}}{hA_s} = 20\text{°C} - \frac{10.23\text{ W}}{(2.923\text{ W/m}^2.\text{C})(0.64\text{ m}^2)} = 14.53\text{°C}$$

which is almost identical to the assumed value of 15°C used in the evaluation of properties and h. Therefore, there is no need to repeat the calculations.

The rate at which the ice will melt is

$$\dot{Q} = \dot{m}h_{if} \rightarrow \dot{m} = \frac{\dot{Q}}{h_{if}} = \frac{10.23\times10^{-3}\ \text{kJ/s}}{333.7\ \text{kJ/kg}} = 3.066\times10^{-5}\ \text{kg/s}$$

Therefore, the melting of the ice in the chest completely will take

$$m = \dot{m}\Delta t \longrightarrow \Delta t = \frac{m}{\dot{m}} = \frac{30\ \text{kg}}{3.066\times10^{-5}\ \text{kg/s}} = 9.786\times10^{5}\ \text{s} = \mathbf{271.8\ h} = \mathbf{11.3\ days}$$

(b) The temperature drop across the styrofoam will be much greater in this case than that across thermal boundary layer on the surface. Thus we assume outer surface temperature of the styrofoam to be 19 °C. Radiation heat transfer will be neglected. The properties of air at 1 atm and the film temperature of $(T_s+T_\infty)/2 = (19+20)/2 = 19.5$°C are (Table A-22)

$$k = 0.0251\ \text{W/m.°C}$$
$$\nu = 1.511\times10^{-5}\ \text{m}^2\text{/s}$$
$$\text{Pr} = 0.7310$$
$$\beta = \frac{1}{T_f} = \frac{1}{(19.5+273)\text{K}} = 0.00342\ \text{K}^{-1}$$

The characteristic length in this case is the width of the chest, $L_c = W$ =0.4 m. Then,

$$\text{Re} = \frac{VW}{\nu} = \frac{(50\times1000/3600\ \text{m/s})(0.4\ \text{m})}{1.511\times10^{-5}\ \text{m}^2\text{/s}} = 367{,}700$$

which is less than critical Reynolds number (5×10^5). Therefore the flow is laminar, and the Nusselt number is determined from

$$Nu = \frac{hW}{k} = 0.664\,\text{Re}^{0.5}\,\text{Pr}^{1/3} = 0.664(367{,}700)^{0.5}(0.7310)^{1/3} = 362.7$$

$$h = \frac{k}{W}Nu = \frac{0.0251\ \text{W/m.°C}}{0.4\ \text{m}}(362.7) = 22.76\ \text{W/m}^2\text{.°C}$$

Then the rate of heat transfer becomes

$$\dot{Q} = \frac{T_\infty - T_{s,i}}{R_{wall} + R_{conv,o}} = \frac{T_\infty - T_{s,i}}{\frac{L}{kA_s} + \frac{1}{hA_s}} = \frac{(20-0)\text{°C}}{\frac{0.03\ \text{m}}{(0.033\ \text{W/m.°C})(0.64\ \text{m}^2)} + \frac{1}{(22.76\ \text{W/m}^2\text{.°C})(0.64\ \text{m}^2)}} = 13.43\ \text{W}$$

The outer surface temperature of the ice chest is determined from Newton's law of cooling to be

$$\dot{Q} = hA_s(T_\infty - T_s) \rightarrow T_s = T_\infty - \frac{\dot{Q}}{hA_s} = 20\text{°C} - \frac{13.43\ \text{W}}{(22.76\ \text{W/m}^2\text{.C})(0.64\ \text{m}^2)} = 19.1\text{°C}$$

which is almost identical to the assumed value of 19°C used in the evaluation of properties and h. Therefore, there is no need to repeat the calculations. Then the rate at which the ice will melt becomes

$$\dot{Q} = \dot{m}h_{if} \rightarrow \dot{m} = \frac{\dot{Q}}{h_{if}} = \frac{13.43\times10^{-3}\ \text{kJ/s}}{333.7\ \text{kJ/kg}} = 4.025\times10^{-5}\ \text{kg/s}$$

Therefore, the melting of the ice in the chest completely will take

$$m = \dot{m}\Delta t \longrightarrow \Delta t = \frac{m}{\dot{m}} = \frac{30}{4.025\times10^{-5}} = 7.454\times10^{5}\ \text{s} = \mathbf{207.05\ h} = \mathbf{8.6\ days}$$