7-131 An expression for the COP of a completely reversible heat pump in terms of the thermal-energy reservoir temperatures, T_L and T_H is to be derived.

Assumptions The heat pump operates steadily.

Analysis Application of the first law to the completely reversible heat pump yields

$$W_{net,in} = Q_H - Q_L$$

This result may be used to reduce the coefficient of performance,

$$COP_{HP,rev} = \frac{Q_H}{W_{net,in}} = \frac{Q_H}{Q_H - Q_L} = \frac{1}{1 - Q_L / Q_H}$$

Since this heat pump is completely reversible, the thermodynamic definition of temperature tells us that,

$$\frac{Q_L}{Q_H} = \frac{T_L}{T_H}$$

When this is substituted into the COP expression, the result is

$$COP_{HP,rev} = \frac{1}{1 - T_L / T_H} = \frac{T_H}{T_H - T_L}$$