15-27 A glass window transmits 90% of the radiation in a specified wavelength range and is opaque for radiation at other wavelengths. The rate of radiation transmitted through this window is to be determined for two cases.

Assumptions The sources behave as a black body.

Analysis The surface area of the glass window is

$$A_{\rm s} = 4 \, \mathrm{m}^2$$

(a) For a blackbody source at 5800 K, the total blackbody radiation emission is

$$E_b(T) = \sigma T^4 A_s = (5.67 \times 10^{-8} \text{ kW/m}^2 \cdot \text{K})^4 (5800 \text{ K})^4 (4 \text{ m}^2) = 2.567 \times 10^5 \text{ kW}$$

The fraction of radiation in the range of 0.3 to 3.0 µm is

$$\lambda_1 T = (0.30 \ \mu\text{m})(5800 \ \text{K}) = 1740 \ \mu\text{mK} \longrightarrow f_{\lambda_1} = 0.03345$$

$$\lambda_2 T = (3.0 \ \mu\text{m})(5800 \ \text{K}) = 17,400 \ \mu\text{mK} \longrightarrow f_{\lambda_2} = 0.97875$$

$$\Delta f = f_{\lambda_2} - f_{\lambda_1} = 0.97875 - 0.03345 = 0.9453$$

Noting that 90% of the total radiation is transmitted through the window,

$$E_{\text{transmit}} = 0.90 \Delta f E_b(T)$$

= $(0.90)(0.9453)(2.567 \times 10^5 \text{ kW}) = 2.184 \times 10^5 \text{ kW}$

(b) For a blackbody source at 1000 K, the total blackbody emissive power is

$$E_b(T) = \sigma T^4 A_s = (5.67 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4)(1000 \text{ K})^4 (4 \text{ m}^2) = 226.8 \text{ kW}$$

The fraction of radiation in the visible range of 0.3 to 3.0 µm is

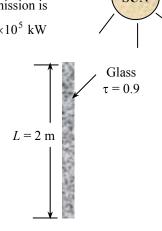
$$\lambda_1 T = (0.30 \ \mu\text{m})(1000 \ \text{K}) = 300 \ \mu\text{mK} \longrightarrow f_{\lambda_1} = 0.0000$$

$$\lambda_2 T = (3.0 \ \mu\text{m})(1000 \ \text{K}) = 3000 \ \mu\text{mK} \longrightarrow f_{\lambda_2} = 0.273232$$

$$\Delta f = f_{\lambda_2} - f_{\lambda_1} = 0.273232 - 0$$

and

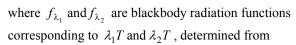
$$E_{\text{transmit}} = 0.90 \Delta f E_h(T) = (0.90)(0.273232)(226.8 \text{ kW}) = 55.8 \text{ kW}$$

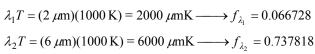


15-33 The variation of emissivity of a surface at a specified temperature with wavelength is given. The average emissivity of the surface and its emissive power are to be determined.

Analysis The average emissivity of the surface can be determined from

$$\begin{split} \varepsilon(T) &= \frac{\varepsilon_1 \int\limits_0^{\lambda_1} E_{b_{\lambda}}(T) d\lambda}{\sigma T^4} + \frac{\varepsilon_2 \int\limits_{\lambda_1}^{\lambda_2} E_{b_{\lambda}}(T) d\lambda}{\sigma T^4} + \frac{\varepsilon_3 \int\limits_{\lambda_2}^{\infty} E_{b_{\lambda}}(T) d\lambda}{\sigma T^4} \\ &= \varepsilon_1 f_{0 - \lambda_1} + \varepsilon_2 f_{\lambda_1 - \lambda_2} + \varepsilon_3 f_{\lambda_2 - \infty} \\ &= \varepsilon_1 f_{\lambda_1} + \varepsilon_2 (f_{\lambda_2} - f_{\lambda_1}) + \varepsilon_3 (1 - f_{\lambda_2}) \end{split}$$





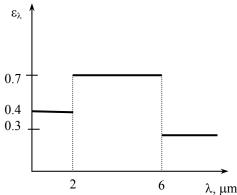
$$f_{0-\lambda_1}=f_{\lambda 1}-f_0=f_{\lambda_1} \text{ since } f_0=0 \text{ and } \mathbf{f}_{\lambda_2-\infty}=f_\infty-f_{\lambda_2} \text{ since } f_\infty=1.$$

and,

$$\varepsilon = (0.4)0.066728 + (0.7)(0.737818 - 0.066728) + (0.3)(1 - 0.737818) = 0.575$$

Then the emissive power of the surface becomes

$$E = \varepsilon \sigma T^4 = 0.575(5.67 \times 10^{-8} \text{ W/m}^2.\text{K}^4)(1000 \text{ K})^4 = 32.6 \text{ kW/m}^2$$



L=2D=4r

(2)

D=2r

(3)

(1)

15-50 A cylindrical enclosure is considered. The view factor from the side surface of this cylindrical enclosure to its base surface is to be determined.

Assumptions The surfaces are diffuse emitters and reflectors.

Analysis We designate the surfaces as follows:

Base surface by (1), top surface by (2), and side surface by (3).

Then from Fig. 15-7

$$\frac{L}{r_1} = \frac{4r_1}{r_1} = 4$$

$$\frac{r_2}{L} = \frac{r_2}{4r_2} = 0.25$$

$$F_{12} = F_{21} = 0.05$$

summation rule: $F_{11} + F_{12} + F_{13} = 1$

$$0 + 0.05 + F_{13} = 1 \longrightarrow F_{13} = 0.95$$

reciprocity rule:
$$A_1 F_{13} = A_3 F_{31} \longrightarrow F_{31} = \frac{A_1}{A_3} F_{13} = \frac{\pi r_1^2}{2\pi r_1 L} F_{13} = \frac{\pi r_1^2}{8\pi r_1^2} F_{13} = \frac{1}{8} (0.95) = 0.119$$

Discussion This problem can be solved more accurately by using the view factor relation from Table 15-3 to be

$$R_{1} = \frac{r_{1}}{L} = \frac{r_{1}}{4r_{1}} = 0.25$$

$$R_{2} = \frac{r_{2}}{L} = \frac{r_{2}}{4r_{2}} = 0.25$$

$$S = 1 + \frac{1 + R_{2}^{2}}{R_{1}^{2}} = 1 + \frac{1 + 0.25^{2}}{0.25^{2}} = 18$$

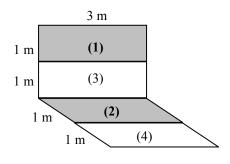
$$F_{12} = \frac{1}{2} \left\{ S - \left[S^{2} - 4 \left(\frac{R_{2}}{R_{1}} \right)^{2} \right]^{0.5} \right\} = \frac{1}{2} \left\{ 18 - \left[18^{2} - 4 \left(\frac{1}{1} \right)^{2} \right]^{0.5} \right\} = 0.056$$

$$F_{13} = 1 - F_{12} = 1 - 0.056 = 0.944$$

reciprocity rule:
$$A_1 F_{13} = A_3 F_{31} \longrightarrow F_{31} = \frac{A_1}{A_3} F_{13} = \frac{\pi r_1^2}{2\pi r_1 L} F_{13} = \frac{\pi r_1^2}{8\pi r_1^2} F_{13} = \frac{1}{8} (0.944) = 0.118$$

15-57 The view factors between the rectangular surfaces shown in the figure are to be determined. **Assumptions** The surfaces are diffuse emitters and reflectors.

Analysis We designate the different surfaces as follows: shaded part of perpendicular surface by (1), bottom part of perpendicular surface by (3), shaded part of horizontal surface by (2), and front part of horizontal surface by (4).



$$\frac{L_2}{W} = \frac{1}{3} \begin{cases} F_{23} = 0.25 & \text{and} & \frac{L_2}{W} = \frac{2}{3} \\ \frac{L_1}{W} = \frac{1}{3} \end{cases} F_{2 \to (1+3)} = 0.32$$

superposition rule:
$$F_{2\rightarrow(1+3)} = F_{21} + F_{23} \longrightarrow F_{21} = F_{2\rightarrow(1+3)} - F_{23} = 0.32 - 0.25 = 0.07$$

reciprocity rule:
$$A_1 = A_2 \longrightarrow A_1 F_{12} = A_2 F_{21} \longrightarrow F_{12} = F_{21} = \mathbf{0.07}$$

(b) From Fig. 15-6,

$$\frac{L_2}{W} = \frac{1}{3}$$
 and $\frac{L_1}{W} = \frac{2}{3} \left\{ F_{(4+2)\to 3} = 0.15 \text{ and } \frac{L_2}{W} = \frac{2}{3} \text{ and } \frac{L_1}{W} = \frac{2}{3} \right\} F_{(4+2)\to (1+3)} = 0.22$

superposition rule:
$$F_{(4+2)\to(1+3)} = F_{(4+2)\to1} + F_{(4+2)\to3} \longrightarrow F_{(4+2)\to1} = 0.22 - 0.15 = 0.07$$

reciprocity rule :
$$A_{(4+2)}F_{(4+2)\to 1} = A_1F_{1\to (4+2)}$$

$$\longrightarrow F_{1 \to (4+2)} = \frac{A_{(4+2)}}{A_1} F_{(4+2) \to 1} = \frac{6}{3} (0.07) = 0.14$$

superposition rule: $F_{1\to(4+2)} = F_{14} + F_{12}$

$$\longrightarrow F_{14} = 0.14 - 0.07 = 0.07$$

since $F_{12} = 0.07$ (from part a). Note that F_{14} in part (b) is equivalent to F_{12} in part (a).



shaded part of top surface by (1), remaining part of top surface by (3), remaining part of bottom surface by (4), and shaded part of bottom surface by (2).



$$\frac{L_2}{D} = \frac{2}{2}$$

$$\begin{cases}
\frac{L_1}{D} = \frac{2}{2} \\
F_{(2+4)\to(1+3)} = 0.20
\end{cases} \text{ and } \frac{\frac{L_2}{D}}{\frac{L_1}{D}} = \frac{2}{2}$$

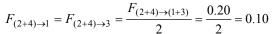
$$\begin{cases}
\frac{L_1}{D} = \frac{1}{2}
\end{cases}$$

superposition rule: $F_{(2+4)\to(1+3)} = F_{(2+4)\to1} + F_{(2+4)\to3}$

symmetry rule : $F_{(2+4)\to 1} = F_{(2+4)\to 3}$

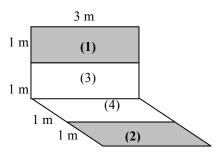
Substituting symmetry rule gives

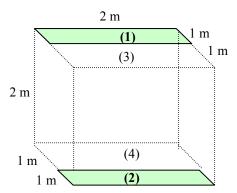
$$F_{(2+4)\to 1} = F_{(2+4)\to 3} = \frac{F_{(2+4)\to(1+3)}}{2} = \frac{0.20}{2} = 0.10$$



reciprocity rule:
$$A_1 F_{1 \to (2+4)} = A_{(2+4)} F_{(2+4) \to 1} \longrightarrow (2) F_{1 \to (2+4)} = (4)(0.10) \longrightarrow F_{1 \to (2+4)} = 0.20$$

superposition rule:
$$F_{1\to(2+4)} = F_{12} + F_{14} \longrightarrow 0.20 = F_{12} + 0.12 \longrightarrow F_{12} = 0.20 - 0.12 =$$
0.08





15-77 A furnace shaped like a long equilateral-triangular duct is considered. The temperature of the base surface is to be determined.

Assumptions 1 Steady operating conditions exist 2 The surfaces are opaque, diffuse, and gray. 3 Convection heat transfer is not considered. 4 End effects are neglected.

Properties The emissivities of surfaces are given to be $\varepsilon_1 = 0.8$ and $\varepsilon_2 = 0.5$.

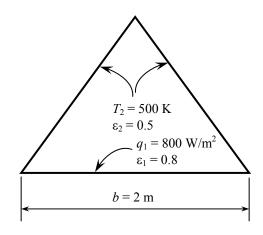
Analysis This geometry can be treated as a two surface enclosure since two surfaces have identical properties. We consider base surface to be surface 1 and other two surface to be surface 2. Then the view factor between the two becomes $F_{12} = 1$. The temperature of the base surface is determined from

$$\dot{Q}_{12} = \frac{\sigma(T_1^4 - T_2^4)}{\frac{1 - \varepsilon_1}{A_1 \varepsilon_1} + \frac{1}{A_1 F_{12}} + \frac{1 - \varepsilon_2}{A_2 \varepsilon_2}}$$

$$800 \text{ W} = \frac{(5.67 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4)[(T_1)^4 - (500 \text{ K})^4]}{\frac{1 - 0.8}{(1 \text{ m}^2)(0.8)} + \frac{1}{(1 \text{ m}^2)(1)} + \frac{1 - 0.5}{(2 \text{ m}^2)(0.5)}}$$

$$T_1 = \mathbf{543} \text{ K}$$

Note that $A_1 = 1 \,\text{m}^2$ and $A_2 = 2 \,\text{m}^2$.



15-79 The floor and the ceiling of a cubical furnace are maintained at uniform temperatures. The net rate of radiation heat transfer between the floor and the ceiling is to be determined.

Assumptions 1 Steady operating conditions exist 2 The surfaces are opaque, diffuse, and gray. 3 Convection heat transfer is not considered.

Properties The emissivities of all surfaces are $\varepsilon = 1$ since they are black or reradiating.

Analysis We consider the ceiling to be surface 1, the floor to be surface 2 and the side surfaces to be surface 3. The furnace can be considered to be three-surface enclosure. We assume that steady-state conditions exist. Since the side surfaces are reradiating, there is no heat transfer through them, and the entire heat lost by the ceiling must be gained by the floor. The view factor from the ceiling to the floor of the furnace is $F_{12} = 0.2$. Then the rate of heat loss from the ceiling can be determined from

$$\dot{Q}_{1} = \frac{E_{b1} - E_{b2}}{\left(\frac{1}{R_{12}} + \frac{1}{R_{13} + R_{23}}\right)^{-1}}$$

where

$$E_{b1} = \sigma T_1^4 = (5.67 \times 10^{-8} \text{ W/m}^2.\text{K}^4)(1100 \text{ K})^4 = 83,015 \text{ W/m}^2$$

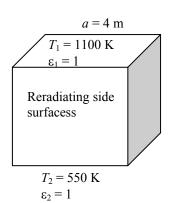
 $E_{b2} = \sigma T_2^4 = (5.67 \times 10^{-8} \text{ W/m}^2.\text{K}^4)(550 \text{ K})^4 = 5188 \text{ W/m}^2$

and

$$A_1 = A_2 = (4 \text{ m})^2 = 16 \text{ m}^2$$

$$R_{12} = \frac{1}{A_1 F_{12}} = \frac{1}{(16 \text{ m}^2)(0.2)} = 0.3125 \text{ m}^{-2}$$

$$R_{13} = R_{23} = \frac{1}{A_1 F_{13}} = \frac{1}{(16 \text{ m}^2)(0.8)} = 0.078125 \text{ m}^{-2}$$



Substituting,

$$\dot{Q}_{12} = \frac{(83,015 - 5188) \text{ W/m}^2}{\left(\frac{1}{0.3125 \text{ m}^{-2}} + \frac{1}{2(0.078125 \text{ m}^{-2})}\right)^{-1}} = 7.47 \times 10^5 \text{ W} = 747 \text{ kW}$$

15-96 The temperature of air in a duct is measured by a thermocouple. The radiation effect on the temperature measurement is to be quantified, and the actual air temperature is to be determined.

Assumptions The surfaces are opaque, diffuse, and gray.

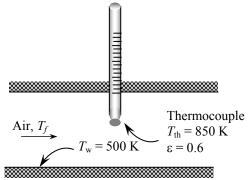
Properties The emissivity of thermocouple is given to be ε =0.6.

Analysis The actual temperature of the air can be determined from

$$T_f = T_{th} + \frac{\varepsilon_{th} \sigma (T_{th}^4 - T_w^4)}{h}$$

$$= 850 \text{ K} + \frac{(0.6)(5.67 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4)[(850 \text{ K})^4 - (500 \text{ K})^4]}{60 \text{ W/m}^2 \cdot ^{\circ}\text{C}}$$

$$= 1111 \text{K}$$



15-107 A cylindrical furnace with specified top and bottom surface temperatures and specified heat transfer rate at the bottom surface is considered. The emissivity of the top surface and the net rates of heat transfer between the top and the bottom surfaces, and between the bottom and the side surfaces are to be determined.

Assumptions 1 Steady operating conditions exist 2 The surfaces are opaque, diffuse, and gray. 3 Convection heat transfer is not considered.

Properties The emissivity of the bottom surface is 0.90.

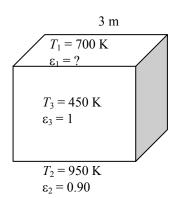
Analysis We consider the top surface to be surface 1, the base surface to be surface 2, and the side surface to be surface 3. This system is a three-surface enclosure. The view factor from the base to the top surface of the cube is from Fig. 15-5 $F_{12} = 0.2$. The view factor from the base or the top to the side surfaces is determined by applying the summation rule to be

$$F_{11} + F_{12} + F_{13} = 1 \longrightarrow F_{13} = 1 - F_{12} = 1 - 0.2 = 0.8$$

since the base surface is flat and thus $F_{11} = 0$. Other view factors are

$$F_{21} = F_{12} = 0.20,$$
 $F_{23} = F_{13} = 0.80,$ $F_{31} = F_{32} = 0.20$

We now apply Eq. 9-35 to each surface



$$\sigma T_1^4 = J_1 + \frac{1 - \varepsilon_1}{\varepsilon_1} [F_{12}(J_1 - J_2) + F_{13}(J_1 - J_3)]$$

Surface 1:

$$(5.67 \times 10^{-8} \text{ W/m}^2.\text{K}^4)(700 \text{ K})^4 = J_1 + \frac{1 - \varepsilon_1}{\varepsilon_1} [0.20(J_1 - J_2) + 0.80(J_1 - J_3)]$$

Surface 2:
$$\sigma T_2^{\ 4} = J_2 + \frac{1 - \varepsilon_2}{\varepsilon_2} \left[F_{21} (J_2 - J_1) + F_{23} (J_2 - J_3) \right]$$

$$(5.67 \times 10^{-8} \text{ W/m}^2.\text{K}^4) (950 \text{ K})^4 = J_2 + \frac{1 - 0.90}{0.90} \left[0.20 (J_2 - J_1) + 0.80 (J_2 - J_3) \right]$$

Surface 3:

$$\sigma T_3^{-} = J_3$$
(5.67×10⁻⁸ W/m².K⁴)(450 K)⁴ = J_3

We now apply Eq. 9-34 to surface 2

$$\dot{Q}_2 = A_2 [F_{21}(J_2 - J_1) + F_{23}(J_2 - J_3)] = (9 \text{ m}^2) [0.20(J_2 - J_1) + 0.80(J_2 - J_3)]$$

Solving the above four equations, we find

$$\varepsilon_1 =$$
0.44, $J_1 = 11,736 \text{ W/m}^2$, $J_2 = 41,985 \text{ W/m}^2$, $J_3 = 2325 \text{ W/m}^2$

The rate of heat transfer between the bottom and the top surface is

$$A_1 = A_2 = (3 \text{ m})^2 = 9 \text{ m}^2$$

 $\dot{Q}_{21} = A_2 F_{21} (J_2 - J_1) = (9 \text{ m}^2)(0.20)(41,985 - 11,736) \text{W/m}^2 = 54.4 \text{ kW}$

The rate of heat transfer between the bottom and the side surface is

$$A_3 = 4A_1 = 4(9 \text{ m}^2) = 36 \text{ m}^2$$

 $\dot{Q}_{23} = A_2 F_{23} (J_2 - J_3) = (9 \text{ m}^2)(0.8)(41,985 - 2325) \text{W/m}^2 = 285.6 \text{ kW}$

Discussion The sum of these two heat transfer rates are 54.4 + 285.6 = 340 kW, which is equal to 340 kW heat supply rate from surface 2.

15-108 Radiation heat transfer occurs between two square parallel plates. The view factors, the rate of radiation heat transfer and the temperature of a third plate to be inserted are to be determined.

Assumptions 1 Steady operating conditions exist 2 The surfaces are opaque, diffuse, and gray. 3 Convection heat transfer is not considered.

Properties The emissivities of plate a, b, and c are given to be $\varepsilon_a = 0.8$, $\varepsilon_b = 0.4$, and $\varepsilon_c = 0.1$, respectively. **Analysis** (a) The view factor from surface a to surface b is determined as follows

$$A = \frac{a}{L} = \frac{20}{40} = 0.5, \ B = \frac{b}{L} = \frac{60}{40} = 1.5$$

$$F_{ab} = \frac{1}{2A} \left\{ \left[(B+A)^2 + 4 \right]^{0.5} - \left[(B-A)^2 + 4 \right]^{0.5} \right\} = \frac{1}{2(0.5)} \left\{ \left[(1.5+0.5)^2 + 4 \right]^{0.5} - \left[(1.5-0.5)^2 + 4 \right]^{0.5} \right\} = \mathbf{0.592}$$

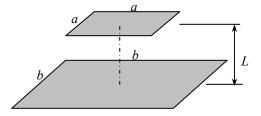
The view factor from surface b to surface a is determined from reciprocity relation:

$$A_a = (0.2 \text{ m})(0.2 \text{ m}) = 0.04 \text{ m}^2$$

$$A_b = (0.6 \text{ m})(0.6 \text{ m}) = 0.36 \text{ m}^2$$

$$A_a F_{ab} = A_b F_{ba}$$

$$(0.04)(0.592) = (0.36)F_{ba} \longrightarrow F_{ba} = \mathbf{0.0658}$$



(b) The net rate of radiation heat transfer between the surfaces can be determined from

$$\dot{Q}_{ab} = \frac{\sigma \left(T_a^4 - T_b^4\right)}{\frac{1 - \varepsilon_a}{A_a \varepsilon_a} + \frac{1}{A_a F_{ab}} + \frac{1 - \varepsilon_b}{A_b \varepsilon_b}} = \frac{(5.67 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4) \left[(1073 \text{ K})^4 - (473 \text{ K})^4 \right]}{\frac{1 - 0.8}{(0.04 \text{ m}^2)(0.8)} + \frac{1}{(0.04 \text{ m}^2)(0.592)} + \frac{1 - 0.4}{(0.36 \text{ m}^2)(0.4)}} = \mathbf{1374 \text{ W}}$$

(c) In this case we have

$$A = \frac{a}{L} = \frac{0.2 \text{ m}}{0.2 \text{ m}} = 1, \quad C = \frac{c}{L} = \frac{2.0 \text{ m}}{0.2 \text{ m}} = 10$$

$$F_{ac} = \frac{1}{2A} \left\{ \left[(C + A)^2 + 4 \right]^{0.5} - \left[(C - A)^2 + 4 \right]^{0.5} \right\} = \frac{1}{2(0.5)} \left\{ \left[(10 + 0.5)^2 + 4 \right]^{0.5} - \left[(10 - 0.5)^2 + 4 \right]^{0.5} \right\} = 0.981$$

$$B = \frac{b}{L} = \frac{0.6 \text{ m}}{0.2 \text{ m}} = 3, \quad C = \frac{c}{L} = \frac{2.0 \text{ m}}{0.2 \text{ m}} = 10$$

$$F_{bc} = \frac{1}{2A} \left\{ \left[(C + B)^2 + 4 \right]^{0.5} - \left[(C - B)^2 + 4 \right]^{0.5} \right\}$$

$$= \frac{1}{2(3)} \left\{ \left[(10 + 3)^2 + 4 \right]^{0.5} - \left[(10 - 3)^2 + 4 \right]^{0.5} \right\} = 0.979$$

$$A_b F_{bc} = A_c F_{cb}$$

$$(0.36)(0.979) = (4.0) F_{cb} \longrightarrow F_{ba} = 0.0881$$

An energy balance gives

$$\frac{\dot{Q}_{ac} = \dot{Q}_{cb}}{\frac{\sigma\left(T_a^4 - T_c^4\right)}{\frac{1 - \varepsilon_a}{A_a \varepsilon_a} + \frac{1}{A_a F_{ac}} + \frac{1 - \varepsilon_c}{A_c \varepsilon_c}} = \frac{\sigma\left(T_c^4 - T_b^4\right)}{\frac{1 - \varepsilon_c}{A_c \varepsilon_c} + \frac{1}{A_c F_{cb}} + \frac{1 - \varepsilon_b}{A_b \varepsilon_b}}$$

$$\frac{(1073 \text{ K})^4 - T_c^4}{\frac{1 - 0.8}{(0.04 \text{ m}^2)(0.8)} + \frac{1}{(0.04 \text{ m}^2)(0.981)} + \frac{1 - 0.1}{(4 \text{ m}^2)(0.1)}} = \frac{T_c^4 - (473 \text{ K})^4}{(4 \text{ m}^2)(0.1)} + \frac{1}{(4 \text{ m}^2)(0.0881)} + \frac{1 - 0.4}{(0.36 \text{ m}^2)(0.4)}$$

Solving the equation with an equation solver such as EES, we obtain $T_c = 754 \text{ K} = 481^{\circ}\text{C}$