

15-107 A cylindrical furnace with specified top and bottom surface temperatures and specified heat transfer rate at the bottom surface is considered. The emissivity of the top surface and the net rates of heat transfer between the top and the bottom surfaces, and between the bottom and the side surfaces are to be determined.

Assumptions 1 Steady operating conditions exist 2 The surfaces are opaque, diffuse, and gray. 3 Convection heat transfer is not considered.

Properties The emissivity of the bottom surface is 0.90.

Analysis We consider the top surface to be surface 1, the base surface to be surface 2, and the side surface to be surface 3. This system is a three-surface enclosure. The view factor from the base to the top surface of the cube is from Fig. 15-5 $F_{12} = 0.2$. The view factor from the base or the top to the side surfaces is determined by applying the summation rule to be

$$F_{11} + F_{12} + F_{13} = 1 \longrightarrow F_{13} = 1 - F_{12} = 1 - 0.2 = 0.8$$

since the base surface is flat and thus $F_{11} = 0$. Other view factors are

$$F_{21} = F_{12} = 0.20, \quad F_{23} = F_{13} = 0.80, \quad F_{31} = F_{32} = 0.20$$

We now apply Eq. 9-35 to each surface

$$\sigma T_1^4 = J_1 + \frac{1 - \varepsilon_1}{\varepsilon_1} [F_{12}(J_1 - J_2) + F_{13}(J_1 - J_3)]$$

Surface 1:

$$(5.67 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4)(700 \text{ K})^4 = J_1 + \frac{1 - \varepsilon_1}{\varepsilon_1} [0.20(J_1 - J_2) + 0.80(J_1 - J_3)]$$

$$\sigma T_2^4 = J_2 + \frac{1 - \varepsilon_2}{\varepsilon_2} [F_{21}(J_2 - J_1) + F_{23}(J_2 - J_3)]$$

Surface 2:

$$(5.67 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4)(950 \text{ K})^4 = J_2 + \frac{1 - 0.90}{0.90} [0.20(J_2 - J_1) + 0.80(J_2 - J_3)]$$

$$\sigma T_3^4 = J_3$$

Surface 3:

$$(5.67 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4)(450 \text{ K})^4 = J_3$$

We now apply Eq. 9-34 to surface 2

$$\dot{Q}_2 = A_2 [F_{21}(J_2 - J_1) + F_{23}(J_2 - J_3)] = (9 \text{ m}^2) [0.20(J_2 - J_1) + 0.80(J_2 - J_3)]$$

Solving the above four equations, we find

$$\varepsilon_1 = \mathbf{0.44}, \quad J_1 = 11,736 \text{ W/m}^2, \quad J_2 = 41,985 \text{ W/m}^2, \quad J_3 = 2325 \text{ W/m}^2$$

The rate of heat transfer between the bottom and the top surface is

$$A_1 = A_2 = (3 \text{ m})^2 = 9 \text{ m}^2$$

$$\dot{Q}_{21} = A_2 F_{21} (J_2 - J_1) = (9 \text{ m}^2)(0.20)(41,985 - 11,736) \text{ W/m}^2 = \mathbf{54.4 \text{ kW}}$$

The rate of heat transfer between the bottom and the side surface is

$$A_3 = 4A_1 = 4(9 \text{ m}^2) = 36 \text{ m}^2$$

$$\dot{Q}_{23} = A_2 F_{23} (J_2 - J_3) = (9 \text{ m}^2)(0.8)(41,985 - 2325) \text{ W/m}^2 = \mathbf{285.6 \text{ kW}}$$

Discussion The sum of these two heat transfer rates are $54.4 + 285.6 = 340 \text{ kW}$, which is equal to 340 kW heat supply rate from surface 2.

