6-60 Air is expanded in an adiabatic turbine. The mass flow rate of the air and the power produced are to be determined.

Assumptions **1** This is a steady-flow process since there is no change with time. **2** The turbine is well-insulated, and thus there is no heat transfer. **3** Air is an ideal gas with constant specific heats.

Properties The constant pressure specific heat of air at the average temperature of (500+150)/2=325°C=598 K is c_p = 1.051 kJ/kg·K (Table A-2*b*). The gas constant of air is R = 0.287 kPa·m^3/kg·K (Table A-1).

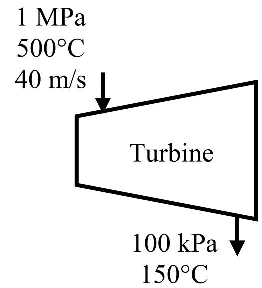
Analysis (*a*) There is only one inlet and one exit, and thus $\dot{m}_1 = \dot{m}_2 = \dot{m}$. We take the turbine as the system, which is a control volume since mass crosses the boundary. The energy balance for this steady-flow system can be expressed in the rate form as

$$\underbrace{\dot{E}_{in} - \dot{E}_{out}}_{\substack{\text{Rate of net energy transfer}\\ \text{by heat, work, and mass}}} = \underbrace{\Delta \dot{E}_{system}^{\nearrow 0\text{ (steady)}}}_{\substack{\text{Rate of change in internal, kinetic,}\\ \text{potential, etc. energies}}} = 0$$

$$\dot{E}_{in} = \dot{E}_{out}$$

$$\dot{m}\left(h_1 + \frac{V_1^2}{2}\right) = \dot{m}\left(h_2 + \frac{V_2^2}{2}\right) + \dot{W}_{out}$$

$$\dot{W}_{out} = \dot{m}\left(h_1 - h_2 + \frac{V_1^2 - V_2^2}{2}\right) = \dot{m}\left(c_p(T_1 - T_2) + \frac{V_1^2 - V_2^2}{2}\right)$$



The specific volume of air at the inlet and the mass flow rate are

$$\boldsymbol{v}_1 = \frac{RT_1}{P_1} = \frac{(0.287\text{ kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(500+273\text{ K})}{1000\text{ kPa}} = 0.2219\text{ m}^3/\text{kg}$$

$$\dot{m} = \frac{A_1 V_1}{\boldsymbol{v}_1} = \frac{(0.2\text{ m}^2)(40\text{ m/s})}{0.2219\text{ m}^3/\text{kg}} = \mathbf{36.06\text{ kg/s}}$$

Similarly at the outlet,

$$\boldsymbol{v}_2 = \frac{RT_2}{P_2} = \frac{(0.287\text{ kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(150+273\text{ K})}{100\text{ kPa}} = 1.214\text{ m}^3/\text{kg}$$

$$V_2 = \frac{\dot{m}\boldsymbol{v}_2}{A_2} = \frac{(36.06\text{ kg/s})(1.214\text{ m}^3/\text{kg})}{1\text{ m}^2} = 43.78\text{ m/s}$$

(*b*) Substituting into the energy balance equation gives

$$\dot{W}_{out} = \dot{m}\left(c_p(T_1 - T_2) + \frac{V_1^2 - V_2^2}{2}\right)$$

$$= (36.06\text{ kg/s})\left[(1.051\text{ kJ/kg}\cdot\text{K})(500-150)\text{K} + \frac{(40\text{ m/s})^2 - (43.78\text{ m/s})^2}{2}\left(\frac{1\text{ kJ/kg}}{1000\text{ m}^2/\text{s}^2}\right)\right]$$

$$= \mathbf{13{,}260\text{ kW}}$$

PROPRIETARY MATERIAL. © 2008 The McGraw-Hill Companies, Inc. Limited distribution permitted only to teachers and educators for course preparation. If you are a student using this Manual, you are using it without permission.