15-108 Radiation heat transfer occurs between two square parallel plates. The view factors, the rate of radiation heat transfer and the temperature of a third plate to be inserted are to be determined.

Assumptions **1** Steady operating conditions exist **2** The surfaces are opaque, diffuse, and gray. **3** Convection heat transfer is not considered.

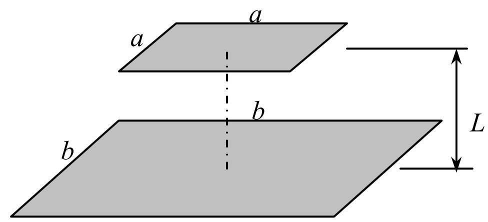
Properties The emissivities of plate a, b, and c are given to be $\varepsilon_a = 0.8$, $\varepsilon_b = 0.4$, and $\varepsilon_c = 0.1$, respectively.

Analysis (*a*) The view factor from surface a to surface b is determined as follows

$$A = \frac{a}{L} = \frac{20}{40} = 0.5, \quad B = \frac{b}{L} = \frac{60}{40} = 1.5$$

$$F_{ab} = \frac{1}{2A}\left\{\left[(B+A)^2+4\right]^{0.5} - \left[(B-A)^2+4\right]^{0.5}\right\} = \frac{1}{2(0.5)}\left\{\left[(1.5+0.5)^2+4\right]^{0.5} - \left[(1.5-0.5)^2+4\right]^{0.5}\right\} = \mathbf{0.592}$$

The view factor from surface b to surface a is determined from reciprocity relation:



$$A_a = (0.2\text{ m})(0.2\text{ m}) = 0.04\text{ m}^2$$

$$A_b = (0.6\text{ m})(0.6\text{ m}) = 0.36\text{ m}^2$$

$$A_a F_{ab} = A_b F_{ba}$$

$$(0.04)(0.592) = (0.36)F_{ba} \longrightarrow F_{ba} = \mathbf{0.0658}$$

(b) The net rate of radiation heat transfer between the surfaces can be determined from

$$\dot{Q}_{ab} = \frac{\sigma\left(T_a^4 - T_b^4\right)}{\frac{1-\varepsilon_a}{A_a\varepsilon_a} + \frac{1}{A_a F_{ab}} + \frac{1-\varepsilon_b}{A_b\varepsilon_b}} = \frac{(5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4)\left[(1073\text{ K})^4 - (473\text{ K})^4\right]}{\frac{1-0.8}{(0.04\text{ m}^2)(0.8)} + \frac{1}{(0.04\text{ m}^2)(0.592)} + \frac{1-0.4}{(0.36\text{ m}^2)(0.4)}} = \mathbf{1374\ W}$$

(c) In this case we have

$$A = \frac{a}{L} = \frac{0.2\text{ m}}{0.2\text{ m}} = 1, \quad C = \frac{c}{L} = \frac{2.0\text{ m}}{0.2\text{ m}} = 10$$

$$F_{ac} = \frac{1}{2A}\left\{\left[(C+A)^2+4\right]^{0.5} - \left[(C-A)^2+4\right]^{0.5}\right\} = \frac{1}{2(0.5)}\left\{\left[(10+0.5)^2+4\right]^{0.5} - \left[(10-0.5)^2+4\right]^{0.5}\right\} = 0.981$$

$$B = \frac{b}{L} = \frac{0.6\text{ m}}{0.2\text{ m}} = 3, \quad C = \frac{c}{L} = \frac{2.0\text{ m}}{0.2\text{ m}} = 10$$

$$F_{bc} = \frac{1}{2A}\left\{\left[(C+B)^2+4\right]^{0.5} - \left[(C-B)^2+4\right]^{0.5}\right\}$$

$$= \frac{1}{2(3)}\left\{\left[(10+3)^2+4\right]^{0.5} - \left[(10-3)^2+4\right]^{0.5}\right\} = 0.979$$

$$A_b F_{bc} = A_c F_{cb}$$

$$(0.36)(0.979) = (4.0)F_{cb} \longrightarrow F_{ba} = 0.0881$$

An energy balance gives

$$\dot{Q}_{ac} = \dot{Q}_{cb}$$

$$\frac{\sigma\left(T_a^4 - T_c^4\right)}{\frac{1-\varepsilon_a}{A_a\varepsilon_a} + \frac{1}{A_a F_{ac}} + \frac{1-\varepsilon_c}{A_c\varepsilon_c}} = \frac{\sigma\left(T_c^4 - T_b^4\right)}{\frac{1-\varepsilon_c}{A_c\varepsilon_c} + \frac{1}{A_c F_{cb}} + \frac{1-\varepsilon_b}{A_b\varepsilon_b}}$$

$$\frac{(1073\text{ K})^4 - T_c^4}{\frac{1-0.8}{(0.04\text{ m}^2)(0.8)} + \frac{1}{(0.04\text{ m}^2)(0.981)} + \frac{1-0.1}{(4\text{ m}^2)(0.1)}} = \frac{T_c^4 - (473\text{ K})^4}{\frac{1-0.1}{(4\text{ m}^2)(0.1)} + \frac{1}{(4\text{ m}^2)(0.0881)} + \frac{1-0.4}{(0.36\text{ m}^2)(0.4)}}$$

Solving the equation with an equation solver such as EES, we obtain $T_c = 754$ K **= 481°C**