ECE 309
Introduction to Thermodynamics and Heat Transfer

Tutorial # 10

Natural Convection

Problem Consider a 15 cm × 20 cm printed circuit board (PCB) that has electronic components on one side. The board is placed in a room at 20ºC. The heat loss from the back surface of the board is negligible. If the circuit board is dissipating 8 W of power in steady operation, determine the average temperature of the hot surface of the board, assuming the board is

(a) vertical
(b) horizontal with hot surface facing up and
(c) horizontal with hot surface facing down.

Take the emissivity of the surface of the board to be 0.8, and assume the surrounding surfaces to be at the same temperature as the air in the room.

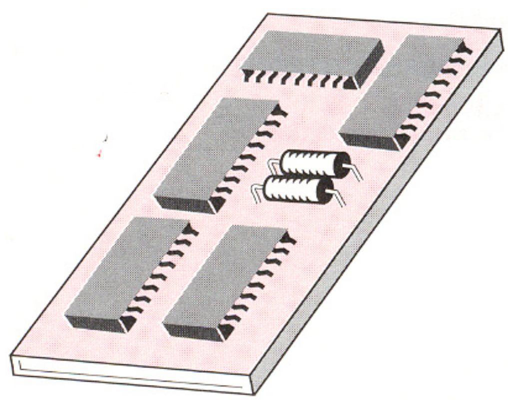

Figure: Printed circuit board (PCB)

Solution:

Step 1: Write the given data from the problem statement:

Size of PCB: 15cm × 20 cm
Room temperature: $T_\infty = 20^\circ C$
Heat dissipated by PCB: $\dot{Q} = 8$ W

Step 2: Write what we are asked to solve for:

Average surface temperature of PCB at different orientations: T_s =?

Step 3: Extract the properties of surrounding fluid

Since the surface temperature of PCB (T_s) is unknown, we will guess it value in order to get the properties of surrounding at the average temperature and to determine Rayleigh (Ra) and Nusselt (Nu) numbers.

(a) Vertical PCB:

$T_{s,guess} = 45^oC$

$$T_{ave} = \frac{T_{s,guess} + T_\infty}{2} = \frac{45+20}{2} = 32.5^o C = 305.5\ K$$

Using Table A-19, get the properties of surrounding fluid at the average temperature, which is air in the present problem

Thermal conductivity: k = 0.0265 W/m^{o}C

Kinematic viscosity: $\nu = 1.62 \times 10^{-5}$ m^2/s

Prandtl number: Pr = 0.711

Volume expansion coefficient: $\beta(\text{for ideal gas}) = \frac{1}{T_{ave}} = \frac{1}{305.5\ K} = 0.00327\ K^{-1}$

Step 4: Perform the calculations:

The characteristic length (δ) in this case (i.e. vertical PCB) is the height of the PCB

Therefore $\delta = L = 0.2m$

When dealing with natural convection problems, we have to determine Rayleigh number (Ra), instead of Reynolds number (Re), which we used to evaluate in forced convection.

Rayleigh number (Ra) is defines as:

$$Ra = Gr\,\Pr = \frac{g\beta\left(T_{s,guess} - T_\infty\right)\delta^3}{\nu^2}\Pr$$

where 'g' is the gravitational acceleration = 9.8 m/s^2

$$Ra = \frac{\left(9.8\ m/s^2\right)\left(0.00327\ K^{-1}\right)\left(318-293\ K\right)\left(0.2\ m\right)^3}{\left(1.62\times10^{-5}\ m^2/s\right)^2}\left(0.711\right) = 1.74\times10^7$$

From Table 11-1, get the Nusselt number (Nu) correlation corresponding the geometry an range of Rayleigh number (Ra)

For the present geometry and the range of Rayleigh number (Ra), we have from Eq. 11-8

$$Nu = 0.59Ra^{1/4}$$

$$Nu = 0.59\left(1.74\times10^7\right)^{1/4} = 38.1$$

From definition of Nusselt number (Nu), we can determine heat transfer coefficient (h) for the present case

$$Nu = \frac{h\delta}{k} \Rightarrow h = \frac{k}{\delta} Nu = \frac{0.0265\, W/m^oC}{0.2\, m}(38.1) = 5.1\, W/m^2.^oC$$

Surface area: $A = (0.15\, m)(0.2\, m) = 0.03\, m^2$

Heat loss due to natural convection and radiation can be expressed as:

$$\dot{Q} = hA(T_s - T_\infty) + \varepsilon A\sigma\left(T_s^4 - T_{surr}^4\right)$$
$$8\, W = \left(5.1\, W/m^2.^oC\right)\left(0.03\, m^2\right)\left[T_s - (20 + 273\, K)\right] + (0.8)\left(0.03\, m^2\right)\left(5.67\times10^{-8}\right)\left[T_s^4 - (20 + 273K)^4\right]$$
$$\boxed{T_s = 319\, K = 46^oC}$$

Since $T_{s,guess} \approx T_s$, we don't have to repeat the calculation.

(b) Horizontal, hot surface facing up:

Again we guess the surface temperature ($T_{s,guess}$) to be 45^{o}C and use the properties evaluated in part (a)

The characteristic length (δ) in this case is

$$\delta = \frac{A}{P} = \left(\frac{(0.2\, m)(0.5\, m)}{2(0.2\, m + 0.15\, m)}\right) = 0.0429\, m$$

$$Ra = \frac{\left(9.8\, m/s^2\right)\left(0.00327\, K^{-1}\right)(318 - 293\, K)(0.0429\, m)^3}{\left(1.62\times10^{-5}\; m^2/s\right)^2}(0.711) = 1.71\times10^5$$

For the present geometry and the range of Rayleigh number (Ra), we have from Eq. 11-11

$$Nu = 0.54 Ra^{1/4}$$

$$Nu = 0.54\left(1.71\times10^5\right)^{1/4} = 11$$

$$Nu = \frac{h\delta}{k} \Rightarrow h = \frac{k}{\delta} Nu = \frac{0.0265\, W/m^oC}{0.0429\, m}(11) = 6.8\, W/m^2.^oC$$

$$\dot{Q} = hA(T_s - T_\infty) + \varepsilon A\sigma\left(T_s^4 - T_{surr}^4\right)$$
$$8\, W = \left(6.8\, W/m^2.^oC\right)\left(0.03\, m^2\right)\left[T_s - (20 + 273\, K)\right] + (0.8)\left(0.03\, m^2\right)\left(5.67\times10^{-8}\right)\left[T_s^4 - (20 + 273K)^4\right]$$
$$\boxed{T_s = 315\, K = 42^oC}$$

The above surface temperature (T_s)is close enough to the guess surface temperature ($T_{s,guess}$)

(c) Horizontal, hot surface facing down:

Assuming $T_{s,guess} = 50^oC$

$$T_{ave} = \frac{T_{s,guess} + T_\infty}{2} = \frac{50 + 20}{2} = 35^o\,C = 308\;K$$

Using Table A-19, get the properties of surrounding fluid at the average temperature, which is air in the present problem

Thermal conductivity: $k = 0.0267\ W/m^oC$

Kinematic viscosity: $\nu = 1.67 \times 10^{-5}\ m^2/s$

Prandtl number: $Pr = 0.710$

Volume expansion coefficient: $\beta(\text{for ideal gas}) = \frac{1}{T_{ave}} = \frac{1}{305.5\ K} = 0.003247\ K^{-1}$

The characteristic length (δ) in this case is same as part (b)

$$Ra = \frac{(9.8\ m/s^2)(0.003247\ K^{-1})(323 - 293\ K)(0.0429\ m)^3}{(1.67\times10^{-5}\ m^2/s)^2}(0.710) = 191{,}879$$

For the present geometry and the range of Rayleigh number (Ra), we have from Eq. 11-13

$$Nu = 0.27 Ra^{1/4}$$

$$Nu = 0.27(191{,}879)^{1/4} = 5.7$$

$$Nu = \frac{h\delta}{k} \Rightarrow h = \frac{k}{\delta} Nu = \frac{0.0265\ W/m^oC}{0.0429\ m}(5.7) = 3.5\ W/m^2.^oC$$

$$\dot{Q} = hA(T_s - T_\infty) + \varepsilon A\sigma(T_s^4 - T_{surr}^4)$$

$$8\ W = (3.5\ W/m^2.^oC)(0.03\ m^2)[T_s - (20 + 273\ K)] + (0.8)(0.03\ m^2)(5.67\times10^{-8})[T_s^4 - (20 + 273K)^4]$$

$$\boxed{T_s = 323\ K = 50^o\,C}$$

The above surface temperature (T_s) is same as the guess surface temperature ($T_{s,guess}$), therefore there is no need to repeat calculations.