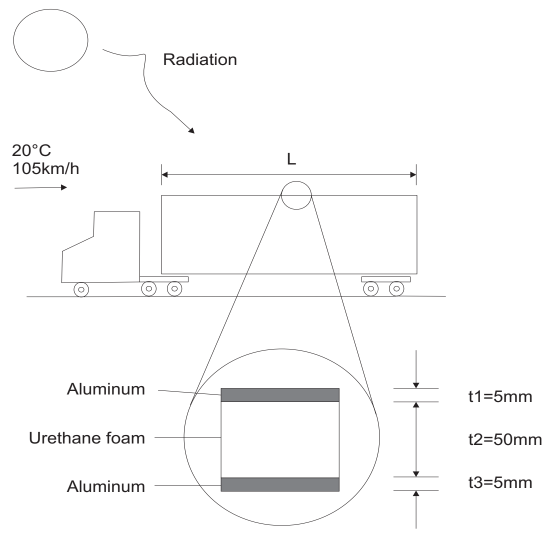
Question

A truck, pulling a refrigerated trailer, travels at 105 km/hr. Solar radiation heats the roof of the truck, with the net radiatvie flux being

$$q''_{rad} = 375 - \varepsilon\sigma T_o^4 \left[\frac{W}{m^2}\right]$$

where $\varepsilon = 0.5$ and T_o is the temperature of the roof. The roof is composed of urethane foam ($t_2 = 50mm, k_{foam} = 0.026\mathrm{W/m\cdot K}$) that is sandwiched between two sheets of aluminum ($t_1 = 5mm, k_{al} = 180\mathrm{W/m\cdot K}$) and the cooling system of the trailer keeps the inner surface of the roof at a temperature of $-10℃$. The air temperature is $T_\infty = 20℃$. If the length of the trailer is 10m and the area of the roof is $35m^2$, find the heat transfer rate through the roof. Assume that the surface of the roof is isothermal.



Known:

- Trailer roof
  - Length, $L = 10m$
  - Area, $A = 35m^2$
  - Aluminum conductivity, $k_{al} = 180\mathrm{W/m\cdot K}$
  - Urethane foam, $k_{foam} = 0.026\mathrm{W/m\cdot K}$
  - Inner wall temperature, $T_i = -10℃ = 263K$
  - Wall thickness
    - Outer aluminum, $t_1 = 5 \times 10^{-3}m$
    - Urethane foam, $t_2 = 50 \times 10^{-3}m$
    - Inner aluminum, $t_3 = 5 \times 10^{-3}m$
- Radiation

$$q''_{rad} = 375 - \varepsilon\sigma T_o^4 \left[\frac{W}{m^2}\right] \qquad \varepsilon = 0.5$$

- Air
  - Velocity, $u_\infty = 105km/h = 29.167m/s$
  - Temperature, $T_\infty = 20℃ = 293K$

Find:

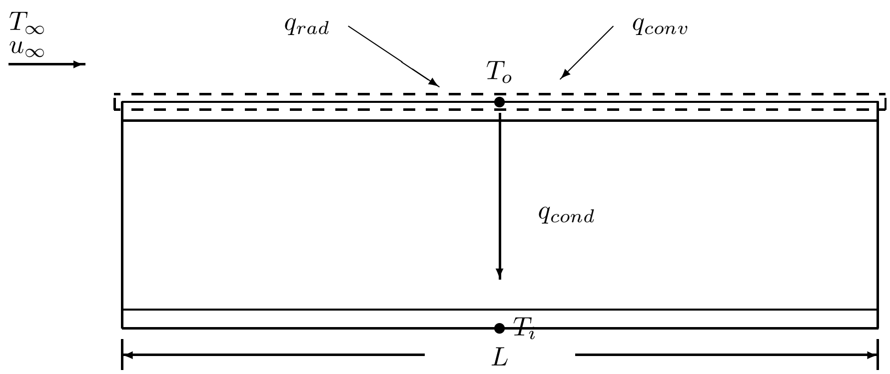
- Heat transfer rate through the roof

Assumptions:

- Steady state condition
- Inner wall temperature is constant throughout the length
- Outer wall temperature is constant throughout the length

Schematic:

☞ The control volume is the top surface of the roof



Analysis:

An energy balance on the roof surface results in:

$$q_{rad} + q_{conv} = q_{cond}$$

The heat tranfer through the roof is q_{cond}. Each term is a function of the surface temperature.

Radiation

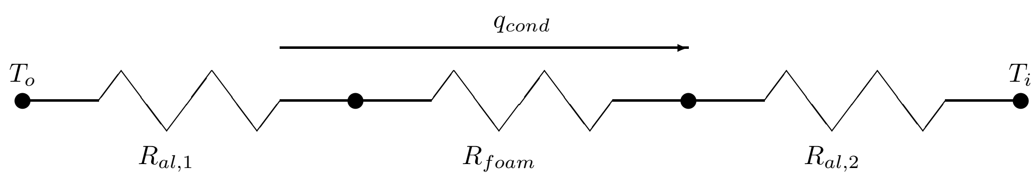
The heat transfer due to radiation was given in the problem:

$$\begin{aligned} q_{rad} &= q''_{rad}A \\ &= A\left(375 - \varepsilon\sigma T_o^4\right), \end{aligned}$$

where $\sigma = 5.67 \times 10^{-8} W/m^2 \cdot K^4$ and $\varepsilon = 0.5$.

Conduction

The heat transfer through the wall can be found using a resistance network:



$$\begin{aligned} q_{cond} &= \frac{T_o - T_i}{R_{eq}}, \\ R_{eq} &= R_{al,1} + R_{foam} + R_{al,2}. \end{aligned}$$

The individual resistance are

$$\begin{aligned}
R_{al,1} &= \frac{t_1}{k_{al}A} \\
&= \frac{5 \times 10^{-3}m}{(180\mathrm{W/m \cdot K})(35m^2)} \\
&= 7.9365 \times 10^{-7}K/W \\
R_{foam} &= \frac{t_2}{k_{foam}A} \\
&= \frac{50 \times 10^{-3}m}{(0.026\mathrm{W/m \cdot K})(35m^2)} \\
&= 0.054945K/W \\
R_{al,2} &= R_{al,1} = 7.9365 \times 10^{-7}K/W
\end{aligned}$$

The equivalent resistance becomes:

$$\begin{aligned}
R_{eq} &= R_{al,1} + R_{foam} + R_{al,2} \\
&= 7.9365 \times 10^{-7}K/W + 0.054945K/W + 7.9365 \times 10^{-7}K/W \\
&= 0.054947K/W
\end{aligned}$$

Convection

The plate is assumed to be constant temperature; thus, the heat transfer due to convection is

$$q_{conv} = \bar{h}A(T_{infty} - T_o),$$

where $\bar{h}$ is the overall heat transfer coefficient. The heat transfer coefficient depends on the fluid velocity, length of the plate, and the properties of the air. The properties of the air must be found at the film temperature, which is defined as

$$T_f = \frac{T_o + T\infty}{2}.$$

However, $T_o$is not known. Therefore, the solution will be iterative and an initial guess for T_f is required. A good first estimate is assume that $T_f \cong T_\infty = 293K$. From Table A-19 (assuming that the air is at $1atm$ pressure),

$$\begin{aligned}
\nu &= 1.51 \times 10^{-5}m^2/s, \\
k_{air} &= 0.02554\mathrm{W/m \cdot K}, \\
Pr &= 0.714.
\end{aligned}$$

Using these values, the Reynolds number for the plate can be calculated:

$$Re_L = \frac{u_\infty L}{\nu} = \frac{(29.167m/s)(10m)}{1.51 \times 10^{-5}m^2/s} = 1.93 \times 10^7$$

Since $Re_L > 5 \times 10^5$, the Nu_L number correlation must be valid for turbulent flow. Equation (10-27) is valid for $5 \times 10^5 \leqslant Re_L \leqslant 10^7, 0.6 \leqslant Pr \leqslant 60$:

$$\bar{Nu}_L = \left(0.037Re_L^{4/5} - 871\right) Pr^{1/3}.$$

For the assumed $T_f = 293, \overline{Nu_L} = 21514.4$. The convection coefficient becomes

$$\begin{aligned}
\bar{h} &= \frac{\overline{Nu_L}k_{air}}{L} \\
&= \frac{(21514.4)(0.02554\mathrm{W/m \cdot K})}{10m} \\
&= 54.95\mathrm{W/m^2 \cdot K}
\end{aligned}$$

Final Solution

The energy balance on the surface of the roof can be expressed as

$$A\left(375-\varepsilon\sigma T_o^4\right)+\bar{h}A(T_\infty-T_o)-\frac{T_o-T_i}{R_{eq}}=0,$$
$$13125-(9.925\times10^{-7})T_o^4+5.6351\times10^5-1923.25T_o-18.199T_o+4786.3=0.$$

Solving the equation results in $T_o=295.58$. The $\bar{h}$ value was calculated assuming that $T_o=T_\infty=293K$; therefore, $\bar{h}$ should be recalculated using a new film temperature,

$$T_f=\frac{293+295.58}{2}=294.3K$$

Get the properties from Table A-19 again,

$$\nu=1.53\times10^{-5}m^2/s,$$
$$k_{air}=0.02574\mathrm{W/m\cdot K},$$
$$Pr=0.713.$$

The iterations are summarized in the table below.

it#	$T_f[K]$	Re_L	Pr	Nu_L	$\bar{h}[\mathrm{W/m^2\cdot K}]$	$T_o[K]$
1	293	1.93×10^7	0.714	21514.4	54.95	295.6
2	294.3	1.91×10^7	0.713	21271.0	54.75	295.6

Thus, the surface temperature is $T_o=295.6K$. The heat transfer rate through the wall becomes

$$q_{cond}=\frac{T_o-T_i}{R_{eq}}$$
$$=\frac{295.6K-263K}{0.05494K/W}$$
$$=593W \qquad \longleftarrow \quad \textbf{Answer}$$