***ECE309***

**INTRODUCTION TO THERMODYNAMICS & HEAT TRANSFER**

3 August 2004

Final Examination — R. Culham

- This is a 3 hour, closed-book examination.
- You are permitted to use one 8.5 in.$\times$ 11 in. crib sheet (both sides), Conversion Factors (inside cover of text) and the Property Tables and Figures from your text book.
- There are 5 questions to be answered. Read the questions very carefully.
- Clearly state all assumptions.
- It is your responsibility to write clearly and legibly.
- When using correlations, it is your responsibility to verify that all limiting conditions are satisfied.

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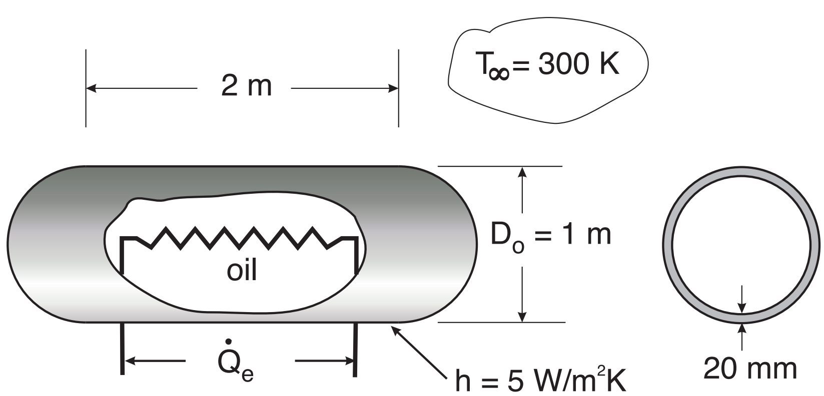
**Question 1** (20 marks)

An oil storage tank is designed to maintain the oil temperature at a uniform $400\ K$ by using a submerged resistance heating element. The storage tank consists of a cylindrical section that has a length of $2\ m$ and an outer diameter of $1\ m$ with the end caps being formed from two hemispherical sections as shown in the figure below. The tank is constructed from $20\ mm$ thick glass (pyrex, $k = 1.4\ W/m{\cdot}K$). The surrounding ambient air temperature is $T_\infty = 300\ K$ and the convective heat transfer coefficient over the full outer surface of the tank is $5\ W/(m^2 \cdot K)$. Assume 1-D conduction in both the cylindrical and hemispherical sections.

a) determine the electrical power, $(W)$, that must be supplied to the heater to maintain these conditions.

b) the critical thickness of insulation for this tank can be determined for both the cylindrical and hemispherical sections. For the cylindrical section we know $r_{cr,cylinder} = k/h$.
Set up the controlling equation used to determine $r_{cr,sphere}$ and show all calculations necessary to derive $r_{cr,sphere}$.



**Part a)**

We can use a 1st law energy balance to determine the rate at which heat must be supplied to the electrical heater.

$$\dot{Q}_e - \dot{Q}_{cyl} - 2\dot{Q}_{hemisph} = 0$$

or we can combine the two hemispherical sections to give

$$\dot{Q}_e = \dot{Q}_{cyl} + \dot{Q}_{sph}$$

Knowing that $\dot{Q} = \Delta T/R$ we can then write

$$\begin{aligned}
\dot{Q}_e &= \frac{T_{s,i} - T_\infty}{\dfrac{\ln(D_o/D_i)}{2\pi L k} + \dfrac{1}{\pi D_o L h}} + \frac{T_{s,i} - T_\infty}{\dfrac{1}{2\pi k}\left(\dfrac{1}{D_i} - \dfrac{1}{D_o}\right) + \dfrac{1}{\pi D_o^2 h}} \\
&= \frac{(400 - 300)\ K}{\dfrac{\ln(1/0.96)}{2\pi(2\ m)(1.4\ W/m\cdot K)} + \dfrac{1}{\pi(1\ m)(2\ m)(5\ W/m^2\cdot K)}} \\
&\quad + \frac{(400 - 300)\ K}{\dfrac{1}{2\pi(1.4\ W/m\cdot K)}\left(\dfrac{1}{0.96} - 1.0\right)\ m^{-1} + \dfrac{1}{\pi(1)^2(5\ W/m^2\cdot K)}} \\
&= 2928.14 + 1462.01 = 4390.15\ W \Leftarrow
\end{aligned}$$

**Part b)**

The total resistance for a spherical section is given as

$$\begin{aligned}
R_{total,sphere} &= R_{cond} + R_{conv} \\
&= \frac{1}{2\pi k}\left[\frac{1}{D_i} - \frac{1}{D_o}\right] + \frac{1}{\pi D_o^2 h} \\
&= \frac{1}{4\pi k}\left[\frac{1}{r_i} - \frac{1}{r_o}\right] + \frac{1}{4\pi r_o^2 h}
\end{aligned}$$

We then need to minimize the total resistance with respect to $r_o$.

$$\frac{dR_{total,sphere}}{dr_o} = \frac{\dfrac{1}{4\pi k}\left[\dfrac{1}{r_i} - \dfrac{1}{r_o}\right] + \dfrac{1}{4\pi r_o^2 h}}{dr_o}$$

Performing the differentiation and setting it equal to zero, we obtain

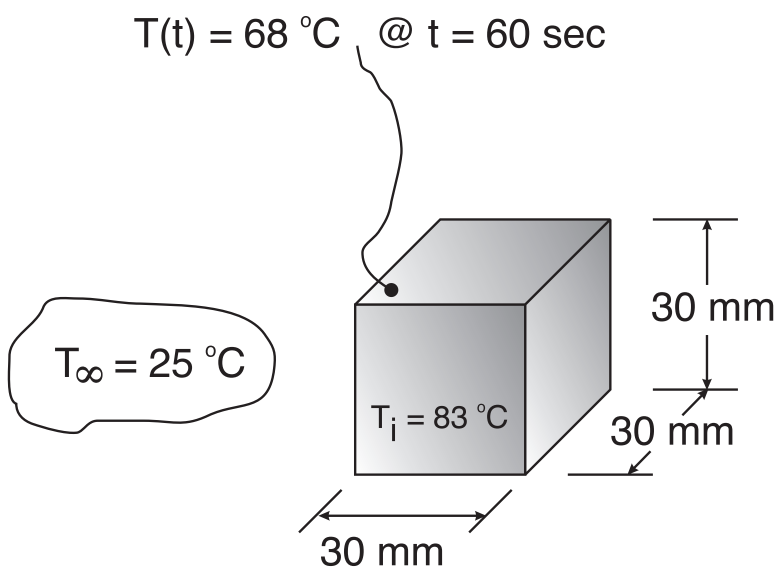
$$\left[\frac{1}{4\pi k}\cdot\frac{1}{r_0^2}\right] + \left[\frac{1}{4\pi h}\cdot\left(-\frac{2}{r_o^3}\right)\right] = 0$$

$$r_0 = r_{cr} = \frac{2k}{h} \Leftarrow$$

**Question 2** (20 marks)

The convective heat transfer coefficient for air flow over a cuboid is to be determined by using the temperature versus time transient response. A pure copper cuboid with a length, width and height each equal to $30\ mm$ is uniformly heated to $83\ °C$ before it is inserted into an air stream having a temperature of $25\ °C$. Using a thermocouple located on the outer surface of the cuboid, we observe a temperature of $68\ °C$, $60\ s$ after the cuboid is inserted into the air stream.

Determine the convective heat transfer coefficient ($W/m^2 \cdot K$), state all assumptions used to arrive at this conclusion and justify any assumptions where necessary.




The properties for pure copper at $300\ K$ are given as:

$$\rho = 8933\ kg/m^3$$

$$C_p = 385\ J/kg \cdot K$$

$$k = 401\ W/m \cdot K$$

First we must assume that the Biot is small, which allows us to proceed with a lumped system analysis. We will verify this once we determine a heat transfer coefficient.
The temperature response for a lumped system analysis is given as

$$\frac{T(t) - T_\infty}{T_i - T_\infty} = e^{-t/\tau}$$

where $\tau = R_t C_t$.

$$
\begin{aligned}
R_t &= \frac{1}{hA_s} = \frac{1}{6(0.03\ m)^2 h} = (185.185/h)\ K/W \\
C_t &= \rho V C_p = (8933\ kg/m^3)(0.03\ m)^3(385\ J/kg\cdot K) = 92.859\ J/K
\end{aligned}
$$

Therefore

$$
\frac{T(t)-T_\infty}{T_i-T_\infty} = \frac{(68-25)\ ^\circ C}{(83-25)\ ^\circ C} = \exp\left(-\frac{t}{\tau}\right) = \exp\left(-\frac{60\ s}{\left(\frac{185.185}{h}\times 92.859\right)\ s}\right)
$$

Solving for $h$

$$
\begin{aligned}
0.7414 &= \exp(-0.003489h) \\
h &= 85.76\ W/m^2\cdot K
\end{aligned}
$$

We can now check to see if our initial assumption was correct.
The characteristic length for the cube is given as

$$
\mathcal{L} = \frac{V}{A} = \frac{(0.03\ m)^3}{6\times(0.03\ m)^2} = 0.005\ m = 5\ mm
$$

The Biot number is

$$
Bi = \frac{h\mathcal{L}}{k} = \frac{(85.76\ W/m^2\cdot K)(0.005\ m)}{401\ W/m\cdot K} = 0.001
$$

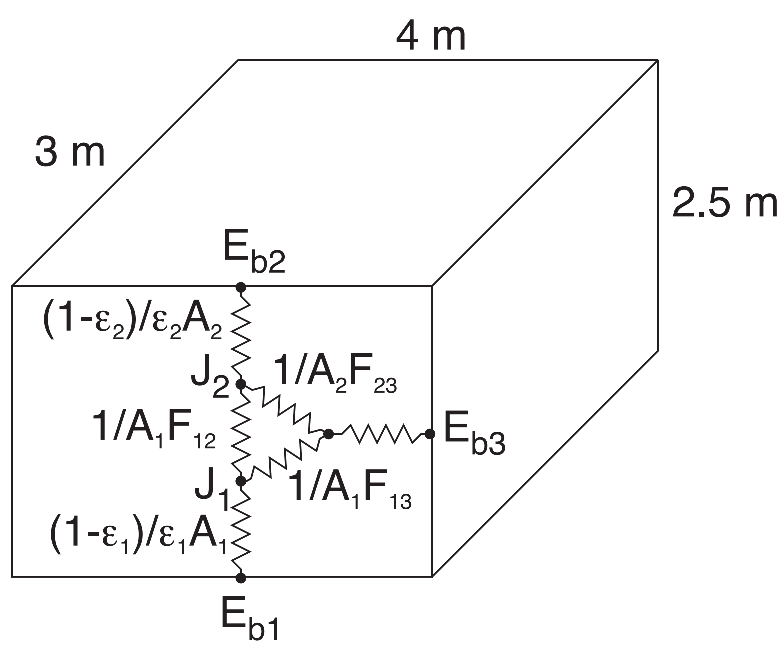
This is much less than $0.1$, therefore our assumption of a lumped system analysis was correct.

**Question 3** (20 marks)

Consider a room that is $4\ m$ long by $3\ m$ wide with a floor-to-ceiling distance of $2.5\ m$. The four walls of the room are well insulated, while the surface of the floor is maintained at a uniform temperature of $30\ °C$ using an electric resistance heater. Heat loss occurs through the ceiling, which has a surface temperature of $12°C$. All surfaces have an emissivity of $0.9$.

a) determine the rate of heat loss, $(W)$, by radiation from the room.
b) determine the temperature, $(K)$, of the walls.

**Part a)**




Modelling the room as a three-surface enclosure with one surface re-radiating, the heat loss equation is given as

$$\dot{Q}_{12} = \frac{E_{b1} - E_{b2}}{R_1 + \left(\frac{1}{R_{12}} + \frac{1}{R_{13} + R_{23}}\right)^{-1} + R_2}$$

We can see from the geometry of the problem

$$A_1 = A_2 = 4 \times 3 = 12m^2$$

From Figure 12-41, for $L_1/D = 4/2.5 = 1.6$ and $L_2/D = 3/2.5 = 1.2$

$$F_{12} \approx 0.29$$

We know that

$$F_{11} + F_{12} + F_{13} = 1 \quad \longrightarrow \quad F_{13} = 1 - F_{12} = 0.71$$

From symmetry we can also conclude that $F_{23} = 0.71$.
Therefore

$$\begin{aligned}
E_{b1} &= \sigma T_1^4 = (5.67 \times 10^{-8}\ W/m^2 \cdot K^4)(30 + 273\ K)^4 = 477.92\ W/m^2 \\
E_{b2} &= \sigma T_2^4 = (5.67 \times 10^{-8}\ W/m^2 \cdot K^4)(12 + 273\ K)^4 = 370.08\ W/m^2 \\
R_1 &= \frac{1-\epsilon_1}{A_1\epsilon_1} = \frac{1-0.9}{12\ m^2 \times 0.9} = 0.00926 \\
R_2 &= \frac{1-\epsilon_2}{A_2\epsilon_2} = \frac{1-0.9}{12\ m^2 \times 0.9} = 0.00926 \\
R_{12} &= \frac{1}{A_1 F_{12}} = \frac{1}{12\ m^2 \times 0.29} = 0.2874 \\
R_{13} &= \frac{1}{A_1 F_{13}} = \frac{1}{12\ m^2 \times 0.71} = 0.1174 \\
R_{23} &= \frac{1}{A_2 F_{23}} = \frac{1}{12\ m^2 \times 0.71} = 0.1174
\end{aligned}$$

Finally

$$\dot{Q}_{12} = \frac{477.92 - 370.08}{0.00926 + \left(\frac{1}{0.2874} + \frac{1}{0.1174 + 0.1174}\right)^{-1} + 0.00926} = 729.9\ W$$

**Part b)**

From the symmetry of the resistance network we can see that

$$J_3 = \frac{E_{b1} + E_{b2}}{2} = \frac{\sigma(T_1^4 + T_2^4)}{2} = \frac{5.67 \times 10^{-8}((273 + 30^4 + (273 + 12)^4)}{2} = 426\ W/m^2$$

The wall temperature can then be calculated as

$$J_3 = E_{b3} = \sigma T_3^4 \ \Rightarrow T_3 = \left(\frac{426\ W/m^2}{5.67 \times 10^{-8}\ W/m^2 \cdot K^4}\right)^{1/4} = 294.4\ K = 21.4\ ^\circ C$$

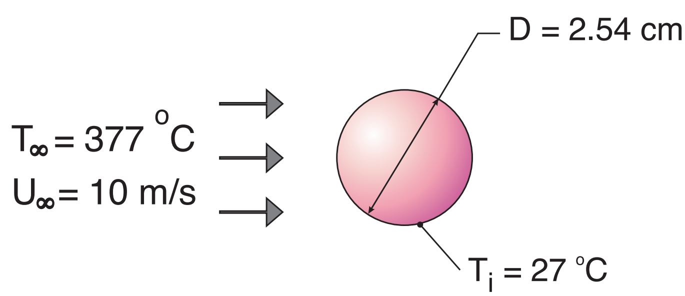
**Question 4** (20 marks)

A new experimental resin is being developed for making canon balls. The resin which is initially formed into a spherical-shaped ball with a diameter of $D = 2.54\ cm$ and a uniform temperature of $27\ ^\circ C$ is cured by suddenly placing it in an air stream with an ambient temperature of $T_\infty = 377\ ^\circ C$ and an ambient flow velocity of $U_\infty = 10\ m/s$. The properties of the resin can be assumed to be:

$$\begin{aligned} \rho_r &= 2500\ kg/m^3 \\ (C_p)_r &= 1100\ J/kg \cdot K \\ k_r &= 7.5\ W/m \cdot K \end{aligned}$$

If the resin cures at $175\ ^\circ C$

a) determine the rate of heat transfer required to cure the resin
b) how long will it take for the sphere to reach the cure temperature?




The fluid properties are calculated at the ambient temperature, $T_\infty = (377 + 273)\ K = 650\ K$

$$\begin{aligned} k &= 0.04845\ W/m \cdot K \\ \nu &= 5.895 \times 10^{-5}\ m^2/s \\ Pr &= 0.70 \end{aligned}$$

we also must calculate

$$\begin{aligned} \mu_\infty &= 3.19 \times 10^{-5}\ kg/m \cdot s \qquad \text{(evaluated at } T_\infty) \\ \mu_w &= 1.85 \times 10^{-5}\ kg/m \cdot s \qquad \text{(evaluated at } T_w) \end{aligned}$$

The Reynolds number is

$$Re_D = \frac{U_\infty D}{\nu} = \frac{10\ m/s \times 0.0254\ m}{5.895 \times 10^{-5}\ m^2/s} = 4308.7$$

Prior to using the correlation for a sphere, we need to check the various limiting conditions

$$Pr = 0.70 \qquad \text{valid for } 0.7 \leq Pr \leq 380 \text{ therefore OK}$$

$$Re_D = 4308.7 \qquad \text{valid for } 3.5 \leq Re_D \leq 80,000 \text{ therefore OK}$$

Therefore we can use the relationship for a sphere

$$\begin{aligned} Nu_D &= 2 + \left(0.4Re_D^{1/2} + 0.06Re_D^{2/3}\right) Pr^{0.4} \left(\frac{\mu_\infty}{\mu_w}\right)^{1/4} \\ &= 2 + \left(0.4 \times (4308.7)^{1/2} + 0.06 \times (4308.7)^{2/3}\right) (0.70)^{0.4} \left(\frac{3.19 \times 10^{-5}}{1.85 \times 10^{-5}}\right)^{1/4} \\ &= 43.87 \end{aligned}$$

The average heat transfer coefficient is

$$\begin{aligned} h &= Nu_D \cdot \frac{k}{D} \\ &= 43.87 \cdot \left[\frac{0.04845\ W/m \cdot K}{0.0254\ m}\right] = 83.69\ W/m^2 \cdot K \end{aligned}$$

**Part a)**

From Newton's law of cooling

$$\begin{aligned} \dot{Q} &= hA(T_{cure} - T_i) \\ &= (83.69\ W/m^2 \cdot K) \times (\pi \cdot 0.0254^2)(175 - 27)\ K \\ &= 25.1\ W \Leftarrow \end{aligned}$$

**Part b)**

The Biot number for a lumped system approach can be determined by using a characteristic length of $r/3$ or $D/6$.

$$Bi = \frac{h \cdot (r/3)}{k_r} = \frac{(83.69\ W/m^2 \cdot K)(0.0254/6)\ m}{7.5\ W/m \cdot K} = 0.047$$

Since this is less than $Bi = 0.1$ we can use a lumped system analysis to find the cure time. The temperature response for a lumped system analysis is given as

$$\frac{T(t) - T_\infty}{T_i - T_\infty} = e^{-t/\tau}$$

where $\boldsymbol{\tau = RC = \frac{\rho_r V (C_p)_r}{h A_s}}$.

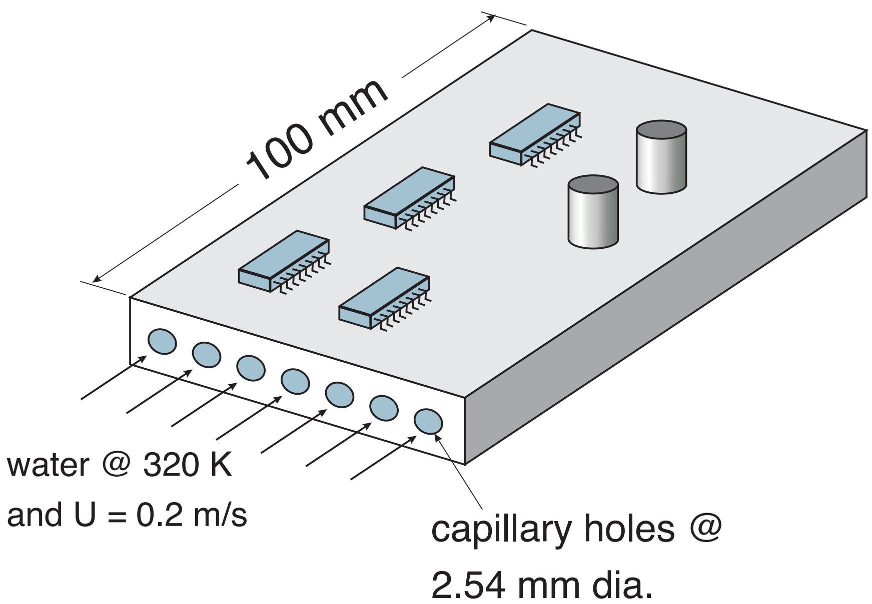
Therefore

$$
\begin{aligned}
\boldsymbol{t} &= \boldsymbol{-\left(\frac{\rho_r V (C_p)_r}{h A_s}\right) \ln\left[\frac{T(t) - T_\infty}{T_i - T_\infty}\right]} \\
&= \boldsymbol{-\left[\frac{\left(2500\ \frac{kg}{m^3}\right)\left(\frac{4}{3}\pi \cdot 0.0127^3\ m^3\right)\left(1100\ \frac{J}{kg \cdot K}\right)}{(83.69\ W/m^2 \cdot K)\,(\pi \cdot 0.0254^2\ m^2)}\right] \times \ln\left[\frac{175 - 377}{27 - 377}\right]} \\
&= \boldsymbol{76.5\ s \Leftarrow}
\end{aligned}
$$

**Question 5** (20 marks)

An electronic device is cooled by water flowing through capillary holes drilled in the casing as shown below. The temperature of the device casing is considered constant at $350\ K$. The capillary holes are $100\ mm$ long and $2.54\ mm$ in diameter. If water enters at a temperature of $320\ K$ and flows at a velocity of $0.2\ m/s$, calculate the outlet temperature of the water.

Note: Since the mean temperature of the fluid stream cannot be determined without already knowing the outlet temperature, use the inlet temperature of the fluid to calculate the fluid properties.




The mean fluid properties will be calculated at the inlet temperature.

From Table A-18, at an inlet temperature of $T_{in} = 320\ K$

$$\begin{aligned}\rho &= 989\ kg/m^3 \\ C_p &= 4176\ J/(kg \cdot K) \\ \mu &= 0.579 \times 10^{-3}\ kg/(m \cdot s) \\ k &= 0.637\ W/(m \cdot K) \\ Pr &= 3.79\end{aligned}$$

and at the wall temperature, $T_w = 350\ K$,

$$\mu_w = 0.3715\ \times 10^{-3}\ kg/(m \cdot s)$$

Check to see if the flow is laminar

$$Re_D = \frac{\rho U D}{\mu} = \frac{(989\,kg/m^3)(0.2\,m/s)(0.00254\,m)}{0.579\times10^{-3}\,kg/(m\cdot s)} = 867.72$$

This Reynolds number is well below the critical Reynolds for a tube of $2300$, therefore the flow is laminar.
We also need to check to see if the flow is in the developing region

$$\begin{aligned} L_h &= 0.05 Re_D D = 0.05\times867.72\times0.00254\,m = 0.1102\,m \\ L_t &= 0.05 Re_D Pr D = 0.05\times867.72\times3.79\times0.00254\,m = 0.4177\,m \end{aligned}$$

Both $L_h$ and $L_t$ are greater that $L$, therefore the entire length of the capillary tube is in the developing region and we can use the correlation

$$\begin{aligned} Nu_D &= 1.86\left(\frac{Re_D Pr D}{L}\right)^{1/3}\left(\frac{\mu_b}{\mu_w}\right)^{0.14} \\ &= 1.86\left(\frac{867.72\times3.79\times0.00254\,m}{0.1\,m}\right)^{1/3}\left(\frac{0.579\times10^{-3}}{0.3715\times10^{-3}}\right)^{0.14} \\ &= 8.65 \end{aligned}$$

and

$$h = \frac{kNu_D}{D} = \frac{0.637\,W/(m\cdot K)\times8.65}{0.00254\,m} = 2169.3\,W/(m^2\cdot K)$$

The heat transfer at the surface of the tube can be determined using Newtons law of cooling as

$$\begin{aligned} \dot{Q} = hA(T_w - T_m) &= h\times(\pi D L)\times\left(T_w - \frac{T_{in}+T_{out}}{2}\right) \\ &= \left(2169.3\,\frac{W}{m^2\cdot K}\right)(\pi\times0.00254\,m\times0.1\,m)\left(350\,K - \frac{320\,K+T_{out}}{2}\right) \\ &= 328.89 - 0.8655T_{out} \end{aligned}$$

The heat transfer in the fluid in the flow direction is

$$\dot{Q} = \dot{m}C_p(T_{out} - T_{in})$$

where the mass flow rate is determined as

$$\dot{m} = \rho\frac{\pi D^2}{4}U = \frac{(989\,kg/m^3)\pi(0.00254\,m)^2(0.2\,m/s)}{4} = 0.001\,kg/s$$

We can then solve for $T_{out}$ as

$$328.89 - 0.8655T_{out} = 0.001\times4176\times(T_{out} - 320)$$

$$T_{out} = \frac{1665.21}{5.0415} = 330.3\,K \Leftarrow$$