University of Waterloo

ECE309

THERMODYNAMICS & HEAT TRANSFER
MIDTERM EXAMINATION

June 13, 2016 2:30 pm - 4:30 pm

Instructor: R. Culham

Name: ______________________________

Student ID Number: ______________________________

Instructions

1. This is a 2 hour, closed-book examination.
2. Permitted aids include:
   - one 8.5 in.× 11 in. crib sheet (one side only),
   - Property Tables Booklet for Introduction to Thermodynamics and Heat Transfer, Yunus Cengel, 2nd ed
   - calculator
3. Answer all questions in the space provided. If additional space is required, use the back of the pages or the blank pages included.
4. It is your responsibility to write clearly and legibly. Clearly state all assumptions. Part marks will be given for part answers, provided that your methodology is clear.

Question	Marks	Grade
1	15	
2	19	
3	16	
TOTAL	50	

Question 1 (15 marks)

A rigid container holds water at the critical point. The container is cooled until the pressure reaches a) $\mathbf{10\ MPa}$, b) $\mathbf{1.0\ MPa}$ and c) $\mathbf{0.1\ MPa}$. Determine the temperature, $\boldsymbol{T_{crit}}$ and pressure, $\boldsymbol{P_{crit}}$ at the critical point. Determine the quality of the water at each value of pressure, the temperature and the heat loss necessary to move between each value of pressure. Enter your calculated values in the table below. Show all calculations.

	State Point	Pressure ($\boldsymbol{MPa}$)	Temperature ($°\boldsymbol{C}$)	Quality ($-$)	$\boldsymbol{\Delta q}$ ($\boldsymbol{kJ/kg}$)
critical point ⇒	1				
	2	**10.0**			
	3	**1.0**			
	4	**0.1**			

Assumptions

1. container is rigid, volume is constant
2. since the mass and the volume are constant, the specific volume is constant
3. quassi steady state between state points

State Point 1
At the critical point of water

$$\boldsymbol{P_{crit} = 22.064\ MPa} \qquad \boldsymbol{T_{crit} = 373.95\ °C}$$

$$\boldsymbol{v_{crit} = 0.003106\ m^3/kg} \qquad \boldsymbol{u_{crit} = 2015.7\ kJ/kg}$$

State Point 2
At $\mathbf{10}\ \boldsymbol{MPa}$ and $\mathbf{0.003106}\ \boldsymbol{m^3/kg}$

$$\begin{aligned}
T_2 &= 311\ °C \\
x_2 &= \frac{v - v_f}{v_g - v_f} = \frac{0.003106 - 0.001452}{0.018028 - 0.001452} = 0.0997828 \\
u_2 &= u_f + x u_{fg} = 1393.3 + 0.0997828 \times 1151.8 = 1508.23\ kJ/kg \\
\Delta q_{1-2} &= u_{crit} - u_2 = 2015.7 - 1508.23 = 507.47\ kJ/kg
\end{aligned}$$

State Point 3
At **1** $\boldsymbol{MPa}$ and **0.003106** $\boldsymbol{m^3/kg}$

$$\begin{aligned}
\boldsymbol{T_3} &= \boldsymbol{179.88\ ^\circ C} \\
\boldsymbol{x_3} &= \frac{v - v_f}{v_g - v_f} = \frac{0.003106 - 0.001127}{0.19436 - 0.001127} = 0.0102415 \\
\boldsymbol{u_3} &= u_f + x u_{fg} = 761.39 + 0.0102415 \times 1821.4 = 780.04\ kJ/kg \\
\boldsymbol{\Delta q_{2-3}} &= u_2 - u_3 = 1508.23 - 780.04 = 728.19\ kJ/kg
\end{aligned}$$

State Point 4
At **0.1** $\boldsymbol{MPa}$ and **0.003106** $\boldsymbol{m^3/kg}$

$$\begin{aligned}
\boldsymbol{T_4} &= \boldsymbol{99.61\ ^\circ C} \\
\boldsymbol{x_4} &= \frac{v - v_f}{v_g - v_f} = \frac{0.003106 - 0.0001043}{0.1.6941 - 0.0001043} = 0.0012185 \\
\boldsymbol{u_4} &= u_f + x u_{fg} = 417.40 + 0.0012185 \times 2088.2 = 419.95\ kJ/kg \\
\boldsymbol{\Delta q_{3-4}} &= u_3 - u_4 = 780.04 - 419.95 = 360.10\ kJ/kg
\end{aligned}$$

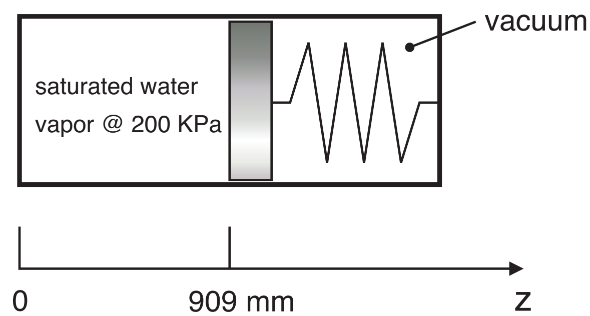
State Point	Pressure ($\boldsymbol{MPa}$)	Temperature ($^\circ\boldsymbol{C}$)	Quality ($-$)	$\boldsymbol{\Delta q}$ ($\boldsymbol{kJ/kg}$)
1	22.064	373.95		
2	10.0	311.0	0.0997828	507.47
3	1.0	179.88	0.0102415	728.19
4	0.1	99.61	0.0012185	360.10

<u>**Question 2**</u> (19 marks)

A cylinder fitted with a frictionless piston and a linear spring initially contains saturated water vapor at $\boldsymbol{200\ kPa}$. The spring side of the piston is completely evacuated. Under these conditions the piston is $\boldsymbol{909\ mm}$ from the left end of the cylinder. The cross-sectional area of the cylinder is $\boldsymbol{0.05\ m^2}$. The spring force is given by

$$\boldsymbol{F_s = k \times z}$$

where the spring constant is given as $\boldsymbol{k = 11\ kN/mm}$ and $\boldsymbol{z}$ is the distance in $\boldsymbol{mm}$ between the piston face and the left end of the cylinder. The contents of the cylinder are then cooled until the pressure reaches $\boldsymbol{100\ kPa}$. Assume that <u>pressure changes in a linear manner between state 1 and state 2</u> and that the surrounding air temperature is $\boldsymbol{20\ °C}$.



a) determine the amount of heat transfer [$\boldsymbol{kJ}$]
b) draw and label a $\boldsymbol{T - s}$ diagram, including all relevant state points, and property values at those state points
c) determine the entropy generated during the process [$\boldsymbol{kJ/K}$]

Assumptions

1. the piston is frictionless
2. $\boldsymbol{\Delta KE = \Delta PE = 0}$
3. the compression process is quassi-equilibrium
4. steady state
5. the surroundings are a TER at $\boldsymbol{20\ °C}$
6. pressure changes in a linear manner between state 1 and state 2

Part a) Let the initial state be state 1 and the final state be state 2.

Force Balance
From the saturated water tables (Table A-5)

$$\boldsymbol{v_1 = 0.88578\ m^3/kg} \qquad \boldsymbol{u_1 = 2529.1\ kJ/kg}$$

Now we need to find $\boldsymbol{v_2}$. Two approaches are possible:

1. establish a relationship between specific volume and pressure
2. establish a relationship between specific volume and distance travelled by the piston

Both approaches will lead to the same result and both are acceptable solutions.

specific volume → pressure

A force balance over the piston gives:

$$P = \frac{F_s}{A} = \frac{k}{A} \times z$$

But the specific volume is

$$v = \frac{V}{m} = \frac{A \cdot z}{m} \quad \Rightarrow z = \frac{v \cdot m}{A}$$

therefore

$$P = \left(\frac{k \cdot m}{A^2}\right) v$$

Since k, m, and A are constant between state 1 and state 2, we can write

$$\frac{P_2}{P_1} = \frac{v_2}{v_1}$$

$$\begin{aligned} v_2 &= \left(\frac{P_2}{P_1}\right) v_1 \\ &= \left(\frac{100\ kPa}{200\ kPa}\right) \times 0.88578\ \frac{m^3}{kg} \\ &= 0.44289\ m^3/kg \end{aligned}$$

specific volume → distance

$z_1 = 0.909\ m$ is given in the problem.

We know that

$$z = \frac{P \cdot A}{k}$$

therefore

$$\begin{aligned} z_2 &= (100\ kPa) \times \frac{\left(1\ \frac{kN/m^2}{kPa}\right)}{11\ kN/m} (0.05\ m^2) \\ &= 0.4545\ m \end{aligned}$$

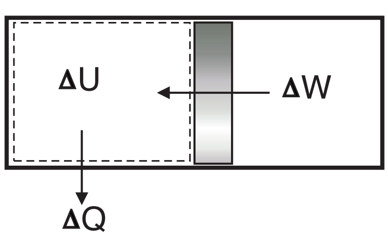
Since A and m are constant between state 1 and state 2, we can write

$$\frac{v_2}{v_1} = \frac{z_2}{z_1}$$

and

$$\begin{aligned} v_2 &= \frac{z_2 v_1}{z_1} \\ &= \left(\frac{0.4545\ m}{0.909\ m}\right) \times 0.88578\ \frac{m^3}{kg} \\ &= 0.44289\ m^3/kg \end{aligned}$$

Energy Balance



$$\begin{aligned} \Delta Q &= \Delta W - \Delta U \\ &= -\int_{V_1}^{V_2} (P \cdot dV) - (U_2 - U_1) \\ &= m \left[\int_{v_2}^{v_1} (P \cdot dv) - u_2 + u_1\right] \end{aligned}$$

$$\begin{aligned} \int_{v_2}^{v_1} P \cdot dv &= \left(\frac{P_1 + P_2}{2}\right)(v_1 - v_2) \\ &= \left(\frac{200 + 100}{2}\right) kPa \times \left(\frac{1\ kJ/m^3}{1\ kPa}\right) \times (0.88578 - 0.44289)\ m^3/kg \\ &= 66.43\ kJ/kg \end{aligned}$$

We know that state point 1 is on the saturated vapor line but we need to find the location of state point 2 under the dome at $\boldsymbol{P_2 = 100\ kPa}$.

$$x_2 = \frac{v_2 - v_{f2}}{v_{g2} - v_{f2}} = \frac{0.44289 - 0.001043}{1.6941 - 0.001043} = 0.261$$

and

$$\begin{aligned} u_2 = u_{f2} + x_2 \times u_{fg_2} &= 417.4 + 0.261 \times 2088.2\ kJ/kg \\ &= 962.42\ kJ/kg \end{aligned}$$

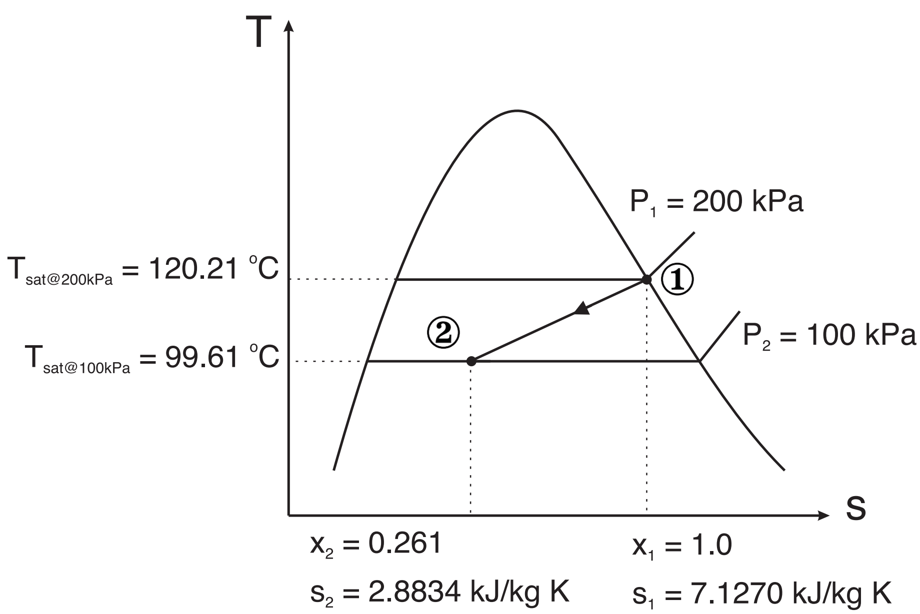
The mass can be determined from our earlier relationship $\quad P = \left(\frac{k \cdot m}{A^2}\right) v$

$$\begin{aligned} m = \frac{P_1 \times A^2}{k \times v_1} &= \frac{(200\ kPa)\left(\frac{1\ kN/m^2}{1\ kPa}\right)(0.05\ m^2) \times (0.05\ m^2)}{(11\ kN/m)(0.88578\ m^3/kg)} \\ &= 0.0513\ kg \end{aligned}$$

Then

$$\begin{aligned} \Delta Q &= m\left[\int_{v_2}^{v_1} (P \cdot dv) - u_2 + u_1\right] \\ &= (0.0513\ kg)\left[(66.43\ kJ/kg) - 962.42\ kJ/kg + 2529.1\ kJ/kg\right] \\ &= 83.8\ kJ \Leftarrow \end{aligned}$$

Part b)



Part c)

Entropy Balance

$$S_1 - \frac{Q}{T_0} + S_{gen} = S_2$$

or

$$S_{gen} = m(s_2 - s_1) + \frac{Q}{T_0}$$

The entropy at state point 2 is calculated as

$$\begin{aligned} s_2 &= s_{f2} + x_2(s_{g2} - s_{f2}) \\ &= 1.3028 + 0.261(7.3589 - 1.3028)\ kJ/kg \\ &= 2.8834\ kJ/kg \cdot K \end{aligned}$$

The entropy at state point i is given as

$$s_1 = s_{f@200\ kPa} = 7.1270\ kJ/kg \cdot K$$

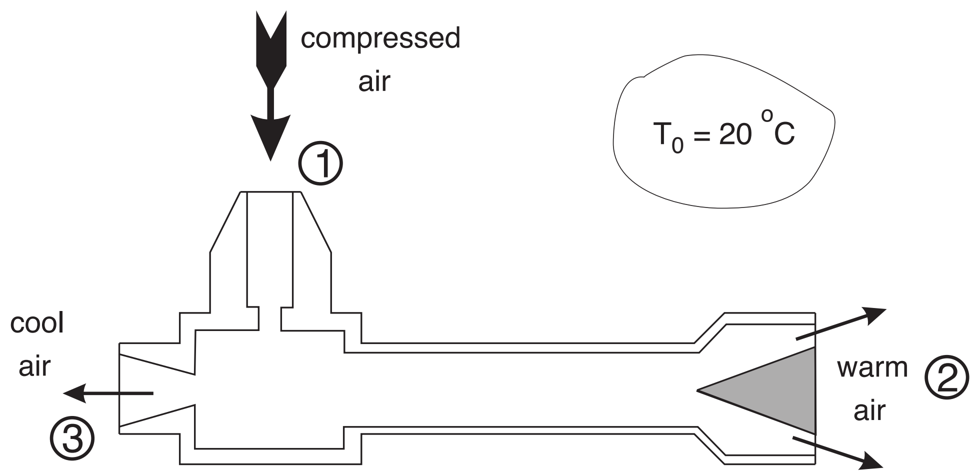
The entropy generated is

$$S_{gen} = (0.0513\ kg)(2.8834 - 7.1270)\ kJ/kg \cdot K + \frac{83.8\ kJ}{293\ K} = 0.068\ kJ/K \Leftarrow$$

Question 3 (16 marks)

A vortex tube is a steady-state device that splits a high pressure gas stream into two streams, one warm and one cool. During a controlled test, it was determined that **85%** of the compressed air entering the vortex tube went to the warm air stream and **15%** went to the cool air stream. Compressed air enters the vortex tube at $\mathbf{19.3\ °C}$, $\mathbf{0.52\ MPa}$ and a mass flow rate of $\mathbf{1.5\ kg/s}$. Warm air leaves the vortex tube at $\mathbf{26.3\ °C}$ while the cool air leaves at $\mathbf{-21.8\ °C}$. Both exit streams are at atmospheric pressure of $\mathbf{101.35\ kPa}$. The temperature of the surroundings can be assumed to be $\mathbf{20\ °C}$. Determine:

a) the rate of heat loss from the vortex tube to the surroundings, $[\mathbf{kW}]$
b) the rate of entropy generation in the vortex tube, $[\mathbf{kW/K}]$



Assumptions

1. steady state, steady flow
2. $\mathbf{\Delta KE = \Delta PE = 0}$
3. quassi equilibrium
4. properties are calculated at $\boldsymbol{T_{avg}}$

Part a)

From conservation of mass we know

$$\dot{m}_2 = 0.85 \times 1.5\ kg/s = 1.275\ kg/s$$
$$\dot{m}_3 = 0.15 \times 1.5\ kg/s = 0.225\ kg/s$$

First calculate the temperature compensated values of specific heat from Table A-2b

$$\begin{aligned}
\text{at} \quad \frac{T_1+T_2}{2} &= \frac{19.3+26.3}{2} = 22.8\ ^\circ C = 295.95\ K \\
\longrightarrow c_p &= 1.0048\ kJ/kg\cdot K \\
\text{at} \quad \frac{T_1+T_3}{2} &= \frac{19.3-21.8}{2} = -1.25\ ^\circ C = 271.90\ K \\
\longrightarrow c_p &= 1.0039\ kJ/kg\cdot K
\end{aligned}$$

From conservation of energy we know

$$\begin{aligned}
\dot{Q}_{out} &= \dot{m}_1\cdot h_1 - \dot{m}_2\cdot h_2 - \dot{m}_3\cdot h_3 \\
&= (\dot{m}_2+\dot{m}_3)\cdot h_1 - \dot{m}_2\cdot h_2 - \dot{m}_3\cdot h_3 \\
&= \dot{m}_2(h_1-h_2) + \dot{m}_3(h_1-h_3) \\
&= \dot{m}_2 c_p(T_1-T_2) + \dot{m}_3 c_p(T_1-T_3) \\
&= (1.275\ kg/s)\times(1.0048\ kJ/kg\cdot K)\times(292.45-299.45)\ ^\circ C \\
&\quad +(0.225\ kg/s)\times(1.0039\ kJ/kg\cdot K)\times(292.45-251.35)\ ^\circ C \\
&= 0.3157\ kW \ \Leftarrow \ \text{part a)}
\end{aligned}$$

Part b)

A second law balance gives

$$\dot{m}_1 s_1 + \dot{S}_{gen} = \dot{m}_2 s_2 + \dot{m}_3 s_3 + \frac{\dot{Q}_{out}}{T_0}$$

or

$$\begin{aligned}
\dot{S}_{gen} &= \dot{m}_2 s_2 + \dot{m}_3 s_3 - \dot{m}_1 s_1 + \frac{\dot{Q}_{out}}{T_0} \\
&= \dot{m}_2 s_2 + \dot{m}_3 s_3 - (\dot{m}_2+\dot{m}_3) s_1 + \frac{\dot{Q}_{out}}{T_0} \\
&= \dot{m}_2(s_2-s_1) + \dot{m}_3(s_3-s_1) + \frac{\dot{Q}_{out}}{T_0}
\end{aligned}$$

where

$$
\begin{aligned}
s_2 - s_1 &= c_p \ln(T_2/T_1) - R\ln(P_2/P_1) \\
&= 1.0048 \ \frac{kJ}{kg \cdot K} \ln(299.45/292.45) - 0.286 \ \frac{kJ}{kg \cdot K} \ln(0.10135/0.52) \\
&= 0.4914 \ \frac{kJ}{kg \cdot K} \\
s_3 - s_1 &= c_p \ln(T_3/T_1) - R\ln(P_3/P_1) \\
&= 1.0039 \ \frac{kJ}{kg \cdot K} \ln(251.35/292.45) - 0.286 \ \frac{kJ}{kg \cdot K} \ln(0.10135/0.52) \\
&= 0.3156 \ \frac{kJ}{kg \cdot K}
\end{aligned}
$$

Therefore the entropy production can be written as

$$
\begin{aligned}
\dot{S}_{gen} &= (1.275 \ kg/s)\left(0.4914 \ \frac{kJ}{kg \cdot K}\right) + (0.225 \ kg/s)\left(0.3156 \ \frac{kJ}{kg \cdot K}\right) \\
&\qquad + \frac{0.3157 \ kW}{(20 + 273.15) \ K} \\
&= 0.6986 \ kW/K \ \Leftarrow \ \text{part b)}
\end{aligned}
$$