ECE – 309
Heat Transfer & Thermodynamics

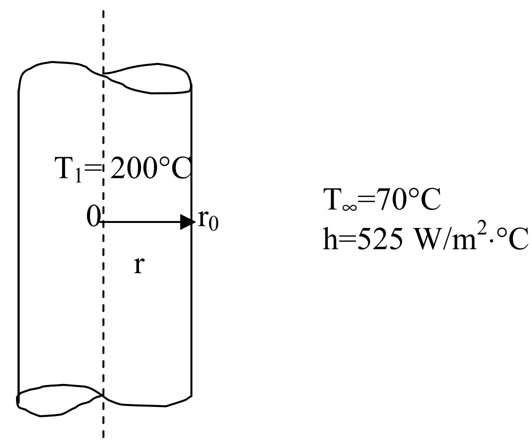
Tutorial # 8

Transient Heat Conduction

Problem 1: A long aluminium cylinder 50 mm in diameter [k=215 W/(m·°C), ρ= 2700 kg/m^3 , Cp = 0.9 kJ/(kg·°C), and α= 8.4 × 10^{-5} m^2/s] and initially at 200°C is suddenly exposed to a convective environment at 70°C and h = 525 W/(m^2·°C). Calculate the temperature at a radius of 12.5 mm and the heat lost 1 min after the cylinder is exposed to the environment.

Solution :

Step-1 : Schematic Diagram



Step-2: Solve for

a) Temperature at a radius of 12.5 mm 1 min after the cylinder is exposed to the environment.
b) Heat transfer from the cylinder during this time period.

Step-3 : List out known data and properties given in the problem statement

Dimensions, initial temperature of the cylinder, surroundings temperature.
Properties of aluminium and the required time.

Step-4: Analysis

The cylinder is said to be very long, and is subjected to uniform thermal conditions. Therefore, it can be modeled as an infinite cylinder in which heat transfer is one-dimensional. The temperature within the cylinder may vary with the radial distance r as well as time, and the temperature at a specified location at a given time can be determined from the Heisler charts. We assume the heat transfer coefficient to be uniform, and the properties of the cylinder to be constant. Noting that the radius of the cylinder is r_0=25 mm,

$$\frac{1}{Bi} = \frac{k}{h r_0} = \frac{(215 W/m\cdot^\circ C)}{(525 W/m^2\cdot^\circ C)(0.025 m)} = 16.38$$

The Fourier number is

$$\tau = \frac{\alpha\ t}{{r_0}^2} = \frac{(8.45\times 10^{-5}\ m^2/s)(60 s)}{(0.025 m)^2} = 8.112$$

This is greater than the value of 0.2. Therefore the Heisler charts can still be used.

From Fig. 9-14a

$$\theta_0 = \frac{T_0 - T_\infty}{T_i - T_\infty} = 0.38$$

$$\frac{r}{r_0} = \frac{12.5}{25} = 0.5$$

From Fig. 9-14 b using the value of r/r_0 and 1/Bi

$$\theta = \frac{T - T_\infty}{T_0 - T_\infty} = 0.98$$

So that $$\theta\,\theta_0 = \frac{T - T_\infty}{T_0 - T_\infty}\,\frac{T_0 - T_\infty}{T_i - T_\infty} = (0.98)(0.38) = 0.372$$

And $$\theta = T - T_\infty = (0.372)(130) = 48.4$$

T=70+48.4=118.4 °C

To determine the actual heat transfer, we first need to calculate the maximum heat that can be transferred from the cylinder, which is the sensible energy of the cylinder relative to the environment. Taking L=1 m,

$$m = \rho V = \rho\pi\, r_0^2 L = (2700 kg/m^3)\left[\pi(0.025 m)^2 (1 m)\right] = 5.30 kg$$

$$Q_{max} = m C_p (T_i - T_\infty) = (5.30 kg)\,[0.9 kJ/(kg\cdot^\circ C)](200 - 70)^\circ C = 620.268\ kJ$$

The dimensionless heat transfer ratio is determined from Fig. 9-14c for a long cylinder to be

$$\mathrm{Bi} = \frac{h\,r_0}{k} = \frac{(525\mathrm{W/m^2\cdot ^\circ C})(0.025\mathrm{m})}{(215\mathrm{W/m\cdot ^\circ C})} = 0.061$$

$$\frac{h^2 \alpha t}{k^2} = \mathrm{Bi}^2 \tau = (0.061)^2 (8.112) = 0.0302$$

$$Q/Q_{max} = 0.65$$

Therefore,

$$Q = 0.65Q_{max} = 0.65\times(620.268\mathrm{kJ}) = 403.174\ \mathrm{kJ}$$

Which is the total heat transfer from the cylinder during the first 1 min of the cooling.

Alternative Solution we could also solve this problem using the one-term solution relation instead of the transient charts. First we find the Biot number

$$\mathrm{Bi} = \frac{h\,r_0}{k} = \frac{(525\mathrm{W/m^2\cdot ^\circ C})(0.025\mathrm{m})}{(215\mathrm{W/m\cdot ^\circ C})} = 0.061$$

The coefficients λ_1 and A_1 for a cylinder corresponding to this Bi are determined from Table 9-1 to be

λ_1=0.3438 and A_1=1.0148

Substituting these values into Eq 9-11 gives

$$\theta = \frac{T(r,t) - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} J_0(\lambda_1 r / r_0) = (1.0148) e^{-(0.3438)^2 (8.112)} 0.992 = 0.386$$

and $\theta = T - T_\infty = (0.386)(130) = 50.167$

$T = 70 + 50.167 = 120.167\ ^\circ C$

The value of $J_1(\lambda_1)$ for λ_1=0.3438 is determined from Table 9-2 to be 0.169.Then the fractional heat transfer is determined from Eq. 9-18 to be

$$\frac{Q}{Q_{max}} = 1 - 2\theta_0 \frac{J_1(\lambda_1)}{\lambda_1}$$

The value of θ_0 can be determined from Eq 9-14

$$\theta_{0,cyl} = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} = (1.0148)e^{-(0.3438)^2(8.112)} = 0.389$$

Therefore, $\frac{Q}{Q_{max}} = 1 - 2\theta_0 \frac{J_1(\lambda_1)}{\lambda_1}$ =1-2×0.389(0.169/0.3438)=0.617

And thus $Q = 0.617Q_{max} = 0.617\times(620.268\text{kJ}) = 383.054$ kJ

The slight difference between the two results is due to the reading of the charts.