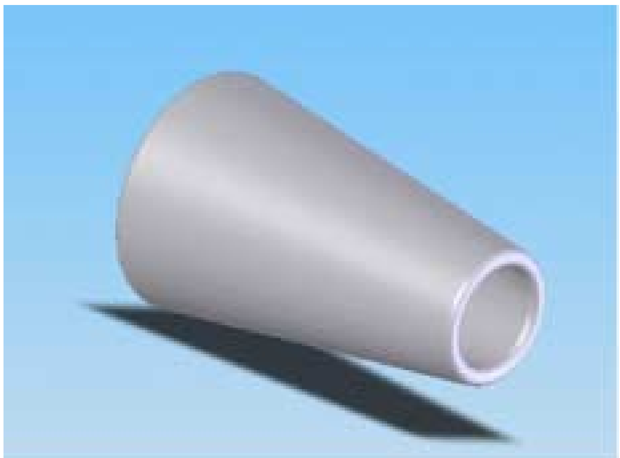
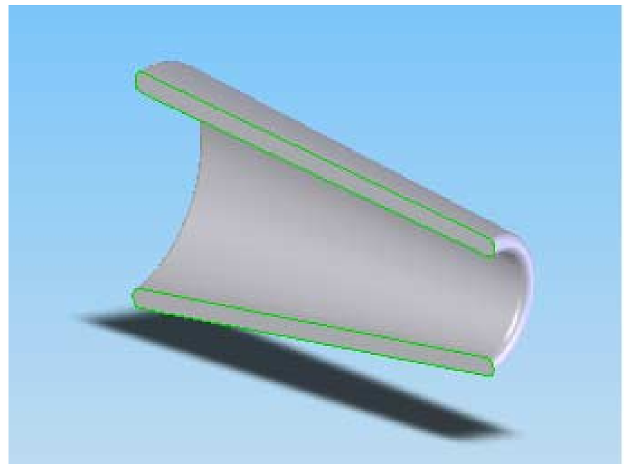
Thermodynamics and Heat Transfer

ECE 309 Tutorial # 4

First Law of Thermodynamics: Control Volumes

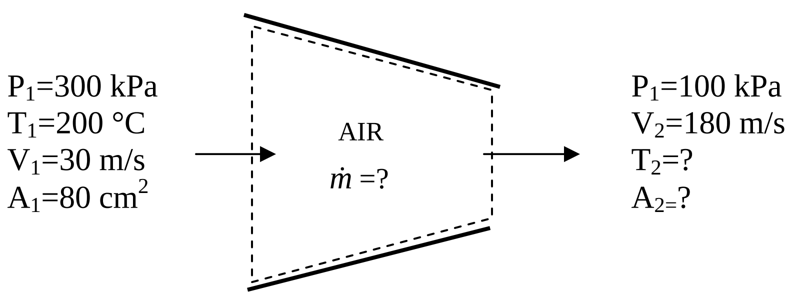
Problem 1: Air enters an adiabatic nozzle steadily at 300 kPa, 200°C, and 30 m/s and leaves at 100 kPa and 180 m/s. The inlet area of the nozzle is 80 cm^2. Determine
(a) the mass flow rate through the nozzle,
(b) the exit temperature of the air, and
(c) the exit area of the nozzle.


3-D view of a nozzle


Cross-sectional view of a nozzle

Solution:

Step -1: Schematic Diagram



Step-2: **Solve for**

(a) The mass flow rate through the nozzle,
(b) The exit temperature T_2 of the air,
(c) The exit area A_2 of the nozzle.

Step-3: Make table of values for the substance or fluid given

	Pressure (kPa)	Temperature (K)	Area (m^2)	Velocity (m/s)
Initial Condition	300	200+273=473	0.008	30
Final Condition	100	Unknown	Unknown	180

Step-4: **Analysis**

The region within the nozzle is selected as the system, and its boundaries are indicated by the dashed lines in the schematic diagram. Mass is crossing the boundaries, thus it is a control volume. And since there is no observable change within the control volume with time, it is a steady-flow system. As the specified conditions, the air can be treated as an ideal gas since it is at a high temperature and low pressure relative to its critical values (T_{cr}= – 147 °C and P_{cr}=3390 kPa for nitrogen, the main constituent of air).

(a) To determine the mass flow rate, we need to find the specific volume of the air first. This is determined from the ideal-gas relation at the inlet conditions:

$$\text{Specific volume at inlet: } \upsilon_1 = \frac{RT_1}{P_1} = \frac{\left(0.287\ \text{kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K}\right)\ \left(473\ \text{K}\right)}{300\ \text{kPa}} = 0.4525\ \text{m}^3/\text{kg}$$

$$\text{Mass flow rate: } \dot{m} = \frac{1}{\upsilon_1} A_1 V_1 = \frac{1}{0.4525\ \text{m}^3/\text{kg}}\left(0.008\ \text{m}^2\right)\left(30\ \text{m/s}\right) = 0.5304\ \text{kg/s}$$

Since the flow is steady, the mass flow rate through the entire nozzle will remain constant at this value.

(b) A nozzle normally involves
(i) no shaft or electrical work (w=0),
(ii) negligible heat transfer ($q \approx 0$), and
(iii) a small (if any) elevation change between the inlet and exit ($\Delta pe \approx o$).

Then the conservation of energy relation on a unit mass basis for this single-stream steady-flow system reduces to

$$q - w = \Delta h + \Delta ke + \Delta pe$$

$$0 - 0 = \Delta h + \Delta ke + 0$$

$$0 = h_2 - h_1 + \frac{V_2{}^2 - V_1{}^2}{2}$$

$$0 = C_p\left(T_2 - T_1\right) + \frac{V_2^2 - V_1^2}{2}$$

Using specific heats at the anticipated average temperature of 450K (C_p=1.020 kJ/kg·K, C_v=0.733 kJ/kg·K, and R=C_p–C_v=0.287 kJ/kg·K; see Table A-2), the exit temperature of air is determined to be

$$T_2 = T_1 - \frac{V_2^2 - V_1^2}{2C_p} = 200^\circ\text{C} - \frac{180^2 \text{ m}^2/\text{s}^2 - 30^2 \text{ m}^2/\text{s}^2}{2\times(1020 \text{ J/kg}\cdot^\circ\text{C})}$$

$$= 200^\circ\text{C} - \frac{180^2 \text{ m}^2/\text{s}^2 - 30^2 \text{ m}^2/\text{s}^2}{2\times(1020 \text{ m}^2/\text{s}^2\cdot^\circ\text{C})} = 184.6^\circ\text{C}$$

$$\left(\text{Note}: 1\frac{\text{J}}{\text{kg}} = 1\frac{\text{N}\cdot\text{m}}{\text{kg}} = 1\frac{\text{kg}\cdot\text{m}}{\text{s}^2}\frac{\text{m}}{\text{kg}} = 1\frac{\text{m}^2}{\text{s}^2}\right)$$

This shows that the temperature of the air is decreased by about 15°C. The temperature drop of the air is mainly due to the conversion of internal energy to the kinetic energy.

(c) To determine the exit area, we need to find the specific volume of the exit air from the ideal- gas relation.

$$\upsilon_2 = \frac{RT_2}{P_2} = \frac{(0.287 \text{ kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(184.6+273)\text{K}}{100 \text{ kPa}} = 1.313 \text{ m}^3/\text{kg}$$

Since the mass flow rate of the air is constant, exit area can be found from the mass flow rate equation.

$$\dot{m} = \frac{1}{\upsilon_2}A_2V_2 \rightarrow 0.5304 \text{ kg/s} = \frac{1}{1.313 \text{ m}^3/\text{kg}}A_2(180 \text{ m/s})$$

$$A_2 = 0.00387 \text{ m}^2 = 38.7 \text{ cm}^2$$

Problem2: A hair dryer is basically a duct in which a few layers of electric resistors are placed. A small fan pulls the air in and forces it through the resistors where it is heated. Air enters a 1200 watt hair dryer at 100 kPa and 22°C and leaves at 47°C. The cross-sectional area of the hair dryer at the exit is 60cm^2. The power consumed by the fan is 2 watt and the heat losses through the walls of the hair dryer is 5 watt, determine

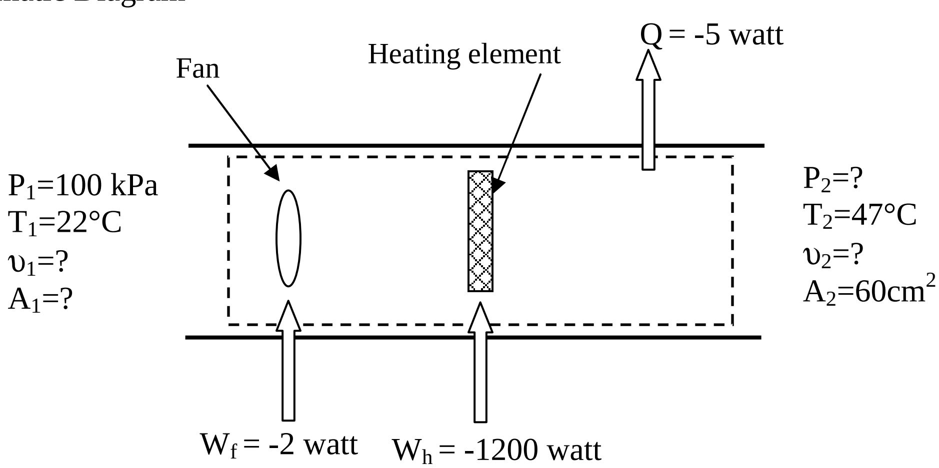
(a) the volume flow rate of the air at inlet
(b) the velocity of the air at the exit

3-D view of hair dryer

Cross-section of hair dryer showing heating element and fan

Solution:

Step -1: Schematic Diagram



Step-2: **Solve for**

(a) the volume flow rate of the air at inlet, $\dot{\forall}_{in}$
(b) the velocity of air at the exit, V_2

Step-3: Make table of values for the substance or fluid given

	Pressure (kPa)	Temperature (K)	Area (m^2)	Velocity (m/s)
Inlet condition	100	22+273=295	?	?
Exit condition	?	47+273=320	0.006	?

Step-4: **Analysis**

The region within the hair dryer is selected as the system, and its boundaries are indicated by the dashed lines in the schematic diagram. Mass is crossing the boundaries, thus it is a control volume. And since there is no observable change within the control volume with time, it is a steady-flow system. As the specified conditions, the air can be treated as an ideal gas since it is at a high temperature and low pressure relative to its critical values (T_{cr}= – 147 °C and P_{cr}=3390 kPa for nitrogen, the main constituent of air).

(a) The conservation of energy relation in the rate form for this single-stream steady-flow system is

$$\dot{Q} - \dot{W} = \dot{m}\left[\Delta h + \Delta KE + \Delta PE\right]$$
$$\Rightarrow \dot{Q} - \dot{W} = \dot{m}\left[(h_2 - h_1) + \left(\frac{V_2^2 - V_1^2}{2}\right) + g(z_2 - z_1)\right]$$

Assumptions:
(a) fixed elevation; that is, negligible potential energy ($\Delta PE \approx 0$)
(b) change in kinetic energy is small ($\Delta KE \approx 0$)
(c) inlet and exit sections of the hair dryer are exposed to atmosphere; that is, $P_1 = P_2$

Therefore, the reduced form of the energy equation is

$$\dot{Q} - \dot{W} = \dot{m}(h_2 - h_1) = \dot{m}C_p(T_2 - T_1)$$
$$\Rightarrow (-5 \text{ watt}) - (-1200 \text{ watt} - 2 \text{ watt}) = \dot{m}\ (1.005 \text{ kJ/kg}\cdot\text{K})\ (320 \text{ K} - 295 \text{ K})$$
$$\Rightarrow \dot{m} = \frac{1197 \text{ watt}}{(1.005 \text{ kJ/kg}\cdot\text{K})\ (25 \text{ K})} = \frac{1197 \text{ J/s}}{(1005 \text{ J/kg}\cdot\text{K})\ (25 \text{ K})} = 0.047641 \text{ kg/s}$$

Note: In the above calculation specific heats are calculated at the anticipated average temperature of 300K (C_p=1.005 kJ/kg·K, C_v=0.718 kJ/kg·K, and $R=C_p-C_v$=0.287 kJ/kg·K; see Table A-2).

Now, using the equation of state: $P\upsilon = RT$

$$\text{Specific volume at inlet: } \upsilon_1 = \frac{RT_1}{P_1} = \frac{\left(0.287 \text{ kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K}\right)\left(295 \text{ K}\right)}{100 \text{ kPa}} = 0.84665 \text{ m}^3/\text{kg}$$

Using the mass conservation equation

$$\text{Mass flow rate: } \dot{m} = \rho_1 V_1 A_1 = \frac{V_1 A_1}{\upsilon_1} = \frac{\dot{\forall}_1}{\upsilon_1}$$

where, $\dot{\forall}_1$ is the volume flow rate at inlet.

$$\text{Volume flow rate in inlet: } \dot{\forall}_1 = \upsilon_1 \dot{m} = (0.84665 \text{ m}^3/\text{kg})\,(0.047641 \text{ kg/s}) = 0.04033 \text{ m}^3/\text{s}$$

(b) mass flow rate must be constant at the inlet and outlet, therefore

$$\dot{m} = \rho_2 V_2 A_2 = \frac{V_2 A_2}{\upsilon_2} \quad \Rightarrow V_2 = \frac{\dot{m}\upsilon_2}{A_2}$$

The only unknown in above equation is υ_2 and it is calculated from

$$\upsilon_2 = \frac{RT_2}{P_2} = \frac{\left(0.287 \text{ kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K}\right)\left(320 \text{ K}\right)}{100 \text{ kPa}} = 0.9184 \text{ m}^3/\text{kg}$$

Now the velocity at the exit is

$$V_2 = \frac{\dot{m}\upsilon_2}{A_2} = \frac{(0.047641 \text{ kg/s})\,(0.9184 \text{ m}^3/\text{kg})}{(0.006 \text{ m}^2)} = 7.2922 \text{ m/s}$$