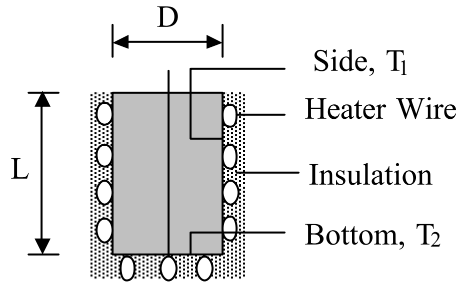
ECE-309

Heat Transfer & Thermodynamics

Tutorial # 12

Radiation heat transfer-exchanged between surfaces

Problem 1: A furnace cavity, which is in the form of a cylinder of 75 -mm diameter and 150-mm length, is open at one end to large surroundings that are at 27 °C. The sides and bottom may be approximated as blackbodies, are heated electrically, are well insulated, and are maintained at temperatures of 1350 and 1650 °C, respectively. How much power is required to maintain the furnace conditions?

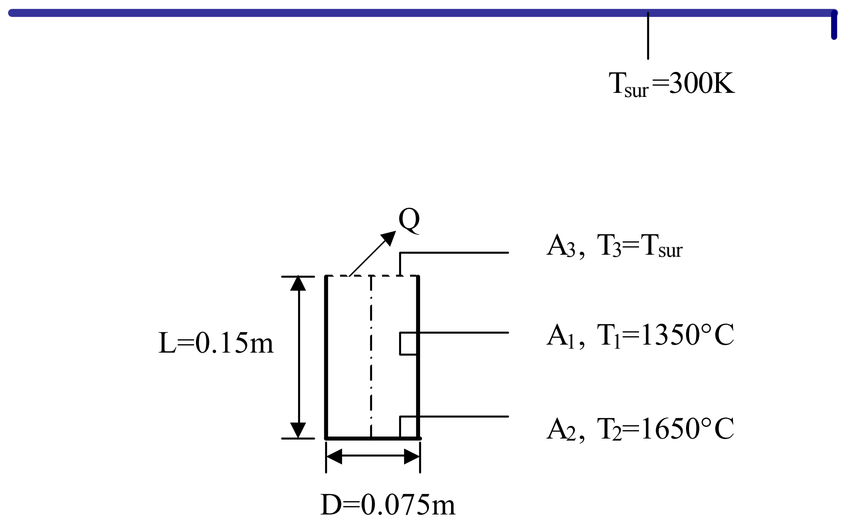


Solution

Known: Surface temperatures of the cylindrical furnace.

Find: Power required to maintain prescribed temperatures.

Schematic:



Assumptions :

1. Interior surfaces behave as black bodies.
2. Heat transfer by convection is negligible.
3. Outer surface of furnace is adiabatic.

Analysis: The power needed to operate the furnace at the prescribed conditions must be balanced by the heat losses from the fu rnace. Subject to the foregoing assumptions, the only heat loss is by radiation through the opening, which may be treated as a hypothetical surface of area A_3. Because the surroundings are large, radiation exchange between the furnace and the surroundings may be treated by approximating the surface as a black body at T_3=T_{sur}. The heat loss may then be expressed as

$$Q = Q_{13} + Q_{23}$$

or, from Equation 12 -37,

$$Q = A_1 F_{13} s\,(T_1^4 - T_3^4) + A_2 F_{23} s\,(T_2^4 - T_3^4)$$

From Figure 12 -43, it follows that, with $\left(\frac{r_2}{L}\right) = \left(\frac{0.0375m}{0.15m}\right) = 0.25$ and $\left(\frac{L}{r_1}\right) = \left(\frac{0.15m}{0.0375\,m}\right) = 4$, F_{23}=0.06. From the summation rule

F_{21}=1-F_{23}=1-0.06=0.94

And from reciprocity

$$F_{12} = \frac{A_2}{A_1} F_{21} = \frac{p\,(0.075m)^2 / 4}{p\,(0.075m)(0.15m)} \times 0.94 = 0.118$$

Hence, since F_{13}=F_{12} from symmetry,

$$Q = (p \times 0.075m \times 0.15m)0.118 \times 5.67 \times 10^{-8} W / m^2.K^4 \times [(1623\,K)^4 - (300\,K)^4] +$$
$$\left(\frac{p}{4}\right)(0.075\,m)^2 \times 0.06 \times 5.67 \times 10^{-8} W / m^2.K^4 [(1923\,K)^4 - (300\,K)^4]$$

Q=1639W+205W=1844W