ECE309

INTRODUCTION TO THERMODYNAMICS & HEAT TRANSFER

16 June 2004

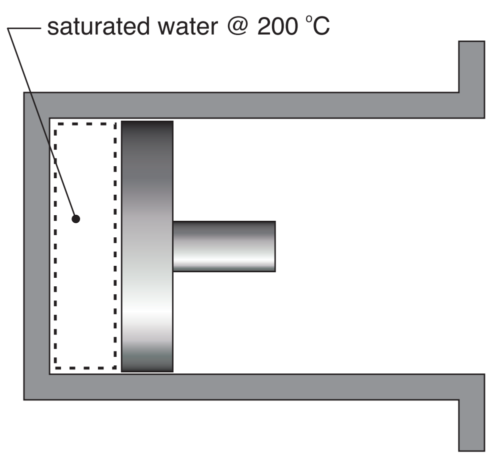
Midterm Examination R. Culham

- This is a 90 minute, closed-book examination.
- You are permitted to use one 8.5 in.$\times$ 11 in. crib sheet (one side only), Conversion Factors (inside cover of text) and the Property Tables and Figures from your text book.
- There are 4 questions to be answered. Read the questions very carefully.
- Clearly state all assumptions.
- It is your responsibility to write clearly and legibly.

Question 1 (20 marks)

Initially saturated water at $200\ ^\circ C$ is contained in a piston-cylinder device as shown below. The water is then heated isothermally until its volume is 100 times larger than its initial volume.

a) Determine the increase in energy $[kJ/kg]$ of the water
b) Determine the work transfer $[kJ/kg]$ and indicate whether it is into or out of the water.
c) Determine the heat transfer $[kJ/kg]$ and indicate whether it is into or out the water.



Part a)

From Table A-4, the specific volume of saturated liquid water at $200\ ^\circ C$ is

$$v_1 = 0.001157\ m^3/kg$$

The volume at state 2 is $100\times$ larger than state 1

$$v_2 = 100 \times (0.001157\ m^3/kg) = 0.1157\ m^3/kg$$

The quality at state 2 is

$$x_2 = \frac{v_2 - v_f}{v_g - v_f} = \frac{0.1157 - 0.001157}{0.12736 - 0.001157} = 0.9076$$

The internal energy at state 2 is

$$u_2 = u_f + x_2(u_g - u_f) = 850.65 + 0.9076 \cdot (2595.3 - 850.65) = 2434.09 \, \frac{kJ}{kg}$$

The internal energy at state 1 is

$$u_1 = u_f = 850.65 \, kJ/kg$$

The increase in energy of the water is

$$u_2 - u_1 = (2434.09 - 850.65) \, kJ/kg = \underline{1583.44 \, kJ/kg} \Leftarrow$$

Part b)

The work transfer by way of expansion is given as

$$\Delta W = -\int_{V_1}^{V_2} P \, dV = P(V_1 - V_2) = P \times m \times (v_1 - v_2)$$

Noting that the saturation pressure at $T = 200 \, °C$ is $P = 1.5538 \, MPa$
The work per unit mass is given as

$$\begin{aligned} w = \frac{W}{m} &= P(v_1 - v_2) \\ &= (1553.8 \, kPa) \left(\frac{1 \, kJ/m^3}{1 \, kPa} \right) \times (0.001157 - 0.1157) \, m^3/kg \\ &= \underline{-177.98 \, kJ/kg} \Leftarrow \end{aligned}$$

where negative work implies the work is out of the water.

Part c)

From an energy balance on the water, we can write

$$\begin{aligned} \Delta Q &= \Delta W + \Delta U \\ \frac{\Delta Q}{m} &= \frac{\Delta W}{m} + (u_2 - u_1) \\ &= (177.98 + 1583.44) \, kJ/kg = \underline{1761.42 \, kJ/kg} \Leftarrow \end{aligned}$$

where positive heat transfer implies energy addition to the water.

Question 2 (20 marks)

Air at $40\ °C$ and $0.6\ MPa$ enters a $25\ mm$ diameter pipeline at a mass flow rate of $0.01\ kg/s$. The air flows through several valves and leaves the pipeline at a pressure of $110\ kPa$. The pipeline and the valves can be assumed to be adiabatic.

a) Determine the velocity of the air $[m/s]$ entering the pipeline.
b) Neglecting changes in kinetic energy, determine the exit state, i.e. T and v of air.
c) Based on your answer to part b), determine the exit velocity $[m/s]$ of the air.
d) Determine the exit state, T and v of the air if the change in kinetic energy is accounted for by assuming the exit velocity is the value found in part c). How much difference is there between your new state and the state found in part b)? What conclusion do you reach regarding the importance of including kinetic energy terms in your analysis?

Part a)

Assume that the air behaves as an ideal gas and use the gas constant for air at a constant temperature of $300\ K$

$$R_{air} = 0.287\ kJ/(kg \cdot K)$$

The specific volume of the air at the entrance can be determined as

$$v_1 = \frac{R_{air} \times T_1}{P_1} = \frac{0.287\ \dfrac{kJ}{(kg \cdot K)} \times (40 + 273.15)\ K}{600\ kPa\ \left(\dfrac{1\ kJ/m^3}{1\ kPa}\right)} = 0.1498\ m^3/kg$$

The velocity at the entrance is

$$\mathcal{V}_1 = \frac{\dot{m} \times v_1}{A} = \frac{(0.01\ kg/s) \times (0.1498\ m^3/kg)}{\dfrac{\pi (0.025\ m)^2}{4}} = \underline{3.05\ m/s} \Leftarrow$$

Part b)

If kinetic energy is considered negligible and the pipe and valves are considered adiabatic, the energy balance between the inlet and the outlet can be written as

$$\dot{m} h_1 = \dot{m} h_2 \qquad \longrightarrow \qquad h_1 = h_2$$

Since the air is considered an ideal gas we can write

$$h_1 = h_2 \qquad \longrightarrow \qquad \underline{T_1 = T_2 = 40\ °C} \Leftarrow$$

The specific volume at the exit can then be written as

$$v_2 = \frac{R_{air} \times T_2}{P_2} = \frac{0.287\ \dfrac{kJ}{(kg \cdot K)} \times (40 + 273.15)\ K}{110\ kPa\ \left(\dfrac{1\ kJ/m^3}{1\ kPa}\right)} = \underline{0.8170\ m^3/kg} \Leftarrow$$

Part c)

The velocity at the exit is given as

$$\mathcal{V}_2 = \frac{\dot{m} \times v_2}{A} = \frac{(0.01\ kg/s) \times (0.8170\ m^3/kg)}{\dfrac{\pi(0.025\ m)^2}{4}} = \underline{16.64\ m/s} \Leftarrow$$

Part d)

If we now include the effects of kinetic energy, the energy balance equation is written as

$$\begin{aligned}
\dot{m}\left(h_1 + \frac{\mathcal{V}_1^2}{2}\right) &= \dot{m}\left(h_2 + \frac{\mathcal{V}_2^2}{2}\right) \\
C_p(T_2 - T_1) &= \left(\frac{\mathcal{V}_1^2 - \mathcal{V}_2^2}{2}\right) \\
T_2 - T_1 &= \left(\frac{\mathcal{V}_1^2 - \mathcal{V}_2^2}{2C_p}\right) \\
&= \frac{[(3.05)^2 - (16.64)^2]\ m^2/s^2\ \left(\dfrac{1\ kJ/kg}{1000\ m^2.s^2}\right)}{2 \times 1.005\ kJ/(kg \cdot K)} = -0.133\ K
\end{aligned}$$

Therefore

$$T_2 = (40 + 273.15)\ K - 0.133\ K = \underline{313.017\ K} \Leftarrow$$

and

$$v_2 = \frac{R_{air} \times T_2}{P_2} = \frac{0.287\ \dfrac{kJ}{(kg \cdot K)} \times 313.017\ K}{110\ kPa\ \left(\dfrac{1\ kJ/m^3}{1\ kPa}\right)} = \underline{0.81669\ m^3/kg} \Leftarrow$$

Both the temperature and the specific volume are with 0.04% of the solution when kinetic energy was ignored. Therefore kinetic energy is insignificant in this calculation.

Question 3 (15 marks)

Air is contained in a rigid, adiabatic $0.3\ m^3$ container at $20\ ^\circ C$ and $101.325\ kPa$. A paddle wheel sticking into the container then does $50\ kJ$ of work. Determine the entropy produced $[kJ/K]$.

Since we are working with air we can assume ideal gas and therefore

$$m = \frac{P_1\, V_1}{R\, T_1} = \frac{(101.325\ kPa)\left(\frac{1\ kJ/m^3}{1\ kPa}\right)(0.3\ m^3)}{(0.287\ kJ/(kg \cdot K))(20 + 273.15)\ K} = 0.3613\ kg$$

From a 1st law energy balance we know

$$\Delta W = \Delta U = m\, C_v\, (T_2 - T_1)$$

Therefore we can solve for T_2 as

$$T_2 = \frac{\Delta W}{m\, C_v} + T_1 = \frac{50\ kJ}{(0.3613\ kg)\ (0.718\ kJ/(kg \cdot K))} + 293.15\ K$$

$$= 485.89\ K = 212.74\ ^\circ C$$

The change in entropy is given as

$$s_2 - s_1 = C_v \ln\left(\frac{T_2}{T_1}\right) + R \ln\left(\frac{v_2}{v_1}\right)$$

Since the container is rigid, $v_1 = v_2$ and the second term in the above equation goes to zero.

$$s_2 - s_1 = C_v \ln\left(\frac{T_2}{T_1}\right) = 0.718\ kJ/(kg \cdot K) \ln\left(\frac{485.89\ K}{293.15\ K}\right) = 0.3628\ \frac{kJ}{kg \cdot K}$$

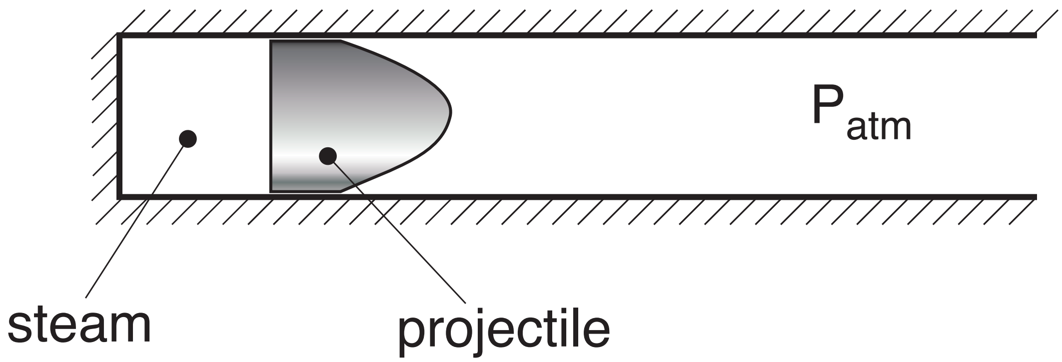
The entropy produced is

$$\Delta S = m \cdot (s_2 - s_1) = (0.3613\ kg)\left(0.3628\ \frac{kJ}{kg \cdot K}\right) = \underline{0.131\ kJ/K} \Leftarrow$$

Question 4 (20 marks)

One kilogram of superheated water vapour (steam) at $700\ °C$ and $2.0\ MPa$ is contained in an ejection tube behind a $50\ kg$ projectile that is initially held in place by a pin. The pin is then removed and the vapour pushes the projectile forward into the reference atmosphere at $T_{atm} = 20\ °C$ and $P_{atm} = 100\ kPa$. The process is adiabatic and occurs without friction.

a) determine the expansion ratio V_2/V_1 necessary to obtain the maximum possible projectile velocity. Hint: a process with no irreversibilities.
b) determine the maximum possible projectile velocity $[m/s]$.



Part a)

For maximum velocity the expansion process must be reversible. Since the process is adiabatic, an entropy balance gives

$$m_s(s_2 - s_1) = 0$$

where m_s is the mass of the steam.

$$s_2 = s_1$$

From the superheated water vapour tables at $700\ °C$ and $2.0\ MPa$,

$$\begin{aligned} s_2 = s_1 &= 7.9487\ kJ/kg \cdot K \\ v_1 &= 0.2232\ m^3/kg \\ u_1 &= 3470.9\ kJ/kg \end{aligned}$$

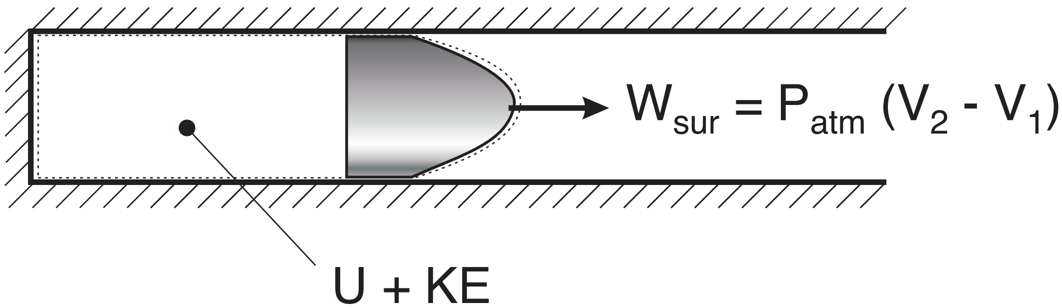
At $s_2 = 7.9487\ kJ/kg \cdot K$ and atmospheric pressure of $100\ kPa$

$$\begin{aligned} v_2 &= 2.307\ m^3/kg \\ u_2 &= 2701.6\ kJ/kg \end{aligned}$$

Therefore

$$\frac{V_2}{V_1} = \frac{m_s \cdot v_2}{m_s \cdot v_1} = \frac{v_2}{v_1} = \frac{2.307}{0.2232} = \underline{10.336} \Leftarrow$$

Part b)



If we draw a control volume around the steam and the bullet, the 1st law energy balance gives

$$E_1 - W_{sur} = E_2$$

since work must be directed out of the control volume to overcome the ambient resistance.

$$\begin{aligned} 0 &= (E_2 - E_1) + W_{sur} \\ 0 &= (U_2 - U_1) + \frac{1}{2} m \mathcal{V}_2^2 + P_{atm}(V_2 - V_1) \end{aligned}$$

since, $\mathcal{V}_1 = 0$ (starting in a stationary position) and potential energy is considered negligible.

Since the mass of the projectile (m_p) is significantly larger than the mass of the steam (m_s), we can assume that the kinetic energy term is only a function of the mass of the projectile and $KE = m_p \mathcal{V}_2^2/2$.

Solving for the velocity at state 2 gives

$$m_p \frac{\mathcal{V}^2}{2} = m_s[-P_{atm}(v_2 - v_1) - (u_2 - u_1)]$$

where

$$\begin{aligned} P_{atm}(v_2 - v_1) &= 100\, kPa \times 1000\, \frac{kg \cdot m/s^2}{kPa} [2.307 - 0.2232] \frac{m^3}{kg} \\ &= 208,308\, m^2/s^2 \\ u_2 - u_1 &= (2701.6 - 3470.9)\, kJ/kg \times 1000\, \frac{m^2/s^2}{kJ/kg} \\ &= -769,300\, m^2/s^2 \end{aligned}$$

$$
\begin{aligned}
\mathcal{V}_2 &= \sqrt{2 \cdot \frac{m_s}{m_p} \left[P_{atm}(v_2 - v_1) - (u_2 - u_1)\right]} \\
&= \sqrt{2 \cdot \frac{1}{50} \left[-208,308 - (-769,300)\right]} \\
&= 149.8 \; m/s
\end{aligned}
$$

Therefore the maximum velocity is given as

$$
\mathcal{V}_2 = \underline{149.8 \; m/s} \Leftarrow
$$