**5-63** The internal energy change of hydrogen gas during a heating process is to be determined using an empirical specific heat relation, constant specific heat at average temperature, and constant specific heat at room temperature.

***Analysis*** (*a*) Using the empirical relation for $\bar{c}_p(T)$ from Table A-2c and relating it to $\bar{c}_v(T)$,

$$\bar{c}_v(T) = \bar{c}_p - R_u = (a - R_u) + bT + cT^2 + dT^3$$

where a = 29.11, b = -0.1916×10$^{-2}$, c = 0.4003×10$^{-5}$, and d = -0.8704×10$^{-9}$. Then,

$$\begin{aligned}
\Delta\bar{u} &= \int_1^2 \bar{c}_v(T)dT = \int_1^2 \left[(a - R_u) + bT + cT^2 + dT^3\right]dT \\
&= (a - R_u)(T_2 - T_1) + \tfrac{1}{2}b(T_2^2 + T_1^2) + \tfrac{1}{3}c(T_2^3 - T_1^3) + \tfrac{1}{4}d(T_2^4 - T_1^4) \\
&= (29.11 - 8.314)(800 - 200) - \tfrac{1}{2}(0.1961\times10^{-2})(800^2 - 200^2) \\
&\quad + \tfrac{1}{3}(0.4003\times10^{-5})(800^3 - 200^3) - \tfrac{1}{4}(0.8704\times10^{-9})(800^4 - 200^4) \\
&= 12{,}487 \text{ kJ/kmol}
\end{aligned}$$

$$\Delta u = \frac{\Delta\bar{u}}{M} = \frac{12{,}487 \text{ kJ/kmol}}{2.016 \text{ kg/kmol}} = \mathbf{6194\ kJ/kg}$$

(*b*) Using a constant $c_p$ value from Table A-2b at the average temperature of 500 K,

$$\begin{aligned}
c_{v,\text{avg}} &= c_{v@500\text{ K}} = 10.389 \text{ kJ/kg}\cdot\text{K} \\
\Delta u &= c_{v,\text{avg}}(T_2 - T_1) = (10.389 \text{ kJ/kg}\cdot\text{K})(800 - 200)\text{K} = \mathbf{6233\ kJ/kg}
\end{aligned}$$

(*c*) Using a constant $c_p$ value from Table A-2a at room temperature,

$$\begin{aligned}
c_{v,\text{avg}} &= c_{v@300\text{ K}} = 10.183 \text{ kJ/kg}\cdot\text{K} \\
\Delta u &= c_{v,\text{avg}}(T_2 - T_1) = (10.183 \text{ kJ/kg}\cdot\text{K})(800 - 200)\text{K} = \mathbf{6110\ kJ/kg}
\end{aligned}$$