11-14 The temperature of a gas stream is to be measured by a thermocouple. The time it takes to register 99 percent of the initial ΔT is to be determined.

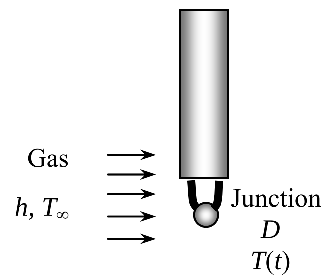
Assumptions **1** The junction is spherical in shape with a diameter of D = 0.0012 m. **2** The thermal properties of the junction are constant. **3** The heat transfer coefficient is constant and uniform over the entire surface. **4** Radiation effects are negligible. **5** The Biot number is Bi < 0.1 so that the lumped system analysis is applicable (this assumption will be verified).

Properties The properties of the junction are given to be $k = 35\ \text{W/m.°C}$, $\rho = 8500\ \text{kg/m}^3$, and $c_p = 320\ \text{J/kg.°C}$.

Analysis The characteristic length of the junction and the Biot number are

$$L_c = \frac{V}{A_{\text{surface}}} = \frac{\pi D^3/6}{\pi D^2} = \frac{D}{6} = \frac{0.0012\ \text{m}}{6} = 0.0002\ \text{m}$$

$$Bi = \frac{hL_c}{k} = \frac{(90\ \text{W/m}^2.°\text{C})(0.0002\ \text{m})}{(35\ \text{W/m.°C})} = 0.00051 < 0.1$$



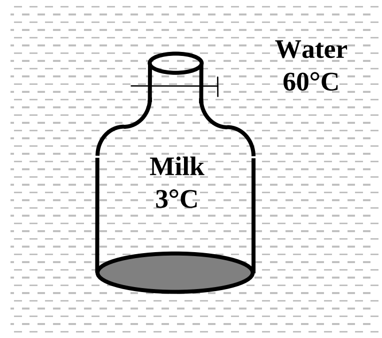
Since $Bi < 0.1$, the lumped system analysis is applicable. Then the time period for the thermocouple to read 99% of the initial temperature difference is determined from

$$\frac{T(t)-T_\infty}{T_i - T_\infty} = 0.01$$

$$b = \frac{hA}{\rho c_p V} = \frac{h}{\rho c_p L_c} = \frac{90\ \text{W/m}^2.°\text{C}}{(8500\ \text{kg/m}^3)(320\ \text{J/kg.°C})(0.0002\ \text{m})} = 0.1654\ \text{s}^{-1}$$

$$\frac{T(t)-T_\infty}{T_i - T_\infty} = e^{-bt} \longrightarrow 0.01 = e^{-(0.1654\ \text{s}^{-1})t} \longrightarrow t = \mathbf{27.8\ s}$$

11-17 Milk in a thin-walled glass container is to be warmed up by placing it into a large pan filled with hot water. The warming time of the milk is to be determined.



Assumptions **1** The glass container is cylindrical in shape with a radius of r_0 = 3 cm. **2** The thermal properties of the milk are taken to be the same as those of water. **3** Thermal properties of the milk are constant at room temperature. **4** The heat transfer coefficient is constant and uniform over the entire surface. **5** The Biot number in this case is large (much larger than 0.1). However, the lumped system analysis is still applicable since the milk is stirred constantly, so that its temperature remains uniform at all times.

Properties The thermal conductivity, density, and specific heat of the milk at 20°C are k = 0.598 W/m.°C, ρ = 998 kg/m^3, and c_p = 4.182 kJ/kg.°C (Table A-15).

Analysis The characteristic length and Biot number for the glass of milk are

$$L_c = \frac{V}{A_s} = \frac{\pi r_o^2 L}{2\pi r_o L + 2\pi r_o^2} = \frac{\pi (0.03\text{ m})^2 (0.07\text{ m})}{2\pi (0.03\text{ m})(0.07\text{ m}) + 2\pi (0.03\text{ m})^2} = 0.01050\text{ m}$$

$$Bi = \frac{hL_c}{k} = \frac{(120\text{ W/m}^2.°\text{C})(0.0105\text{ m})}{(0.598\text{ W/m.°C})} = 2.107 > 0.1$$

For the reason explained above we can use the lumped system analysis to determine how long it will take for the milk to warm up to 38°C:

$$b = \frac{hA_s}{\rho c_p V} = \frac{h}{\rho c_p L_c} = \frac{120\text{ W/m}^2.°\text{C}}{(998\text{ kg/m}^3)(4182\text{ J/kg.°C})(0.0105\text{ m})} = 0.002738\text{ s}^{-1}$$

$$\frac{T(t) - T_\infty}{T_i - T_\infty} = e^{-bt} \longrightarrow \frac{38-60}{3-60} = e^{-(0.002738\text{ s}^{-1})t} \longrightarrow t = \mathbf{348\ s = 5.8\ min}$$

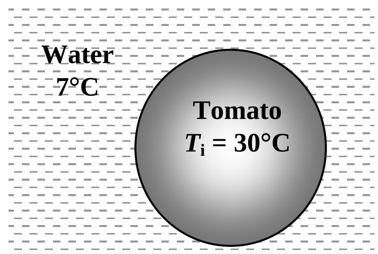
Therefore, it will take about 6 minutes to warm the milk from 3 to 38°C.

11-36 Tomatoes are placed into cold water to cool them. The heat transfer coefficient and the amount of heat transfer are to be determined.

Assumptions **1** The tomatoes are spherical in shape. **2** Heat conduction in the tomatoes is one-dimensional because of symmetry about the midpoint. **3** The thermal properties of the tomatoes are constant. **4** The heat transfer coefficient is constant and uniform over the entire surface. **5** The Fourier number is $\tau > 0.2$ so that the one-term approximate solutions (or the transient temperature charts) are applicable (this assumption will be verified).

Properties The properties of the tomatoes are given to be $k = 0.59$ W/m.°C, $\alpha = 0.141\times10^{-6}$ m^2/s, $\rho = 999$ kg/m^3 and $c_p = 3.99$ kJ/kg.°C.

Analysis The Fourier number is



$$\tau = \frac{\alpha t}{r_o^2} = \frac{(0.141\times10^{-6}\ \text{m}^2/\text{s})(2\times3600\ \text{s})}{(0.04\ \text{m})^2} = 0.635$$

which is greater than 0.2. Therefore one-term solution is applicable. The ratio of the dimensionless temperatures at the surface and center of the tomatoes are

$$\frac{\theta_{s,\text{sph}}}{\theta_{0,\text{sph}}} = \frac{\dfrac{T_s - T_\infty}{T_i - T_\infty}}{\dfrac{T_0 - T_\infty}{T_i - T_\infty}} = \frac{T_s - T_\infty}{T_0 - T_\infty} = \frac{A_1 e^{-\lambda_1^2\tau}\dfrac{\sin(\lambda_1)}{\lambda_1}}{A_1 e^{-\lambda_1^2\tau}} = \frac{\sin(\lambda_1)}{\lambda_1}$$

Substituting,

$$\frac{7.1-7}{10-7} = \frac{\sin(\lambda_1)}{\lambda_1} \longrightarrow \lambda_1 = 3.0401$$

From Table 11-2, the corresponding Biot number and the heat transfer coefficient are

$$\text{Bi} = 31.1$$

$$\text{Bi} = \frac{hr_o}{k} \longrightarrow h = \frac{kBi}{r_o} = \frac{(0.59\ \text{W/m.°C})(31.1)}{(0.04\ \text{m})} = \mathbf{459\ W/m^2.°C}$$

The maximum amount of heat transfer is

$$m = 8\rho V = 8\rho\pi D^3/6 = 8(999\ \text{kg/m}^3)[\pi(0.08\ \text{m})^3/6] = 2.143\ \text{kg}$$

$$Q_{\max} = mc_p[T_i - T_\infty] = (2.143\ \text{kg})(3.99\ \text{kJ/kg.°C})(30-7)\text{°C} = 196.6\ \text{kJ}$$

Then the actual amount of heat transfer becomes

$$\left(\frac{Q}{Q_{\max}}\right)_{\text{cyl}} = 1 - 3\left(\frac{T_0 - T_\infty}{T_i - T_\infty}\right)\frac{\sin\lambda_1 - \lambda_1\cos\lambda_1}{\lambda_1^3} = 1 - 3\left(\frac{10-7}{30-7}\right)\frac{\sin(3.0401) - (3.0401)\cos(3.0401)}{(3.0401)^3} = 0.9565$$

$$Q = 0.9565 Q_{\max}$$

$$Q = 0.9565(196.6\ \text{kJ}) = \mathbf{188\ kJ}$$

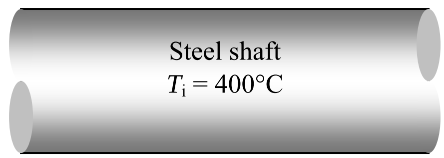
11-41 A long cylindrical shaft at 400°C is allowed to cool slowly. The center temperature and the heat transfer per unit length of the cylinder are to be determined.

Assumptions **1** Heat conduction in the shaft is one-dimensional since it is long and it has thermal symmetry about the center line. **2** The thermal properties of the shaft are constant. **3** The heat transfer coefficient is constant and uniform over the entire surface. **4** The Fourier number is $\tau > 0.2$ so that the one-term approximate solutions (or the transient temperature charts) are applicable (this assumption will be verified).

Properties The properties of stainless steel 304 at room temperature are given to be k = 14.9 W/m.°C, ρ = 7900 kg/m^3, c_p = 477 J/kg.°C, $\alpha = 3.95\times10^{-6}$ m^2/s

Analysis First the Biot number is calculated to be

$$Bi = \frac{hr_o}{k} = \frac{(60\text{ W/m}^2.°\text{C})(0.175\text{ m})}{(14.9\text{ W/m.°C})} = 0.705$$



The constants λ_1 and A_1 corresponding to this Biot number are, from Table 11-2,

$$\lambda_1 = 1.0904 \quad \text{and} \quad A_1 = 1.1548$$

The Fourier number is

$$\tau = \frac{\alpha t}{L^2} = \frac{(3.95\times10^{-6}\text{ m}^2/\text{s})(20\times60\text{ s})}{(0.175\text{ m})^2} = 0.1548$$

which is very close to the value of 0.2. Therefore, the one-term approximate solution (or the transient temperature charts) can still be used, with the understanding that the error involved will be a little more than 2 percent. Then the temperature at the center of the shaft becomes

$$\theta_{0,cyl} = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2\tau} = (1.1548)e^{-(1.0904)^2(0.1548)} = 0.9607$$

$$\frac{T_0 - 150}{400-150} = 0.9607 \longrightarrow T_0 = \mathbf{390\ °C}$$

The maximum heat can be transferred from the cylinder per meter of its length is

$$m = \rho V = \rho\pi r_o^2 L = (7900\text{ kg/m}^3)[\pi(0.175\text{ m})^2(1\text{ m})] = 760.1\text{ kg}$$

$$Q_{\max} = mc_p[T_\infty - T_i] = (760.1\text{ kg})(0.477\text{ kJ/kg.°C})(400-150)°\text{C} = 90{,}640\text{ kJ}$$

Once the constant J_1 = 0.4679 is determined from Table 11-3 corresponding to the constant λ_1 =1.0904, the actual heat transfer becomes

$$\left(\frac{Q}{Q_{\max}}\right)_{cyl} = 1 - 2\left(\frac{T_o - T_\infty}{T_i - T_\infty}\right)\frac{J_1(\lambda_1)}{\lambda_1} = 1 - 2\left(\frac{390-150}{400-150}\right)\frac{0.4679}{1.0904} = 0.1761$$

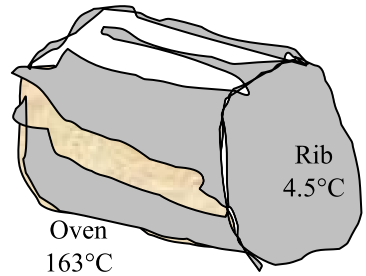
$$Q = 0.1761(90{,}640\text{ kJ}) = \mathbf{15{,}960\ kJ}$$

11-46 A rib is roasted in an oven. The heat transfer coefficient at the surface of the rib, the temperature of the outer surface of the rib and the amount of heat transfer when it is rare done are to be determined. The time it will take to roast this rib to medium level is also to be determined.

Assumptions **1** The rib is a homogeneous spherical object. **2** Heat conduction in the rib is one-dimensional because of symmetry about the midpoint. **3** The thermal properties of the rib are constant. **4** The heat transfer coefficient is constant and uniform over the entire surface. **5** The Fourier number is $\tau > 0.2$ so that the one-term approximate solutions (or the transient temperature charts) are applicable (this assumption will be verified).

Properties The properties of the rib are given to be $k = 0.45$ W/m.°C, $\rho = 1200$ kg/m^3, $c_p = 4.1$ kJ/kg.°C, and $\alpha = 0.91\times10^{-7}$ m^2/s.

Analysis (*a*) The radius of the roast is determined to be



$$m = \rho V \longrightarrow V = \frac{m}{\rho} = \frac{3.2\text{ kg}}{1200\text{ kg/m}^3} = 0.002667\text{ m}^3$$

$$V = \frac{4}{3}\pi r_o^3 \longrightarrow r_o = \sqrt[3]{\frac{3V}{4\pi}} = \sqrt[3]{\frac{3(0.002667\text{ m}^3)}{4\pi}} = 0.08603\text{ m}$$

The Fourier number is

$$\tau = \frac{\alpha t}{r_o^2} = \frac{(0.91\times10^{-7}\text{ m}^2\text{/s})(2\times3600+45\times60)\text{s}}{(0.08603\text{ m})^2} = 0.1217$$

which is somewhat below the value of 0.2. Therefore, the one-term approximate solution (or the transient temperature charts) can still be used, with the understanding that the error involved will be a little more than 2 percent. Then the one-term solution can be written in the form

$$\theta_{0,sph} = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2\tau} \longrightarrow \frac{60-163}{4.5-163} = 0.65 = A_1 e^{-\lambda_1^2(0.1217)}$$

It is determined from Table 11-2 by trial and error that this equation is satisfied when $Bi = 30$, which corresponds to $\lambda_1 = 3.0372$ and $A_1 = 1.9898$. Then the heat transfer coefficient can be determined from

$$Bi = \frac{hr_o}{k} \longrightarrow h = \frac{kBi}{r_o} = \frac{(0.45\text{ W/m.°C})(30)}{(0.08603\text{ m})} = \mathbf{156.9\ W/m^2.°C}$$

This value seems to be larger than expected for problems of this kind. This is probably due to the Fourier number being less than 0.2.

(*b*) The temperature at the surface of the rib is

$$\theta(r_o,t)_{sph} = \frac{T(r_o,t)-T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2\tau}\frac{\sin(\lambda_1 r_o/r_o)}{\lambda_1 r_o/r_o} = (1.9898)e^{-(3.0372)^2(0.1217)}\frac{\sin(3.0372\text{ rad})}{3.0372}$$

$$\frac{T(r_o,t)-163}{4.5-163} = 0.0222 \longrightarrow T(r_o,t) = \mathbf{159.5\ °C}$$

(*c*) The maximum possible heat transfer is

$$Q_{max} = mc_p(T_\infty - T_i) = (3.2\text{ kg})(4.1\text{ kJ/kg.°C})(163-4.5)\text{°C} = 2080\text{ kJ}$$

Then the actual amount of heat transfer becomes

$$\frac{Q}{Q_{max}} = 1 - 3\theta_{o,sph}\frac{\sin(\lambda_1)-\lambda_1\cos(\lambda_1)}{\lambda_1^3} = 1-3(0.65)\frac{\sin(3.0372)-(3.0372)\cos(3.0372)}{(3.0372)^3} = 0.783$$

$$Q = 0.783Q_{max} = (0.783)(2080\text{ kJ}) = \mathbf{1629\ kJ}$$

(*d*) The cooking time for medium-done rib is determined to be

$$\theta_{0,sph} = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2\tau} \longrightarrow \frac{71-163}{4.5-163} = (1.9898)e^{-(3.0372)^2\tau} \longrightarrow \tau = 0.1336$$

$$t = \frac{\tau r_o^2}{\alpha} = \frac{(0.1336)(0.08603\text{ m})^2}{(0.91\times10^{-7}\text{ m}^2\text{/s})} = 10{,}866\text{ s} = 181\text{ min} \cong \mathbf{3\ hr}$$

This result is close to the listed value of 3 hours and 20 minutes. The difference between the two results is due to the Fourier number being less than 0.2 and thus the error in the one-term approximation.

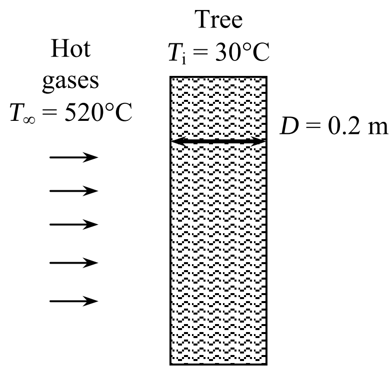
Discussion The temperature of the outer parts of the rib is greater than that of the inner parts of the rib after it is taken out of the oven. Therefore, there will be a heat transfer from outer parts of the rib to the inner parts as a result of this temperature difference. The recommendation is logical.

11-97 CD EES The trunks of some dry oak trees are exposed to hot gases. The time for the ignition of the trunks is to be determined.

Assumptions **1** Heat conduction in the trunks is one-dimensional since it is long and it has thermal symmetry about the center line. **2** The thermal properties of the trunks are constant. **3** The heat transfer coefficient is constant and uniform over the entire surface. **4** The Fourier number is $\tau > 0.2$ so that the one-term approximate solutions (or the transient temperature charts) are applicable (this assumption will be verified).

Properties The properties of the trunks are given to be $k = 0.17$ W/m.°C and $\alpha = 1.28\times10^{-7}$ m^2/s.

Analysis We treat the trunks of the trees as an infinite cylinder since heat transfer is primarily in the radial direction. Then the Biot number becomes



$$Bi = \frac{hr_o}{k} = \frac{(65 \text{ W/m}^2.°\text{C})(0.1 \text{ m})}{(0.17 \text{ W/m.°C})} = 38.24$$

The constants λ_1 and A_1 corresponding to this Biot number are, from Table 11-2,

$$\lambda_1 = 2.3420 \quad \text{and} \quad A_1 = 1.5989$$

The Fourier number is

$$\tau = \frac{\alpha t}{r_o^2} = \frac{(1.28\times10^{-7} \text{ m}^2/\text{s})(4 \text{ h}\times3600 \text{ s/h})}{(0.1 \text{ m})^2} = 0.184$$

which is slightly below 0.2 but close to it. Therefore, assuming the one-term approximate solution for transient heat conduction to be applicable, the temperature at the surface of the trees in 4 h becomes

$$\theta(r_o,t)_{cyl} = \frac{T(r_o,t)-T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2\tau} J_0(\lambda_1 r / r_o)$$

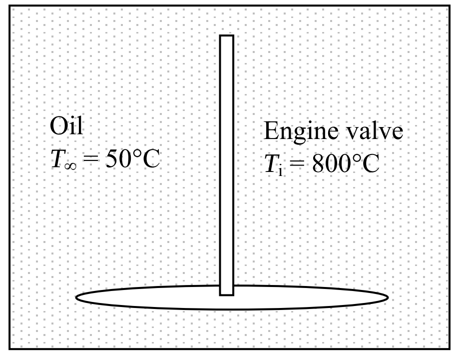
$$\frac{T(r_o,t)-520}{30-520} = (1.5989)e^{-(2.3420)^2(0.184)}(0.0332) = 0.01935 \longrightarrow T(r_o,t) = \mathbf{511\,°C} \;\; > \;\; 410°\text{C}$$

Therefore, the trees will ignite. (Note: J_0 is read from Table 11-3).

11-104 Internal combustion engine valves are quenched in a large oil bath. The time it takes for the valve temperature to drop to specified temperatures and the maximum heat transfer are to be determined.

Assumptions **1** The thermal properties of the valves are constant. **2** The heat transfer coefficient is constant and uniform over the entire surface. **3** Depending on the size of the oil bath, the oil bath temperature will increase during quenching. However, an average canstant temperature as specified in the problem will be used. **4** The Biot number is Bi < 0.1 so that the lumped system analysis is applicable (this assumption will be verified).

Properties The thermal conductivity, density, and specific heat of the balls are given to be $k = 48$ W/m.°C, $\rho = 7840$ kg/m^3, and $c_p = 440$ J/kg.°C.



Analysis (*a*) The characteristic length of the balls and the Biot number are

$$L_c = \frac{V}{A_s} = \frac{1.8(\pi D^2 L/4)}{2\pi DL} = \frac{1.8D}{8} = \frac{1.8(0.008\text{ m})}{8} = 0.0018\text{ m}$$

$$Bi = \frac{hL_c}{k} = \frac{(800\text{ W/m}^2.°\text{C})(0.0018\text{ m})}{48\text{ W/m.°C}} = 0.03 < 0.1$$

Therefore, we can use lumped system analysis. Then the time for a final valve temperature of 400°C becomes

$$b = \frac{hA_s}{\rho c_p V} = \frac{8h}{1.8\rho c_p D} = \frac{8(800\text{ W/m}^2.°\text{C})}{1.8(7840\text{ kg/m}^3)(440\text{ J/kg.°C})(0.008\text{ m})} = 0.1288\text{ s}^{-1}$$

$$\frac{T(t)-T_\infty}{T_i - T_\infty} = e^{-bt} \longrightarrow \frac{400-50}{800-50} = e^{-(0.1288\text{ s}^{-1})t} \longrightarrow t = \mathbf{5.9\ s}$$

(*b*) The time for a final valve temperature of 200°C is

$$\frac{T(t)-T_\infty}{T_i - T_\infty} = e^{-bt} \longrightarrow \frac{200-50}{800-50} = e^{-(0.1288\text{ s}^{-1})t} \longrightarrow t = \mathbf{12.5\ s}$$

(*c*) The time for a final valve temperature of 51°C is

$$\frac{T(t)-T_\infty}{T_i - T_\infty} = e^{-bt} \longrightarrow \frac{51-50}{800-50} = e^{-(0.1288\text{ s}^{-1})t} \longrightarrow t = \mathbf{51.4\ s}$$

(*d*) The maximum amount of heat transfer from a single valve is determined from

$$m = \rho V = \rho \frac{1.8\pi D^2 L}{4} = (7840\text{ kg/m}^3)\frac{1.8\pi(0.008\text{ m})^2(0.10\text{ m})}{4} = 0.0709\text{ kg}$$

$$Q = mc_p[T_f - T_i] = (0.0709\text{ kg})(440\text{ J/kg.°C})(800-50)°\text{C} = 23{,}400\text{ J} = \mathbf{23.4\ kJ}\ \text{(per valve)}$$