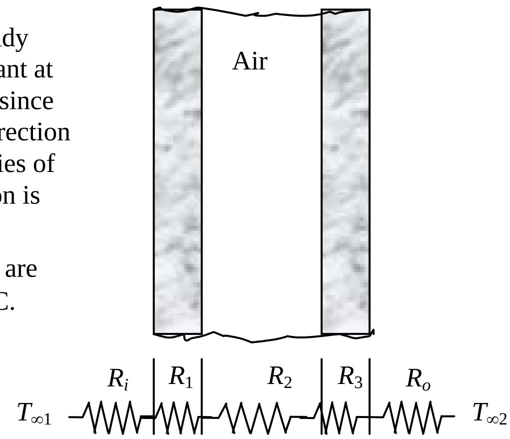
10-20 A double-pane window consists of two layers of glass separated by a stagnant air space. For specified indoors and outdoors temperatures, the rate of heat loss through the window and the inner surface temperature of the window are to be determined.



Assumptions **1** Heat transfer through the window is steady since the indoor and outdoor temperatures remain constant at the specified values. **2** Heat transfer is one-dimensional since any significant temperature gradients will exist in the direction from the indoors to the outdoors. **3** Thermal conductivities of the glass and air are constant. **4** Heat transfer by radiation is negligible.

Properties The thermal conductivity of the glass and air are given to be k_{glass} = 0.78 W/m·°C and k_{air} = 0.026 W/m·°C.

Analysis The area of the window and the individual resistances are

$$A = (1.2\text{ m}) \times (2\text{ m}) = 2.4\text{ m}^2$$

$$R_i = R_{conv,1} = \frac{1}{h_1 A} = \frac{1}{(10\text{ W/m}^2.°\text{C})(2.4\text{ m}^2)} = 0.0417\text{ °C/W}$$

$$R_1 = R_3 = R_{glass} = \frac{L_1}{k_1 A} = \frac{0.003\text{ m}}{(0.78\text{ W/m.°C})(2.4\text{ m}^2)} = 0.0016\text{ °C/W}$$

$$R_2 = R_{air} = \frac{L_2}{k_2 A} = \frac{0.012\text{ m}}{(0.026\text{ W/m.°C})(2.4\text{ m}^2)} = 0.1923\text{ °C/W}$$

$$R_o = R_{conv,2} = \frac{1}{h_2 A} = \frac{1}{(25\text{ W/m}^2.^\circ\text{C})(2.4\text{ m}^2)} = 0.0167\ ^\circ\text{C/W}$$

$$R_{total} = R_{conv,1} + 2R_1 + R_2 + R_{conv,2} = 0.0417 + 2(0.0016) + 0.1923 + 0.0167$$
$$= 0.2539\text{ °C/W}$$

The steady rate of heat transfer through window glass then becomes

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{total}} = \frac{[24 - (-5)]°\text{C}}{0.2539°\text{C/W}} = \mathbf{114\ W}$$

The inner surface temperature of the window glass can be determined from

$$\dot{Q} = \frac{T_{\infty 1} - T_1}{R_{conv,1}} \longrightarrow T_1 = T_{\infty 1} - \dot{Q}R_{conv,1} = 24^\circ\text{C} - (114\text{ W})(0.0417°\text{C/W}) = \mathbf{19.2°C}$$

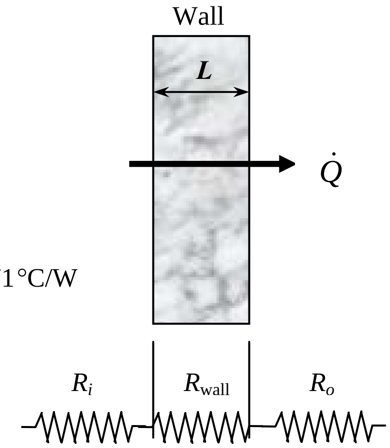
10-35 Two of the walls of a house have no windows while the other two walls have single- or double-pane windows. The average rate of heat transfer through each wall, and the amount of money this household will save per heating season by converting the single pane windows to double pane windows are to be determined.

Assumptions **1** Heat transfer through the window is steady since the indoor and outdoor temperatures remain constant at the specified values. **2** Heat transfer is one-dimensional since any significant temperature gradients will exist in the direction from the indoors to the outdoors. **3** Thermal conductivities of the glass and air are constant. **4** Heat transfer by radiation is disregarded.

Properties The thermal conductivities are given to be k = 0.026 W/m·°C for air, and 0.78 W/m·°C for glass.

Analysis The rate of heat transfer through each wall can be determined by applying thermal resistance network. The convection resistances at the inner and outer surfaces are common in all cases.

Walls without windows:



$$R_i = \frac{1}{h_i A} = \frac{1}{(7\ \text{W/m}^2\cdot{}^\circ\text{C})(10\times 4\ \text{m}^2)} = 0.003571\ ^\circ\text{C/W}$$

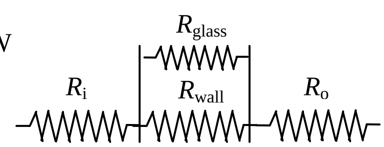
$$R_{\text{wall}} = \frac{L_{\text{wall}}}{kA} = \frac{R-value}{A} = \frac{2.31\ \text{m}^2\cdot{}^\circ\text{C/W}}{(10\times 4\ \text{m}^2)} = 0.05775\ ^\circ\text{C/W}$$

$$R_o = \frac{1}{h_o A} = \frac{1}{(18\ \text{W/m}^2\cdot{}^\circ\text{C})(10\times 4\ \text{m}^2)} = 0.001389^\circ\text{C/W}$$

$$R_{\text{total}} = R_i + R_{\text{wall}} + R_o = 0.003571 + 0.05775 + 0.001389 = 0.06271\ ^\circ\text{C/W}$$

Then $$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{total}} = \frac{(24-8)^\circ\text{C}}{0.06271^\circ\text{C/W}} = \mathbf{255.1\ W}$$

Wall with single pane windows:



$$R_i = \frac{1}{h_i A} = \frac{1}{(7\ \text{W/m}^2\cdot{}^\circ\text{C})(20\times 4\ \text{m}^2)} = 0.001786\ ^\circ\text{C/W}$$

$$R_{\text{wall}} = \frac{L_{\text{wall}}}{kA} = \frac{R-value}{A} = \frac{2.31\ \text{m}^2\cdot{}^\circ\text{C/W}}{(20\times 4) - 5(1.2\times 1.8)\ \text{m}^2} = 0.033382\ ^\circ\text{C/W}$$

$$R_{\text{glass}} = \frac{L_{\text{glass}}}{kA} = \frac{0.005\ \text{m}}{(0.78\ \text{W/m}^2\cdot{}^\text{o}\text{C})(1.2\times 1.8)\text{m}^2} = 0.002968\ ^\circ\text{C/W}$$

$$\frac{1}{R_{\text{eqv}}} = \frac{1}{R_{\text{wall}}} + 5\frac{1}{R_{\text{glass}}} = \frac{1}{0.033382} + 5\frac{1}{0.002968} \rightarrow R_{eqv} = 0.000583\ ^\text{o}\text{C/W}$$

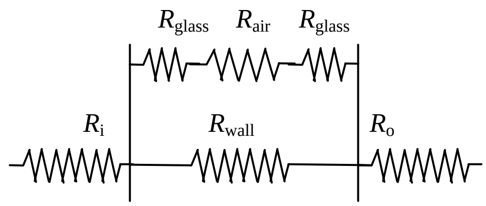
$$R_o = \frac{1}{h_o A} = \frac{1}{(18\ \text{W/m}^2\cdot{}^\circ\text{C})(20\times 4\ \text{m}^2)} = 0.000694\ ^\circ\text{C/W}$$

$$R_{\text{total}} = R_i + R_{\text{eqv}} + R_o = 0.001786 + 0.000583 + 0.000694 = 0.003063\ ^\circ\text{C/W}$$

Then

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{\text{total}}} = \frac{(24-8)^\circ\text{C}}{0.003063^\circ\text{C/W}} = \mathbf{5224\ W}$$

4th wall with double pane windows:



$$R_{\text{wall}} = \frac{L_{\text{wall}}}{kA} = \frac{R-value}{A} = \frac{2.31\,\text{m}^2\cdot{}^\circ\text{C/W}}{(20\times 4)-5(1.2\times 1.8)\text{m}^2} = 0.033382\,^\circ\text{C/W}$$

$$R_{\text{glass}} = \frac{L_{\text{glass}}}{kA} = \frac{0.005\,\text{m}}{(0.78\,\text{W/m}^2\cdot{}^\circ\text{C})(1.2\times 1.8)\text{m}^2} = 0.002968\,^\circ\text{C/W}$$

$$R_{\text{air}} = \frac{L_{\text{air}}}{kA} = \frac{0.015\,\text{m}}{(0.026\,\text{W/m}^2\cdot{}^\text{o}\,\text{C})(1.2\times 1.8)\text{m}^2} = 0.267094\,^\circ\text{C/W}$$

$$R_{\text{window}} = 2R_{\text{glass}} + R_{\text{air}} = 2\times 0.002968 + 0.267094 = 0.27303\,^\circ\text{C/W}$$

$$\frac{1}{R_{\text{eqv}}} = \frac{1}{R_{\text{wall}}} + 5\frac{1}{R_{\text{window}}} = \frac{1}{0.033382} + 5\frac{1}{0.27303} \longrightarrow R_{\text{eqv}} = 0.020717\,^\circ\text{C/W}$$

$$R_{\text{total}} = R_i + R_{\text{eqv}} + R_o = 0.001786 + 0.020717 + 0.000694 = 0.023197\,^\circ\text{C/W}$$

Then $\dot{Q} = \dfrac{T_{\infty 1} - T_{\infty 2}}{R_{\text{total}}} = \dfrac{(24-8)^\circ\text{C}}{0.023197^\circ\text{C/W}} = \mathbf{690\ W}$

The rate of heat transfer which will be saved if the single pane windows are converted to double pane windows is

$$\dot{Q}_{\text{save}} = \dot{Q}_{\substack{\text{single}\\\text{pane}}} - \dot{Q}_{\substack{\text{double}\\\text{pane}}} = 5224 - 690 = 4534\text{ W}$$

The amount of energy and money saved during a 7-month long heating season by switching from single pane to double pane windows become

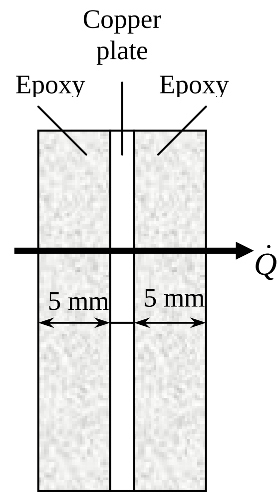
$$Q_{save} = \dot{Q}_{save}\Delta t = (4.534\text{ kW})(7\times 30\times 24\text{ h}) = 22{,}851\text{ kWh}$$

Money savings = (Energy saved)(Unit cost of energy) = (22,851 kWh)($0.08/kWh) = **$1828**

10-49 A thin copper plate is sandwiched between two epoxy boards. The error involved in the total thermal resistance of the plate if the thermal contact conductances are ignored is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat transfer is one-dimensional since the plate is large. **3** Thermal conductivities are constant.

Properties The thermal conductivities are given to be k = 386 W/m·°C for copper plates and k = 0.26 W/m·°C for epoxy boards. The contact conductance at the interface of copper-epoxy layers is given to be h_c = 6000 W/m^2·°C.



Analysis The thermal resistances of different layers for unit surface area of 1 m^2 are

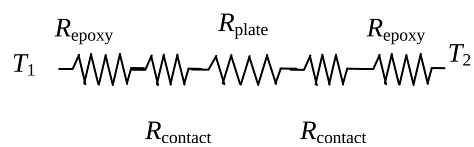
$$R_{\text{contact}} = \frac{1}{h_c A_c} = \frac{1}{(6000\ \text{W/m}^2 \cdot °\text{C})(1\ \text{m}^2)} = 0.00017\ °\text{C/W}$$

$$R_{\text{plate}} = \frac{L}{kA} = \frac{0.001\ \text{m}}{(386\ \text{W/m}\cdot°\text{C})(1\ \text{m}^2)} = 2.6\times10^{-6}\ °\text{C/W}$$

$$R_{\text{epoxy}} = \frac{L}{kA} = \frac{0.005\ \text{m}}{(0.26\ \text{W/m}\cdot°\text{C})(1\ \text{m}^2)} = 0.01923\ °\text{C/W}$$

The total thermal resistance is

$$R_{\text{total}} = 2R_{\text{contact}} + R_{\text{plate}} + 2R_{\text{epoxy}}$$
$$= 2\times0.00017 + 2.6\times10^{-6} + 2\times0.01923 = 0.03880\ °\text{C/W}$$



Then the percent error involved in the total thermal resistance of the plate if the thermal contact resistances are ignored is determined to be

$$\%\text{Error} = \frac{2R_{\text{contact}}}{R_{\text{total}}}\times100 = \frac{2\times0.00017}{0.03880}\times100 = \mathbf{0.88\%}$$

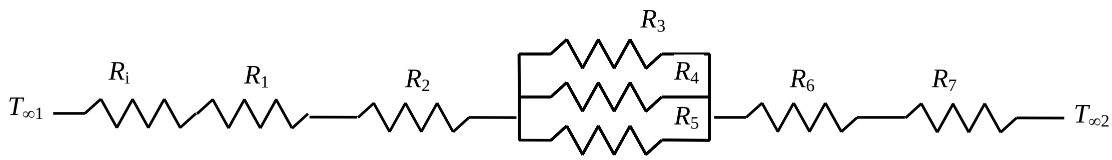
which is negligible.

10-54 A wall consists of horizontal bricks separated by plaster layers. There are also plaster layers on each side of the wall, and a rigid foam on the inner side of the wall. The rate of heat transfer through the wall is to be determined.

Assumptions **1** Heat transfer is steady since there is no indication of change with time. **2** Heat transfer through the wall is one-dimensional. **3** Thermal conductivities are constant. **4** Heat transfer by radiation is disregarded.

Properties The thermal conductivities are given to be k = 0.72 W/m·°C for bricks, k = 0.22 W/m·°C for plaster layers, and k = 0.026 W/m·°C for the rigid foam.

Analysis We consider 1 m deep and 0.33 m high portion of wall which is representative of the entire wall. The thermal resistance network and individual resistances are



$$R_i = R_{conv,1} = \frac{1}{h_1 A} = \frac{1}{(10\ \text{W/m}^2\cdot{}^\circ\text{C})(0.33\times 1\ \text{m}^2)} = 0.303\ {}^\circ\text{C/W}$$

$$R_1 = R_{foam} = \frac{L}{kA} = \frac{0.02\ \text{m}}{(0.026\ \text{W/m}\cdot{}^\circ\text{C})(0.33\times 1\ \text{m}^2)} = 2.33\ {}^\circ\text{C/W}$$

$$R_2 = R_6 = R_{\substack{plaster\\side}} = \frac{L}{kA} = \frac{0.02\ \text{m}}{(0.22\ \text{W/m}\cdot{}^\circ\text{C})(0.33\times 1\ \text{m}^2)} = 0.275\ {}^\circ\text{C/W}$$

$$R_3 = R_5 = R_{\substack{plaster\\center}} = \frac{L}{h_o A} = \frac{0.18\ \text{m}}{(0.22\ \text{W/m}\cdot{}^\circ\text{C})(0.015\times 1\ \text{m}^2)} = 54.55\ {}^\circ\text{C/W}$$

$$R_4 = R_{brick} = \frac{L}{kA} = \frac{0.18\ \text{m}}{(0.72\ \text{W/m}\cdot{}^\circ\text{C})(0.30\times 1\ \text{m}^2)} = 0.833\ {}^\circ\text{C/W}$$

$$R_o = R_{conv,2} = \frac{1}{h_2 A} = \frac{1}{(20\ \text{W/m}\cdot{}^\circ\text{C})(0.33\times 1\ \text{m}^2)} = 0.152\ {}^\circ\text{C/W}$$

$$\frac{1}{R_{mid}} = \frac{1}{R_3} + \frac{1}{R_4} + \frac{1}{R_5} = \frac{1}{54.55} + \frac{1}{0.833} + \frac{1}{54.55} \longrightarrow R_{mid} = 0.81\ {}^\circ\text{C/W}$$

$$R_{total} = R_i + R_1 + 2R_2 + R_{mid} + R_o = 0.303 + 2.33 + 2(0.275) + 0.81 + 0.152 = 4.145\ {}^\circ\text{C/W}$$

The steady rate of heat transfer through the wall per $0.33\ \text{m}^2$ is

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{total}} = \frac{[(22 - (-4)]{}^\circ\text{C}}{4.145{}^\circ\text{C/W}} = 6.27\ \text{W}$$

Then steady rate of heat transfer through the entire wall becomes

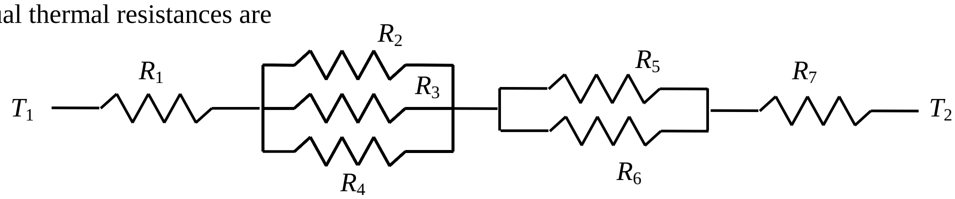
$$\dot{Q}_{total} = (6.27\ \text{W})\frac{(4\times 6)\text{m}^2}{0.33\ \text{m}^2} = \mathbf{456\ W}$$

10-59 A composite wall consists of several horizontal and vertical layers. The left and right surfaces of the wall are maintained at uniform temperatures. The rate of heat transfer through the wall, the interface temperatures, and the temperature drop across the section F are to be determined.

Assumptions **1** Heat transfer is steady since there is no indication of change with time. **2** Heat transfer through the wall is one-dimensional. **3** Thermal conductivities are constant. **4** Thermal contact resistances at the interfaces are disregarded.

Properties The thermal conductivities are given to be $k_A = k_F = 2$, $k_B = 8$, $k_C = 20$, $k_D = 15$, $k_E = 35$ W/m·°C.

Analysis (*a*) The representative surface area is $A = 0.12 \times 1 = 0.12\ \text{m}^2$. The thermal resistance network and the individual thermal resistances are



$$R_1 = R_A = \left(\frac{L}{kA}\right)_A = \frac{0.01\ \text{m}}{(2\ \text{W/m}\cdot°\text{C})(0.12\ \text{m}^2)} = 0.04\ °\text{C/W}$$

$$R_2 = R_4 = R_C = \left(\frac{L}{kA}\right)_C = \frac{0.05\ \text{m}}{(20\ \text{W/m}\cdot°\text{C})(0.04\ \text{m}^2)} = 0.06\ °\text{C/W}$$

$$R_3 = R_B = \left(\frac{L}{kA}\right)_B = \frac{0.05\ \text{m}}{(8\ \text{W/m}\cdot°\text{C})(0.04\ \text{m}^2)} = 0.16\ °\text{C/W}$$

$$R_5 = R_D = \left(\frac{L}{kA}\right)_D = \frac{0.1\ \text{m}}{(15\ \text{W/m}\cdot^{\text{o}}\text{C})(0.06\ \text{m}^2)} = 0.11\ °\text{C/W}$$

$$R_6 = R_E = \left(\frac{L}{kA}\right)_E = \frac{0.1\ \text{m}}{(35\ \text{W/m}\cdot°\text{C})(0.06\ \text{m}^2)} = 0.05\ ^{\text{o}}\text{C/W}$$

$$R_7 = R_F = \left(\frac{L}{kA}\right)_F = \frac{0.06\ \text{m}}{(2\ \text{W/m}\cdot°\text{C})(0.12\ \text{m}^2)} = 0.25\ °\text{C/W}$$

$$\frac{1}{R_{mid,1}} = \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4} = \frac{1}{0.06} + \frac{1}{0.16} + \frac{1}{0.06} \longrightarrow R_{mid,1} = 0.025\ °\text{C/W}$$

$$\frac{1}{R_{mid,2}} = \frac{1}{R_5} + \frac{1}{R_6} = \frac{1}{0.11} + \frac{1}{0.05} \longrightarrow R_{mid,2} = 0.034\ °\text{C/W}$$

$$R_{total} = R_1 + R_{mid,1} + R_{mid,2} + R_7 = 0.04 + 0.025 + 0.034 + 0.25 = 0.349\ °\text{C/W}$$

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{total}} = \frac{(300-100)°\text{C}}{0.349\ °\text{C/W}} = 572\ \text{W}\ \ \text{(for a } 0.12\ \text{m} \times 1\ \text{m section)}$$

Then steady rate of heat transfer through entire wall becomes

$$\dot{Q}_{total} = (572\ \text{W})\frac{(5\ \text{m})(8\ \text{m})}{0.12\ \text{m}^2} = \mathbf{1.91 \times 10^5\ W}$$

(*b*) The total thermal resistance between left surface and the point where the sections B, D, and E meet is

$$R_{total} = R_1 + R_{mid,1} = 0.04 + 0.025 = 0.065\ °\text{C/W}$$

Then the temperature at the point where the sections B, D, and E meet becomes

$$\dot{Q} = \frac{T_1 - T}{R_{total}} \longrightarrow T = T_1 - \dot{Q}R_{total} = 300°\text{C} - (572\ \text{W})(0.065\ °\text{C/W}) = \mathbf{263°C}$$

(*c*) The temperature drop across the section F can be determined from

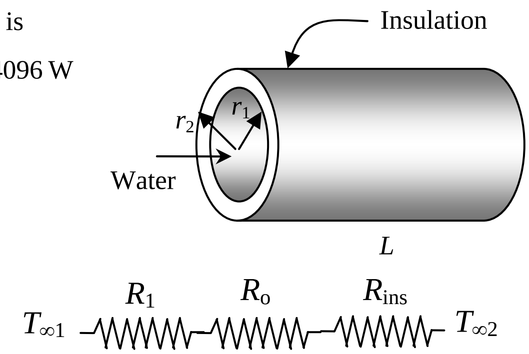
$$\dot{Q} = \frac{\Delta T}{R_F} \rightarrow \Delta T = \dot{Q}R_F = (572\ \text{W})(0.25\ °\text{C/W}) = \mathbf{143°C}$$

10-69 Chilled water is flowing inside a pipe. The thickness of the insulation needed to reduce the temperature rise of water to one-fourth of the original value is to be determined.

Assumptions **1** Heat transfer is steady since there is no indication of any change with time. **2** Heat transfer is one-dimensional since there is thermal symmetry about the centerline and no variation in the axial direction. **3** Thermal conductivities are constant. **4** The thermal contact resistance at the interface is negligible.

Properties The thermal conductivity is given to be k = 0.05 W/m·°C for insulation.

Analysis The rate of heat transfer without the insulation is



$$\dot{Q}_{\text{old}} = \dot{m}c_p\Delta T = (0.98\text{ kg/s})(4180\text{ J/kg}\cdot°\text{C})(8-7)°\text{C} = 4096\text{ W}$$

The total resistance in this case is

$$\dot{Q}_{\text{old}} = \frac{T_\infty - T_w}{R_{\text{total}}}$$

$$4096\text{ W} = \frac{(30-7.5)°\text{C}}{R_{\text{total}}} \longrightarrow R_{\text{total}} = 0.005493°\text{C/W}$$

The convection resistance on the outer surface is

$$R_o = \frac{1}{h_o A_o} = \frac{1}{(9\text{ W/m}^2\cdot°\text{C})\pi(0.05\text{ m})(150\text{ m})} = 0.004716\text{ °C/W}$$

The rest of thermal resistances are due to convection resistance on the inner surface and the resistance of the pipe and it is determined from

$$R_1 = R_{\text{total}} - R_{\text{o}} = 0.005493 - 0.004716 = 0.0007769\text{ °C/W}$$

The rate of heat transfer with the insulation is

$$\dot{Q}_{\text{new}} = \dot{m}c_p\Delta T = (0.98\text{ kg/s})(4180\text{ J/kg}\cdot°\text{C})(0.25°\text{C}) = 1024\text{ W}$$

The total thermal resistance with the insulation is

$$\dot{Q}_{\text{new}} = \frac{T_\infty - T_w}{R_{\text{total,new}}} \longrightarrow 1024\text{ W} = \frac{[30-(7+7.25)/2)]°\text{C}}{R_{\text{total,new}}} \longrightarrow R_{\text{total,new}} = 0.02234°\text{C/W}$$

It is expressed by

$$R_{\text{total,new}} = R_1 + R_{\text{o,new}} + R_{\text{ins}} = R_1 + \frac{1}{h_o A_o} + \frac{\ln(D_2/D_1)}{2\pi k_{\text{ins}} L}$$

$$0.02234°\text{C/W} = 0.0007769 + \frac{1}{(9\text{ W/m}^2\cdot°\text{C})\pi D_2(150\text{ m})} + \frac{\ln(D_2/0.05)}{2\pi(0.05\text{ W/m}\cdot°\text{C})(150\text{ m})}$$

Solving this equation by trial-error or by using an equation solver such as EES, we obtain

$$D_2 = 0.1265\text{ m}$$

Then the required thickness of the insulation becomes

$$t_{\text{ins}} = (D_2 - D_1)/2 = (0.05 - 0.1265)/2 = 0.0382\text{ m} = \mathbf{3.8\text{ cm}}$$

10-71 A spherical container filled with iced water is subjected to convection and radiation heat transfer at its outer surface. The rate of heat transfer and the amount of ice that melts per day are to be determined.
Assumptions **1** Heat transfer is steady since the specified thermal conditions at the boundaries do not change with time. **2** Heat transfer is one-dimensional since there is thermal symmetry about the midpoint. **3** Thermal conductivity is constant.
Properties The thermal conductivity of steel is given to be k = 15 W/m·°C. The heat of fusion of water at 1 atm is $h_{if} = 333.7$ kJ/kg. The outer surface of the tank is black and thus its emissivity is ε = 1.

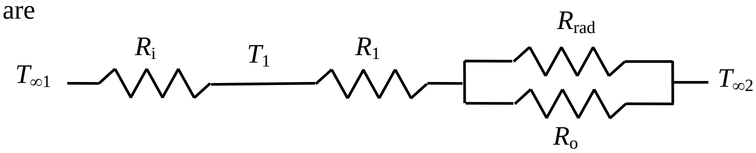
Analysis (*a*) The inner and the outer surface areas of sphere are

$$A_i = \pi D_i^2 = \pi(8\text{ m})^2 = 201.06\text{ m}^2 \qquad A_o = \pi D_o^2 = \pi(8.03\text{ m})^2 = 202.57\text{ m}^2$$

We assume the outer surface temperature T_2 to be 5°C after comparing convection heat transfer coefficients at the inner and the outer surfaces of the tank. With this assumption, the radiation heat transfer coefficient can be determined from

$$\begin{aligned} h_{rad} &= \varepsilon\sigma(T_2^{\,2} + T_{surr}^{\,2})(T_2 + T_{surr}) \\ &= 1(5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4)[(273+5\text{ K})^2 + (273+25\text{ K})^2][(273+25\text{ K})(273+5\text{ K})] \\ &= 5.424\text{ W/m}^2.\text{K} \end{aligned}$$

The individual thermal resistances are



$$R_{conv,i} = \frac{1}{h_i A} = \frac{1}{(80\text{ W/m}^2\cdot°\text{C})(201.06\text{ m}^2)} = 0.000062\text{ °C/W}$$

$$R_1 = R_{sphere} = \frac{r_2 - r_1}{4\pi k r_1 r_2} = \frac{(4.015-4.0)\text{ m}}{4\pi(15\text{ W/m}\cdot°\text{C})(4.015\text{ m})(4.0\text{ m})} = 0.000005\text{ °C/W}$$

$$R_{conv,o} = \frac{1}{h_o A} = \frac{1}{(10\text{ W/m}^2\cdot°\text{C})(202.57\text{ m}^2)} = 0.000494\text{ °C/W}$$

$$R_{rad} = \frac{1}{h_{rad} A} = \frac{1}{(5.424\text{ W/m}^2\cdot°\text{C})(202.57\text{ m}^2)} = 0.000910\text{ °C/W}$$

$$\frac{1}{R_{eqv}} = \frac{1}{R_{conv,o}} + \frac{1}{R_{rad}} = \frac{1}{0.000494} + \frac{1}{0.000910} \longrightarrow R_{eqv} = 0.000320\text{ °C/W}$$

$$R_{total} = R_{conv,i} + R_1 + R_{eqv} = 0.000062 + 0.000005 + 0.000320 = 0.000387\text{ °C/W}$$

Then the steady rate of heat transfer to the iced water becomes

$$\dot{Q} = \frac{T_{\infty1} - T_{\infty2}}{R_{total}} = \frac{(25-0)°\text{C}}{0.000387\text{ °C/W}} = \mathbf{64{,}600\text{ W}}$$

(*b*) The total amount of heat transfer during a 24-hour period and the amount of ice that will melt during this period are

$$Q = \dot{Q}\Delta t = (64.600\text{ kJ/s})(24\times3600\text{ s}) = 5.581\times10^6\text{ kJ}$$

$$m_{ice} = \frac{Q}{h_{if}} = \frac{5.581\times10^6\text{ kJ}}{333.7\text{ kJ/kg}} = \mathbf{16{,}730\text{ kg}}$$

Check: The outer surface temperature of the tank is

$$\dot{Q} = h_{conv+rad} A_o (T_{\infty1} - T_s)$$

$$\rightarrow T_s = T_{\infty1} - \frac{\dot{Q}}{h_{conv+rad} A_o} = 25°\text{C} - \frac{64{,}600\text{ W}}{(10+5.424\text{ W/m}^2\cdot°\text{C})(202.57\text{ m}^2)} = 4.3°\text{C}$$

which is very close to the assumed temperature of 5°C for the outer surface temperature used in the evaluation of the radiation heat transfer coefficient. Therefore, there is no need to repeat the calculations.

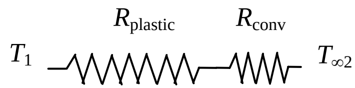
10-92 An electric wire is tightly wrapped with a 1-mm thick plastic cover. The interface temperature and the effect of doubling the thickness of the plastic cover on the interface temperature are to be determined.

Assumptions **1** Heat transfer is steady since there is no indication of any change with time. **2** Heat transfer is one-dimensional since there is thermal symmetry about the centerline and no variation in the axial direction. **3** Thermal properties are constant. **4** The thermal contact resistance at the interface is negligible. **5** Heat transfer coefficient accounts for the radiation effects, if any.

Properties The thermal conductivity of plastic cover is given to be k = 0.15 W/m·°C.

Analysis In steady operation, the rate of heat transfer from the wire is equal to the heat generated within the wire,

$$\dot{Q} = \dot{W}_e = \mathbf{V}I = (8\ \text{V})(13\ \text{A}) = 104\ \text{W}$$



The total thermal resistance is

$$R_{\text{conv}} = \frac{1}{h_o A_o} = \frac{1}{(24\ \text{W/m}^2.^\circ\text{C})[\pi(0.0042\ \text{m})(10\ \text{m})]} = 0.3158\ ^\circ\text{C/W}$$

$$R_{\text{plastic}} = \frac{\ln(r_2 / r_1)}{2\pi k L} = \frac{\ln(2.1/1.1)}{2\pi(0.15\ \text{W/m.}^\circ\text{C})(10\ \text{m})} = 0.0686\ ^\circ\text{C/W}$$

$$R_{\text{total}} = R_{\text{conv}} + R_{\text{plastic}} = 0.3158 + 0.0686 = 0.3844\ ^\circ\text{C/W}$$

Then the interface temperature becomes

$$\dot{Q} = \frac{T_1 - T_{\infty 2}}{R_{\text{total}}} \longrightarrow T_1 = T_\infty + \dot{Q}R_{\text{total}} = 30^\circ\text{C} + (104\ \text{W})(0.3844\ ^\circ\text{C/W}) = \mathbf{70.0^\circ C}$$

The critical radius of plastic insulation is

$$r_{cr} = \frac{k}{h} = \frac{0.15\ \text{W/m.}^\circ\text{C}}{24\ \text{W/m}^2.^\circ\text{C}} = 0.00625\ \text{m} = 6.25\ \text{mm}$$

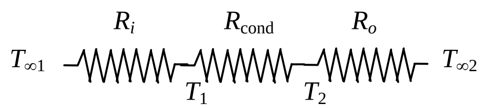
Doubling the thickness of the plastic cover will increase the outer radius of the wire to 3 mm, which is less than the critical radius of insulation. Therefore, doubling the thickness of plastic cover will increase the rate of heat loss and decrease the interface temperature.

10-126 Two cast iron steam pipes are connected to each other through two 1-cm thick flanges exposed to cold ambient air. The average outer surface temperature of the pipe, the fin efficiency, the rate of heat transfer from the flanges, and the equivalent pipe length of the flange for heat transfer are to be determined.

Assumptions **1** Steady operating conditions exist. **2** The temperature along the flanges (fins) varies in one direction only (normal to the pipe). **3** The heat transfer coefficient is constant and uniform over the entire fin surface. **4** The thermal properties of the fins are constant. **5** The heat transfer coefficient accounts for the effect of radiation from the fins.

Properties The thermal conductivity of the cast iron is given to be k = 52 W/m·°C.

Analysis (a) We treat the flanges as fins. The individual thermal resistances are



$$A_i = \pi D_i L = \pi(0.092\text{ m})(6\text{ m}) = 1.73\text{ m}^2$$

$$A_o = \pi D_o L = \pi(0.1\text{ m})(6\text{ m}) = 1.88\text{ m}^2$$

$$R_\text{i} = \frac{1}{h_i A_i} = \frac{1}{(180\text{ W/m}^2.°\text{C})(1.73\text{ m}^2)} = 0.0032\text{ °C/W}$$

$$R_\text{cond} = \frac{\ln(r_2/r_1)}{2\pi kL} = \frac{\ln(5/4.6)}{2\pi(52\text{ W/m.°C})(6\text{ m})} = 0.00004\text{ °C/W}$$

$$R_\text{o} = \frac{1}{h_o A_o} = \frac{1}{(25\text{ W/m}^2.°\text{C})(1.88\text{ m}^2)} = 0.0213\text{ °C/W}$$

$$R_\text{total} = R_\text{i} + R_\text{cond} + R_\text{o} = 0.0032 + 0.00004 + 0.0213 = 0.0245\text{ °C/W}$$

The rate of heat transfer and average outer surface temperature of the pipe are

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_\text{total}} = \frac{(200-12)°\text{C}}{0.0245\text{ °C}} = 7673\text{ W}$$

$$\dot{Q} = \frac{T_2 - T_{\infty 2}}{R_\text{o}} \longrightarrow T_2 = T_{\infty 2} + \dot{Q}R_\text{o} = 12\text{ °C} + (7673\text{ W})(0.0213\text{ °C/W}) = \mathbf{175.4°C}$$

(b) The fin efficiency can be determined from (Fig. 10-43)

$$\left.\begin{aligned} &\frac{r_2 + \frac{t}{2}}{r_1} = \frac{0.1 + \frac{0.02}{2}}{0.05} = 2.2 \\ &\xi = L_c^{3/2}\left(\frac{h}{kA_p}\right)^{1/2} = \left(L + \frac{t}{2}\right)\sqrt{\frac{h}{kt}} = \left(0.05\text{ m} + \frac{0.02}{2}\text{ m}\right)\sqrt{\frac{25\text{ W/m}^2\,^\text{o}\text{C}}{(52\text{ W/m}^\text{o}\text{C})(0.02\text{ m})}} = 0.29 \end{aligned}\right\}\eta_\text{fin} = 0.88$$

$$A_\text{fin} = 2\pi(r_2^2 - r_1^2) + 2\pi r_2 t = 2\pi[(0.1\text{ m})^2 - (0.05\text{ m})^2] + 2\pi(0.1\text{ m})(0.02\text{ m}) = 0.0597\text{ m}^2$$

The heat transfer rate from the flanges is

$$\begin{aligned} \dot{Q}_\text{finned} &= \eta_\text{fin}\dot{Q}_\text{fin,max} = \eta_\text{fin} h A_\text{fin}(T_b - T_\infty) \\ &= 0.88(25\text{ W/m}^2.°\text{C})(0.0597\text{ m}^2)(175.4-12)°\text{C} = \mathbf{215\ W} \end{aligned}$$

(c) A 6-m long section of the steam pipe is losing heat at a rate of 7673 W or 7673/6 = 1279 W per m length. Then for heat transfer purposes the flange section is equivalent to

$$\text{Equivalent length} = \frac{215\text{ W}}{1279\text{ W/m}} = 0.168\text{ m} = \mathbf{16.8\ cm}$$

Therefore, the flange acts like a fin and increases the heat transfer by 16.8/2 = 8.4 times.

10-143 A cylindrical tank filled with liquid propane at 1 atm is exposed to convection and radiation. The time it will take for the propane to evaporate completely as a result of the heat gain from the surroundings for the cases of no insulation and 5-cm thick glass wool insulation are to be determined.

Assumptions **1** Heat transfer is steady. **2** Heat transfer is one-dimensional. **3** The combined heat transfer coefficient is constant and uniform over the entire surface. **4** The temperature of the thin-shelled spherical tank is said to be nearly equal to the temperature of the propane inside, and thus thermal resistance of the tank and the internal convection resistance are negligible.

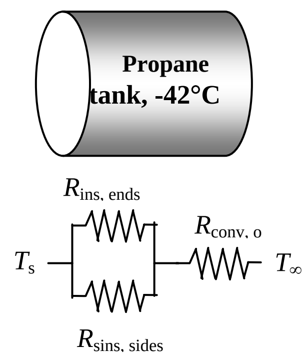
Properties The heat of vaporization and density of liquid propane at 1 atm are given to be 425 kJ/kg and 581 kg/m^3, respectively. The thermal conductivity of glass wool insulation is given to be k = 0.038 W/m·°C.

Analysis (a) If the tank is not insulated, the heat transfer rate is determined to be

$$A_{\text{tank}} = \pi DL + 2\pi(\pi D^2/4) = \pi(1.2\text{ m})(6\text{ m}) + 2\pi(1.2\text{ m})^2/4 = 24.88\text{ m}^2$$

$$\dot{Q} = hA_{\text{tank}}(T_{\infty 1} - T_{\infty 2}) = (25\text{ W/m}^2.°\text{C})(24.88\text{ m}^2)[30-(-42)]°\text{C} = 44{,}787\text{ W}$$

The volume of the tank and the mass of the propane are



$$V = \pi r^2 L = \pi(0.6\text{ m})^2(6\text{ m}) = 6.786\text{ m}^3$$

$$m = \rho V = (581\text{ kg/m}^3)(6.786\text{ m}^3) = 3942.6\text{ kg}$$

The rate of vaporization of propane is

$$\dot{Q} = \dot{m}h_{fg} \rightarrow \dot{m} = \frac{\dot{Q}}{h_{fg}} = \frac{44.787\text{ kJ/s}}{425\text{ kJ/kg}} = 0.1054\text{ kg/s}$$

Then the time period for the propane tank to empty becomes

$$\Delta t = \frac{m}{\dot{m}} = \frac{3942.6\text{ kg}}{0.1054\text{ kg/s}} = 37{,}413\text{ s} = \mathbf{10.4\ hours}$$

(b) We now repeat calculations for the case of insulated tank with 5-cm thick insulation.

$$A_o = \pi DL + 2\pi(\pi D^2/4) = \pi(1.3\text{ m})(6\text{ m}) + 2\pi(1.3\text{ m})^2/4 = 27.16\text{ m}^2$$

$$R_{\text{conv,o}} = \frac{1}{h_o A_o} = \frac{1}{(25\text{ W/m}^2.°\text{C})(27.16\text{ m}^2)} = 0.001473\text{ °C/W}$$

$$R_{\text{insulation,side}} = \frac{\ln(r_2/r_1)}{2\pi kL} = \frac{\ln(65/60)}{2\pi(0.038\text{ W/m.°C})(6\text{ m})} = 0.05587\text{ °C/W}$$

$$R_{\text{insulation,ends}} = 2\frac{L}{kA_{avg}} = \frac{2\times 0.05\text{ m}}{(0.038\text{ W/m.°C})[\pi(1.25\text{ m})^2/4]} = 2.1444\text{ °C/W}$$

Noting that the insulation on the side surface and the end surfaces are in parallel, the equivalent resistance for the insulation is determined to be

$$R_{\text{insulation}} = \left(\frac{1}{R_{\text{insulation,side}}} + \frac{1}{R_{\text{insulation,ends}}}\right)^{-1} = \left(\frac{1}{0.05587\text{ °C/W}} + \frac{1}{2.1444\text{ °C/W}}\right)^{-1} = 0.05445\text{ °C/W}$$

Then the total thermal resistance and the heat transfer rate become

$$R_{\text{total}} = R_{\text{conv,o}} + R_{\text{insulation}} = 0.001473 + 0.05445 = 0.05592\text{ °C/W}$$

$$\dot{Q} = \frac{T_\infty - T_s}{R_{\text{total}}} = \frac{[30-(-42)]°\text{C}}{0.05592\text{ °C/W}} = 1288\text{ W}$$

Then the time period for the propane tank to empty becomes

$$\dot{Q} = \dot{m}h_{fg} \rightarrow \dot{m} = \frac{\dot{Q}}{h_{fg}} = \frac{1.288\text{ kJ/s}}{425\text{ kJ/kg}} = 0.003031\text{ kg/s}$$

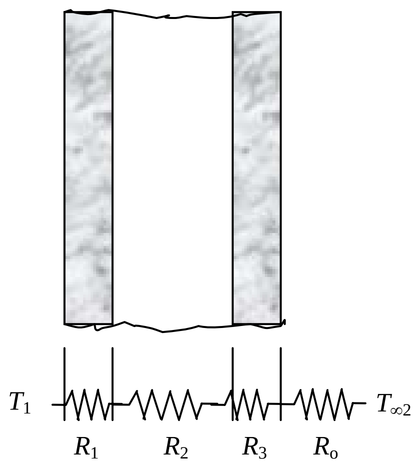
$$\Delta t = \frac{m}{\dot{m}} = \frac{3942.6\text{ kg}}{0.003031\text{ kg/s}} = 1.301\times 10^6\text{ s} = 361.4\text{ hours} = \mathbf{15.1\ days}$$

10-157 A wall constructed of three layers is considered. The rate of hat transfer through the wall and temperature drops across the plaster, brick, covering, and surface-ambient air are to be determined.

Assumptions **1** Heat transfer is steady. **2** Heat transfer is one-dimensional. **3** Thermal conductivities are constant. **4** Heat transfer by radiation is accounted for in the heat transfer coefficient.

Properties The thermal conductivities of the plaster, brick, and covering are given to be k = 0.72 W/m·°C, k = 0.36 W/m·°C, k = 1.40 W/m·°C, respectively.

Analysis The surface area of the wall and the individual resistances are



$$A = (6\text{ m}) \times (2.8\text{ m}) = 16.8\text{ m}^2$$

$$R_1 = R_{\text{plaster}} = \frac{L_1}{k_1 A} = \frac{0.01\text{ m}}{(0.36\text{ W/m.°C})(16.8\text{ m}^2)} = 0.00165\text{ °C/W}$$

$$R_2 = R_{\text{brick}} = \frac{L_2}{k_2 A} = \frac{0.20\text{ m}}{(0.72\text{ W/m.°C})(16.8\text{ m}^2)} = 0.01653\text{ °C/W}$$

$$R_3 = R_{\text{covering}} = \frac{L_3}{k_3 A} = \frac{0.02\text{ m}}{(1.4\text{ W/m.°C})(16.8\text{ m}^2)} = 0.00085\text{ °C/W}$$

$$R_o = R_{\text{conv,2}} = \frac{1}{h_2 A} = \frac{1}{(17\text{ W/m}^2.\text{°C})(16.8\text{ m}^2)} = 0.00350\text{°C/W}$$

$$\begin{aligned} R_{\text{total}} &= R_1 + R_2 + R_3 + R_{\text{conv,2}} \\ &= 0.00165 + 0.01653 + 0.00085 + 0.00350 = 0.02253\text{ °C/W} \end{aligned}$$

The steady rate of heat transfer through the wall then becomes

$$\dot{Q} = \frac{T_1 - T_{\infty 2}}{R_{\text{total}}} = \frac{(23-8)\text{°C}}{0.02253\text{°C/W}} = \mathbf{665.8\ W}$$

The temperature drops are

$$\Delta T_{\text{plaster}} = \dot{Q}R_{\text{plaster}} = (665.8\text{ W})(0.00165\text{°C/W}) = \mathbf{1.1\ °C}$$

$$\Delta T_{\text{brick}} = \dot{Q}R_{\text{brick}} = (665.8\text{ W})(0.01653\text{°C/W}) = \mathbf{11.0\ °C}$$

$$\Delta T_{\text{covering}} = \dot{Q}R_{\text{covering}} = (665.8\text{ W})(0.00085\text{°C/W}) = \mathbf{0.6\ °C}$$

$$\Delta T_{\text{conv}} = \dot{Q}R_{\text{conv}} = (665.8\text{ W})(0.00350\text{°C/W}) = \mathbf{2.3\ °C}$$

10-162 A 6-m-diameter spherical tank filled with liquefied natural gas (LNG) at -160°C is exposed to ambient air. The time for the LNG temperature to rise to -150°C is to be determined.

Assumptions **1** Heat transfer can be considered to be steady since the specified thermal conditions at the boundaries do not change with time significantly. **2** Heat transfer is one-dimensional since there is thermal symmetry about the midpoint. **3** Radiation is accounted for in the combined heat transfer coefficient. **3** The combined heat transfer coefficient is constant and uniform over the entire surface. **4** The temperature of the thin-shelled spherical tank is said to be nearly equal to the temperature of the LNG inside, and thus thermal resistance of the tank and the internal convection resistance are negligible.

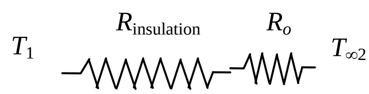
Properties The density and specific heat of LNG are given to be 425 kg/m^3 and 3.475 kJ/kg·°C, respectively. The thermal conductivity of super insulation is given to be k = 0.00008 W/m·°C.

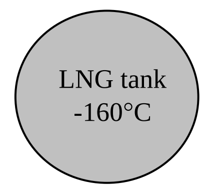
Analysis The inner and outer surface areas of the insulated tank and the volume of the LNG are

$$A_1 = \pi D_1^{\,2} = \pi(4\text{ m})^2 = 50.27\text{ m}^2$$

$$A_2 = \pi D_2^{\,2} = \pi(4.10\text{ m})^2 = 52.81\text{ m}^2$$

$$V_1 = \pi D_1^{\,3}/6 = \pi(4\text{ m})^3/6 = 33.51\text{ m}^3$$





The rate of heat transfer to the LNG is

$$R_{\text{insulation}} = \frac{r_2 - r_1}{4\pi k r_1 r_2} = \frac{(2.05 - 2.0)\text{ m}}{4\pi(0.00008\text{ W/m.°C})(2.0\text{ m})(2.05\text{ m})} = 12.13071\text{ °C/W}$$

$$R_o = \frac{1}{h_o A} = \frac{1}{(22\text{ W/m}^2\text{.°C})(52.81\text{ m}^2)} = 0.00086\text{ °C/W}$$

$$R_{total} = R_o + R_{insulation} = 0.00086 + 12.13071 = 12.13157\text{ °C/W}$$

$$\dot{Q} = \frac{T_{\infty 2} - T_{\text{LNG}}}{R_{\text{total}}} = \frac{[24 - (-155)]\text{°C}}{12.13157\text{ °C/W}} = 14.75\text{ W}$$

We used average LNG temperature in heat transfer rate calculation. The amount of heat transfer to increase the LNG temperature from -160°C to -150°C is

$$m = \rho V_1 = (425\text{ kg/m}^3)(33.51\text{ m}^3) = 14{,}242\text{ kg}$$

$$Q = mc_p\Delta T = (14{,}242\text{ kg})(3.475\text{ kJ/kg.°C})\big[(-150) - (-160)\text{°C}\big] = 4.95\times 10^5\text{ kJ}$$

Assuming that heat will be lost from the LNG at an average rate of 15.17 W, the time period for the LNG temperature to rise to -150°C becomes

$$\Delta t = \frac{Q}{\dot{Q}} = \frac{4.95\times 10^5\text{ kJ}}{0.01475\text{ kW}} = 3.355\times 10^7\text{ s} = 9320\text{ h} = \mathbf{388\ days}$$