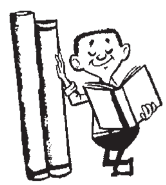
Convection Heat Transfer



Reading	Problems
10-1 → 10-7	10-22, 10-31, 10-33, 10-43, 10-49, 10-51, 10-55, 10-58, 10-83, 10-108,
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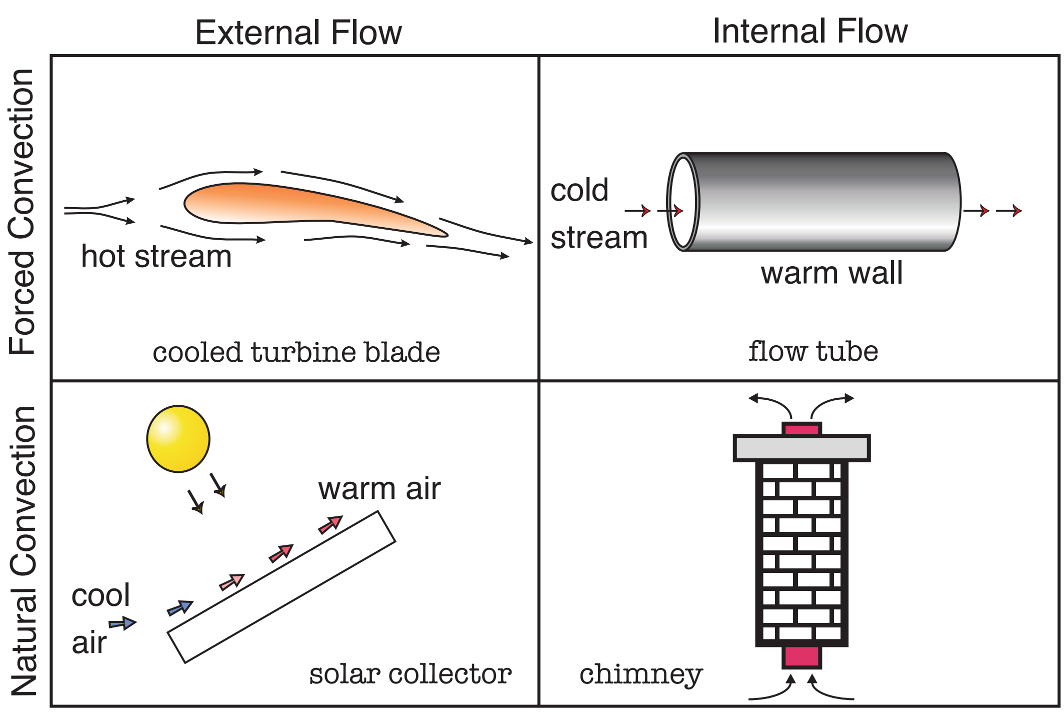
Introduction

- *convection heat transfer* is the transport mechanism made possible through the *motion of fluid*
- the types of flow arrangements and heat transfer configurations associated with convection are extremely diverse including force, natural and mixed convection for both internal and external flow geometries
- in addition to these convection mechanisms, flow can be laminar or turbulent and fluids can be single or multi-phase as in the case of boiling



- the controlling equation for convection is *Newton's Law of Cooling*

$$\dot{Q}_{conv} = \frac{\Delta T}{R_{conv}} = hA(T_w - T_\infty) \qquad \Rightarrow \qquad R_{conv} = \frac{1}{hA}$$

where

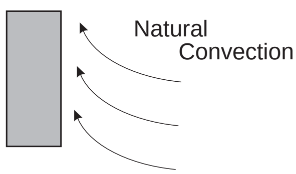
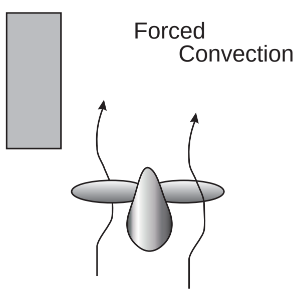
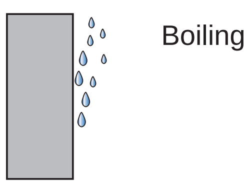
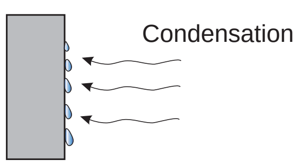
$$\boldsymbol{A} = \text{total convective area}, \; \boldsymbol{m^2}$$

$$\boldsymbol{h} = \text{heat transfer coefficient}, \; \boldsymbol{W/(m^2 \cdot K)}$$

$$\boldsymbol{T_w} = \text{surface temperature}, \; \boldsymbol{^\circ C}$$

$$\boldsymbol{T_\infty} = \text{fluid temperature}, \; \boldsymbol{^\circ C}$$

The following table gives the range of heat transfer coefficient expected for different convection mechanisms and fluid types.

Process	$\boldsymbol{h}$ $[\boldsymbol{W/(m^2 \cdot K)}]$
	• gases 3 - 20 • water 60 - 900
	• gases 30 - 300 • oils 60 - 1 800 • water 100 - 1 500
	• water 3 000 - 100 000
	• steam 3 000 - 100 000

Dimensionless Groups

In the study and analysis of convection processes it is common practice reduce the total number of functional variables by forming dimensionless groups consisting of relevant thermophysical properties, geometry, boundary and flow conditions.

Prandtl number: $Pr = \nu/\alpha$ where $0 < Pr < \infty$ ($Pr \rightarrow 0$ for liquid metals and $Pr \rightarrow \infty$ for viscous oils). A measure of ratio between the diffusion of momentum to the diffusion of heat.

Oils	$Pr \approx 10^3$
Water	$Pr \approx 5$
Air	$Pr \approx 0.7$
Liquid Metals	$Pr \approx 10^{-2}$

Reynolds number: $Re = \rho U \mathcal{L}/\mu \equiv U\mathcal{L}/\nu$ (forced convection). A measure of the balance between the inertial forces and the viscous forces.

Peclet number: $Pe = U\mathcal{L}/\alpha \equiv RePr$

Grashof number: $Gr = g\beta(T_w - T_f)\mathcal{L}^3/\nu^2$ (natural convection)

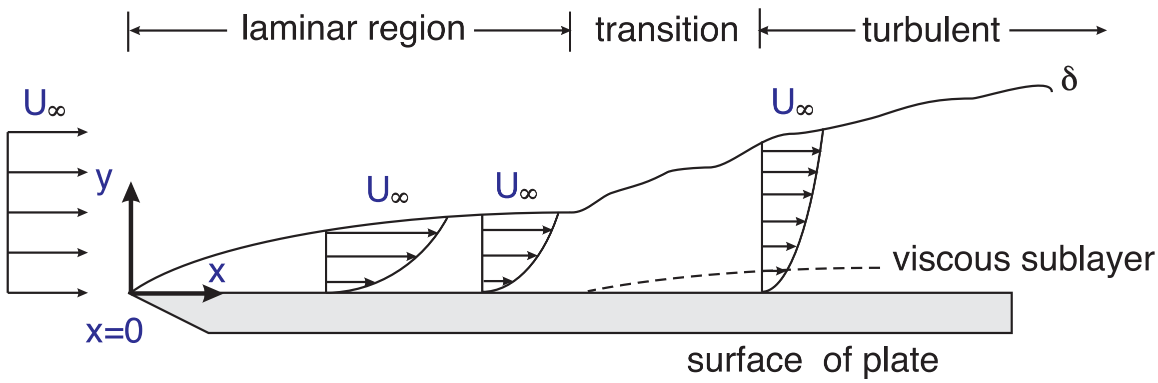
Rayleigh number: $Ra = g\beta(T_w - T_f)\mathcal{L}^3/(\alpha \cdot \nu) \equiv GrPr$

Nusselt number: $Nu = h\mathcal{L}/k_f$ This can be considered as the dimensionless heat transfer coefficient.

Stanton number: $St = h/(U\rho C_p) \equiv Nu/(RePr)$

Forced Convection

The simplest forced convection configuration to consider is the flow of mass and heat near a flat plate as shown below.

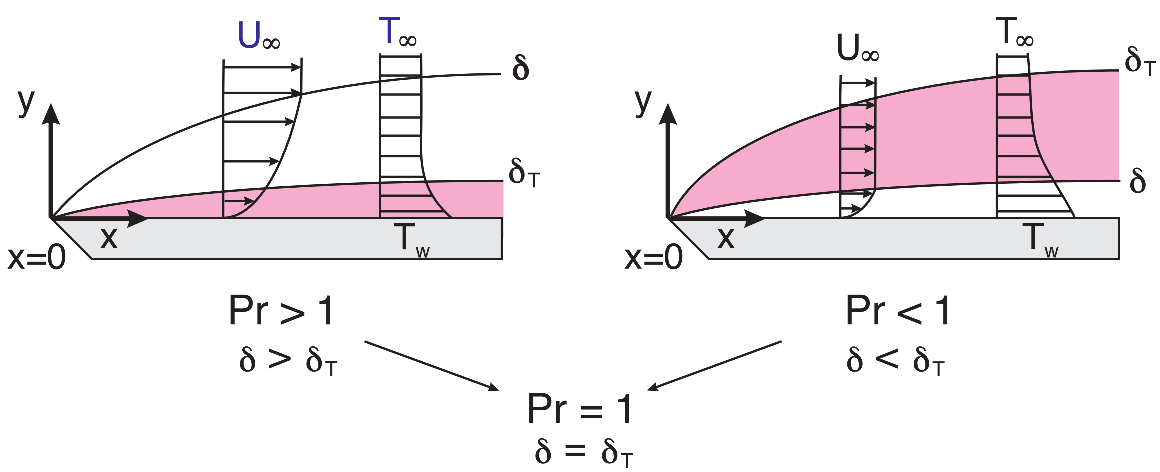


- as Reynolds number increases the flow has a tendency to become more chaotic resulting in disordered motion known as turbulent flow
  - transition from laminar to turbulent is called the critical Reynolds number, $\boldsymbol{Re_{cr}}$

$$\boldsymbol{Re_{cr} = \frac{U_\infty x_{cr}}{\nu}}$$

  - for flow over a flat plate $\boldsymbol{Re_{cr} \approx 500,000}$

Boundary Layers



Velocity Boundary Layer

- the region of fluid flow over the plate where viscous effects dominate is called the *velocity* or *hydrodynamic* boundary layer
- the velocity at the surface of the plate, $\boldsymbol{y = 0}$, is set to zero, $\boldsymbol{U_{@y=0} = 0\ m/s}$ because of the *no slip condition* at the wall
- the velocity of the fluid progressively increases away from the wall until we reach approximately $\boldsymbol{0.99\ U_\infty}$ which is denoted as the $\boldsymbol{\delta}$, the *velocity boundary layer thickness.* Note: **99%** is an arbitrarily selected value.
- the region beyond the velocity boundary layer is denoted as the *inviscid flow* region, where frictional effects are negligible and the velocity remains relatively constant at $\boldsymbol{U_\infty}$

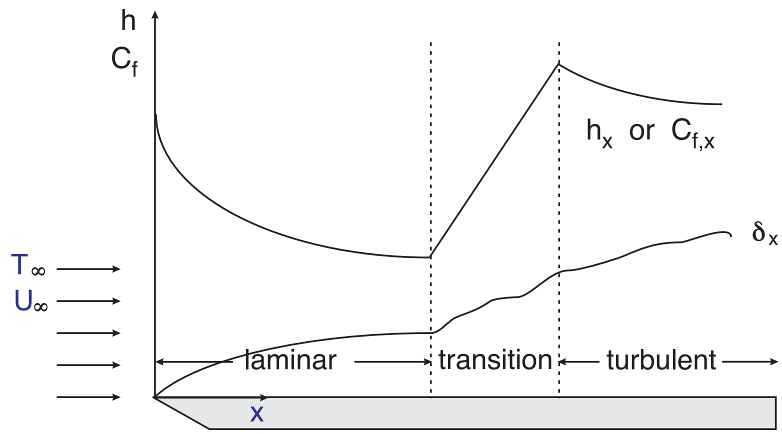
Thermal Boundary Layer

- the thermal boundary layer is arbitrarily selected as the locus of points where

$$\frac{T - T_w}{T_\infty - T_w} = 0.99$$

- for *low Prandtl number* fluids, i.e. liquid metals, momentum diffuses much slower than heat flow (remember $\boldsymbol{Pr = \nu/\alpha}$) and the velocity boundary layer is fully contained within the thermal boundary layer
- conversely, for *high Prandtl number* fluids, i.e. oils, heat diffuses slower than the momentum and the thermal boundary layer is contained with in the velocity boundary layer

Flow Over Plates



1. Laminar Boundary Layer Flow, Isothermal (UWT)

The local values of the skin friction and the Nusselt number are given as

$$\boldsymbol{C_{f,x} = \frac{0.664}{Re_x^{1/2}}}$$

$$\boldsymbol{Nu_x = 0.332\ Re_x^{1/2}\ Pr^{1/3}}$$ $\Rightarrow$ local, laminar, UWT, $\boldsymbol{Pr \geq 0.6}$

where $\boldsymbol{x}$ is the distance from the leading edge of the plate.

$$\boldsymbol{Nu_L = \frac{h_L L}{k_f} = 0.664\ \mathrm{Re}_L^{1/2}\ \mathrm{Pr}^{1/3}}$$ $\Rightarrow$ average, laminar, UWT, $\boldsymbol{Pr \geq 0.6}$

For low Prandtl numbers, i.e. liquid metals

$$\boldsymbol{Nu_x = 0.565\ Re_x^{1/2}\ Pr^{1/2}}$$ $\Rightarrow$ local, laminar, UWT, $\boldsymbol{Pr \leq 0.6}$

2. Turbulent Boundary Layer Flow, Isothermal (UWT)

The local skin friction is given as

$$\boxed{C_{f,x} = \frac{\tau_0}{(1/2)\rho U_\infty^2} = \frac{0.0592}{Re_x^{0.2}}} \quad \Rightarrow \text{local, turbulent, UWT, } Pr \geq 0.6$$

As mentioned previously, we know that

$$Pr^{2/3}\text{St} = \frac{C_{f,x}}{2}$$

Therefore

$$Pr^{2/3} \frac{Nu_x}{Re_x\, Pr} = \frac{1}{2}\left(\frac{0.0592}{Re_x^{0.2}}\right)$$

and

$$\boxed{Nu_x = 0.0296\, Re_x^{0.8}\, Pr^{1/3}} \quad \Rightarrow \begin{array}{l}\text{local, turbulent, UWT,} \\ 0.6 < Pr < 100, \quad Re_x > 500,000\end{array}$$

$$\boxed{Nu_L = 0.037\, Re_x^{0.8}\, Pr^{1/3}} \quad \Rightarrow \begin{array}{l}\text{average, turbulent, UWT,} \\ 0.6 < Pr < 100, \quad Re_x > 500,000\end{array}$$

3. Combined Laminar and Turbulent Boundary Layer Flow, Isothermal (UWT)

When $(T_w - T_\infty)$ constant

$$h_L = \frac{1}{L}\int_0^L h dx = \frac{1}{L}\left\{\int_0^{x_{cr}} h_x^{lam} dx + \int_0^{x_{cr}} h_x^{tur} dx\right\}$$

$$\boxed{Nu_L = \frac{h_L L}{k} = (0.037\, Re_L^{0.8} - 871)\, Pr^{1/3}} \quad \Rightarrow \begin{array}{l}\text{average, turbulent, UWT,} \\ 0.6 < Pr < 100, \quad Re_L > 500,000\end{array}$$

4. Laminar Boundary Layer Flow, Isoflux (UWF)

$$\boxed{Nu_x = 0.453\ Re_x^{1/2}\ Pr^{1/3}} \quad \Rightarrow \text{local, laminar, UWF, } Pr \geq 0.6$$

5. Turbulent Boundary Layer Flow, Isoflux (UWF)

$$\boxed{Nu_x = 0.0308\ Re_x^{4/5}\ Pr^{1/3}} \quad \Rightarrow \text{local, turbulent, UWF, } Pr \geq 0.6$$

Flow Over Cylinders and Spheres

1. Boundary Layer Flow Over Circular Cylinders, Isothermal (UWT)

The Churchill-Berstein (1977) correlation for the average Nusselt number for long ($L/D > 100$) cylinders is

$$\boxed{Nu_D = S_D^* + f(Pr)\ Re_D^{1/2}\left[1 + \left(\frac{Re_D}{28200}\right)^{5/8}\right]^{4/5}} \quad \Rightarrow \begin{array}{l} \text{average, UWT,} \quad Re < 10^7 \\ 0 \leq Pr \leq \infty, \quad Re \cdot Pr > 0.2 \end{array}$$

where $S*_D$ is the diffusive term associated with $Re_D \rightarrow 0$ and is given as

$$S_D^* = \frac{4}{\pi}\left(\frac{1 + 0.869(L/D)^{0.76}}{0.5 + L/D}\right), \qquad 0 \leq \frac{L}{D} \leq 8$$

$$S_D^* = \frac{4}{\sqrt{\pi}}\left(\frac{1}{\sqrt{1 + 0.5D/L}}\right)\left(\frac{1}{\ln(2L/D)}\right), \qquad 0 \leq \frac{L}{D} \leq 8$$

and the Prandtl number function is

$$f(Pr) = \frac{0.62\ Pr^{1/3}}{[1 + (0.4/Pr)^{2/3}]^{1/4}}$$

All fluid properties are evaluated at $T_f = (T_w + T_\infty)/2$.

2. Boundary Layer Flow Over Non-Circular Cylinders, Isothermal (UWT)

The empirical formulations of Zhukauskas and Jakob given in Table 10-3 are commonly used, where

$$\boxed{Nu_D \approx \frac{\overline{h}D}{k} = C\ Re_D^m\ Pr^{1/3}} \quad \Rightarrow \text{see Table 10-3 for conditions}$$

3. Boundary Layer Flow Over a Sphere, Isothermal (UWT)

For flow over an isothermal sphere of diameter D

$$\boxed{Nu_D = S_D^* + 0.4\ Re_D^{1/2} + 0.06\ Re_D^{2/3}\ Pr^{0.4} \left(\frac{\mu_\infty}{\mu_w}\right)^{1/4}} \quad \Rightarrow \quad \begin{array}{l} \text{average, UWT,} \\ 0.7 \le Pr \le 380 \\ 3.5 < Re_D < 80,000 \\ 1.0 < \mu_\infty/\mu_w < 3.2 \end{array}$$

where the diffusive term at $Re_D \to 0$ is

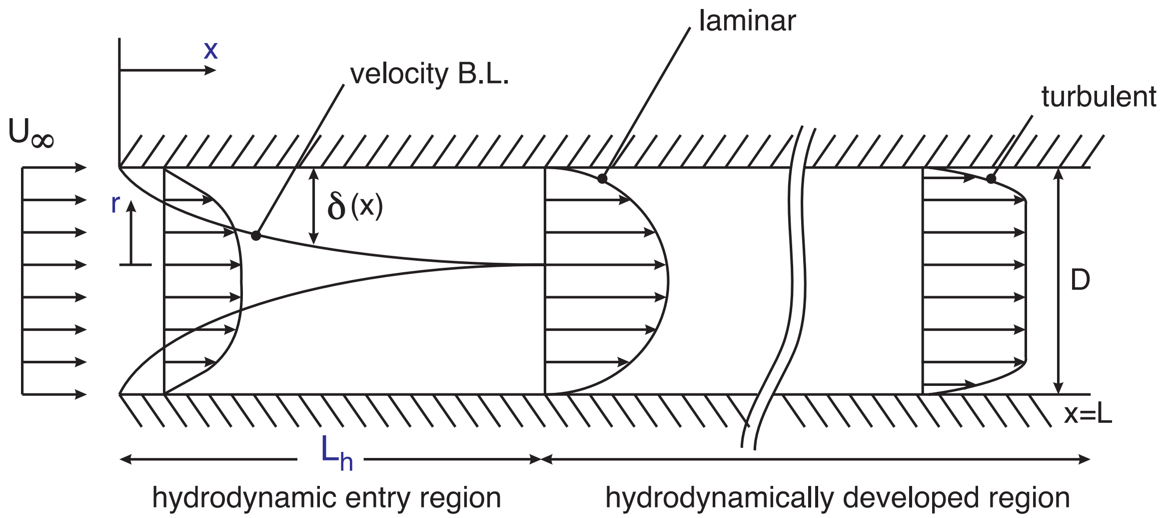
$$S_D^* = 2$$

and the dynamic viscosity of the fluid in the bulk flow, μ_∞ is based on T_∞ and the dynamic viscosity of the fluid at the surface, μ_w, is based on T_w. All other properties are based on T_f.

Internal Flow

Lets consider fluid flow in a duct bounded by a wall that is at a different temperature than the fluid. For simplicity we will examine a round tube of diameter D as shown below

The velocity profile across the tube changes from $U = 0$ at the wall to a maximum value along the center line. The average velocity, obtained by integrating this velocity profile, is



called the *mean velocity* and is given as

$$U_m = \frac{1}{A_c}\int_{A_c} u\, dA = \frac{\dot{m}}{\rho_m A_c}$$

where the area of the tube is given as $A_c = \pi D^2/4$ and the fluid density, ρ_m is evaluated at T_m.

The Reynolds number is given as

$$Re_D = \frac{U_m D}{\nu}$$

For flow in a tube:

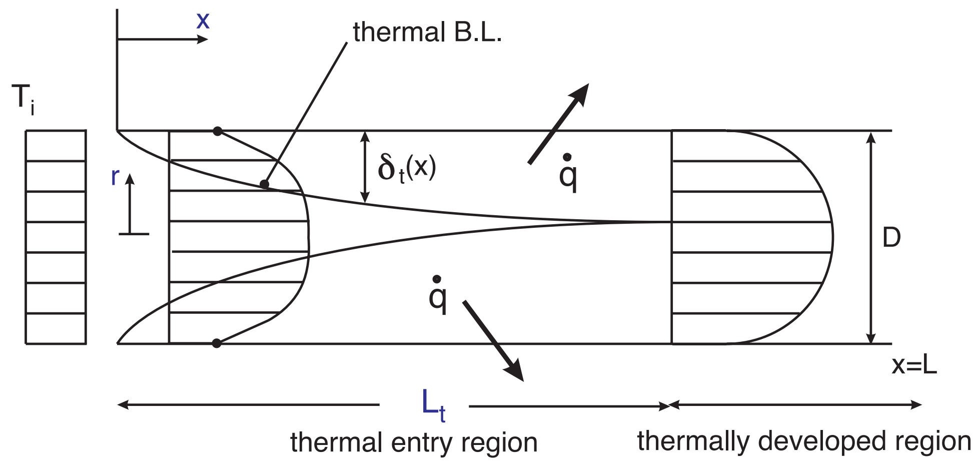
$Re_D < 2300$	laminar flow
$2300 < Re_D < 4000$	transition to turbulent flow
$Re_D > 4000$	turbulent flow

Hydrodynamic (Velocity) Boundary Layer

- the hydrodynamic boundary layer thickness can be approximated as

$$\delta(x) \approx 5x\left(\frac{U_m x}{\nu}\right)^{-1/2} = \frac{5x}{\sqrt{Re_x}}$$

Thermal Boundary Layer



- the thermal entry length can be approximated as

$$\boldsymbol{L_t \approx 0.05 Re_D Pr D} \qquad \text{(laminar flow)}$$

- for turbulent flow $\boldsymbol{L_h \approx L_t \approx 10D}$

1. Laminar Flow in Circular Tubes, Isothermal (UWT) and Isoflux (UWF)

For laminar flow where $\boldsymbol{Re_D \leq 2300}$

$$\boxed{\boldsymbol{Nu_D = 3.66}} \quad \Rightarrow \text{fully developed, laminar, UWT, } \boldsymbol{L > L_t \ \& \ L_h}$$

$$\boxed{\boldsymbol{Nu_D = 4.36}} \quad \Rightarrow \text{fully developed, laminar, UWF, } \boldsymbol{L > L_t \ \& \ L_h}$$

$$\boxed{\boldsymbol{Nu_D = 1.86 \left(\frac{Re_D Pr D}{L}\right)^{1/3} \left(\frac{\mu_b}{\mu_w}\right)^{0.4}}} \quad \Rightarrow \quad \begin{array}{l} \text{developing laminar flow, UWT,} \\ \boldsymbol{Pr > 0.5} \\ \boldsymbol{L < L_h} \text{ or } \boldsymbol{L < L_t} \end{array}$$

For non-circular tubes the hydraulic diameter, $\boldsymbol{D_h = 4A_c/P}$ can be used in conjunction with Table 10-4 to determine the Reynolds number and in turn the Nusselt number.

In all cases the fluid properties are evaluated at the mean fluid temperature given as

$$T_{mean} = \frac{1}{2}\left(T_{m,in} + T_{m,out}\right)$$

except for μ_w which is evaluated at the wall temperature, T_w.

2. Turbulent Flow in Circular Tubes, Isothermal (UWT) and Isoflux (UWF)

For turbulent flow where $Re_D \geq 2300$ the Dittus-Bouler equation (Eq. 10-76) can be used

$$\boxed{Nu_D = 0.023\ Re_D^{0.8}\ Pr^n} \quad \Rightarrow \quad \begin{array}{l} \text{turbulent flow, UWT or UWF,} \\ 0.7 \leq Pr \leq 160 \\ Re_D > 2,300 \\ n = 0.4 \ \ \text{heating} \\ n = 0.3 \ \ \text{cooling} \end{array}$$

For non-circular tubes, again we can use the hydraulic diameter, $D_h = 4A_c/P$ to determine both the Reynolds and the Nusselt numbers.

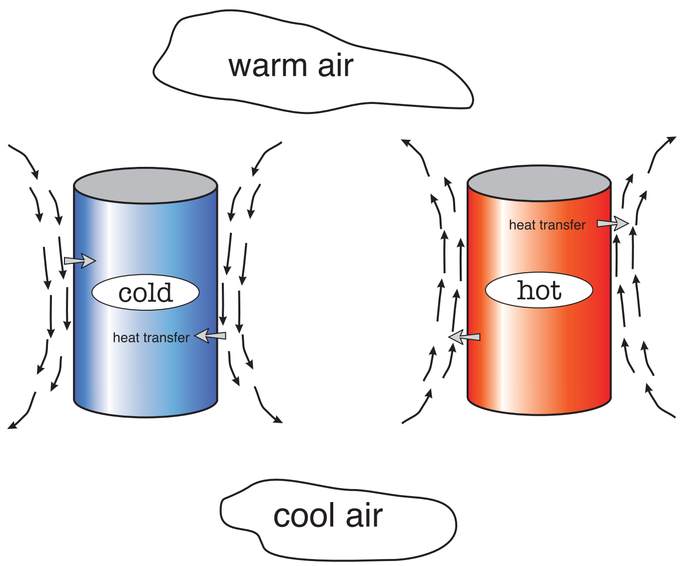
In all cases the fluid properties are evaluated at the mean fluid temperature given as

$$T_{mean} = \frac{1}{2}\left(T_{m,in} + T_{m,out}\right)$$

Natural Convection

What Drives Natural Convection?

- fluid flow is driven by the effects of buoyancy
- therefore a fluid layer adjacent to a surface will become lighter if heated and heavier if cooled by the surface



In natural convection, we do not have a Reynolds number but we have an analogous dimensionless group called the *Grashof number*

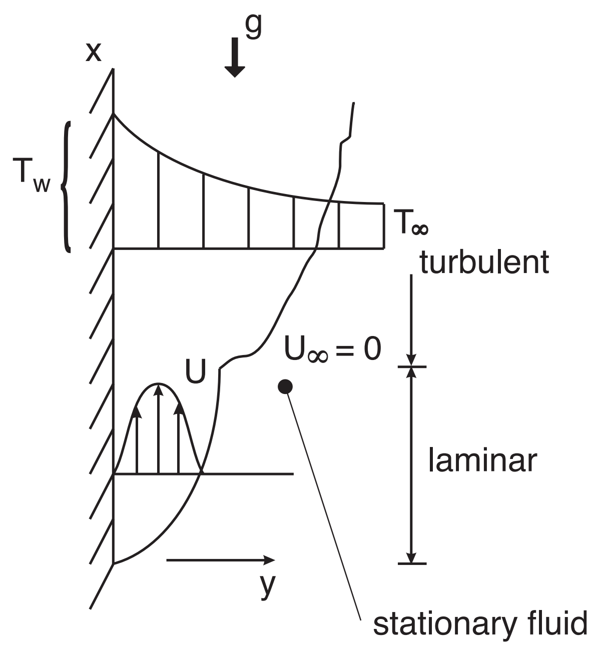
$$Gr = \frac{\text{buouancy force}}{\text{viscous force}} = \frac{g\beta(T_w - T_\infty)\mathcal{L}^3}{\nu^2}$$

where

$$\begin{aligned}
g &= \text{gravitational acceleration, } m/s^2 \\
\beta &= \text{volumetric expansion coefficient, } \ \beta \equiv 1/T \ (T \text{ is ambient temp. in } K) \\
T_w &= \text{wall temperature, } K \\
T_\infty &= \text{ambient temperature, } K \\
\mathcal{L} &= \text{characteristic length, } m \\
\nu &= \text{kinematic viscosity, } m^2/s
\end{aligned}$$

Natural Convection Over Surfaces

- the velocity and temperature profiles within a boundary layer formed on a vertical plate in a stationary fluid looks as follows:



- note that unlike forced convection, the velocity at the edge of the boundary layer goes to zero

Natural Convection Heat Transfer Correlations

The general form of the Nusselt number for natural convection is as follows:

$$\boldsymbol{Nu = f(Gr, Pr) \equiv C Ra^m Pr^n} \qquad \text{where} \quad \boldsymbol{Ra = Gr \cdot Pr}$$

1. Laminar Flow Over a Vertical Plate, Isothermal (UWT)

The general form of the Nusselt number is given as

$$Nu_{\mathcal{L}} = \frac{h\mathcal{L}}{k_f} = C \left(\underbrace{\frac{g\beta(T_w - T_\infty)\mathcal{L}^3}{\nu^2}}_{\equiv Gr} \right)^{1/4} \left(\underbrace{\frac{\nu}{\alpha}}_{\equiv Pr} \right)^{1/4} = C \; \underbrace{Gr_{\mathcal{L}}^{1/4} \; Pr^{1/4}}_{Ra^{1/4}}$$

where

$$Ra_{\mathcal{L}} = Gr_{\mathcal{L}} Pr = \frac{g\beta(T_w - T_\infty)\mathcal{L}^3}{\alpha\nu}$$

for gases $\beta = 1/T_\infty, \quad (1/K)$.

The correlation equation for UWT with a vertical plate and laminar flow is

$$\boxed{Nu_{\mathcal{L}} = \frac{0.67\ Ra_{\mathcal{L}}^{1/4}}{[1 + (0.5/Pr)^{9/16}]^{4/9}}} \quad \Rightarrow \quad \begin{array}{l} \text{average, UWT,} \\ 0 \le Pr \le \infty \\ 10^4 < Gr_{\mathcal{L}} < 10^8 \end{array}$$

2. Laminar Flow Over a Long Horizontal Circular Cylinder, Isothermal (UWT)

The general boundary layer correlation is

$$Nu_D = \frac{hD}{k_f} = C \left(\underbrace{\frac{g\beta(T_w - T_\infty)D^3}{\nu^2}}_{\equiv Gr} \right)^{1/4} \left(\underbrace{\frac{\nu}{\alpha}}_{\equiv Pr} \right)^{1/4} = C \underbrace{Gr_D^{1/4}\ Pr^{1/4}}_{Ra_D^{1/4}}$$

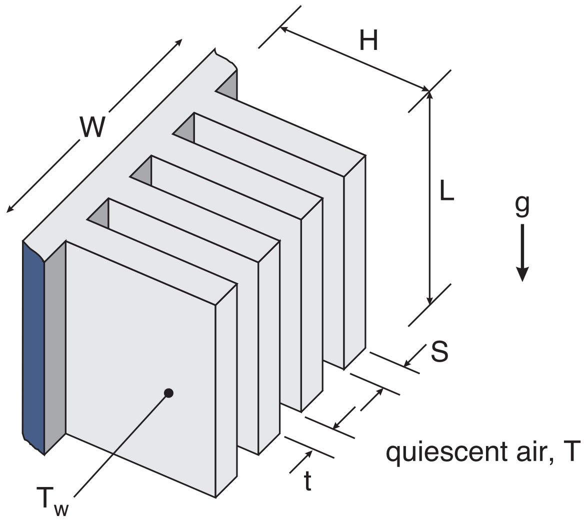
where

$$Ra_D = Gr_D Pr = \frac{g\beta(T_w - T_\infty)\mathcal{L}^3}{\alpha\nu}$$

Natural Convection From Plate Fin Heat Sinks

Plate fin heat sinks are often used in natural convection to increase the heat transfer surface are and in turn reduce the boundary layer resistance

$$R \downarrow = \frac{1}{hA \uparrow}$$



A basic optimization of the fin spacing can be obtained as follows:

$$\dot{Q} = hA(T_w - T_\infty)$$

where the fins are assumed to be isothermal and the surface area is $2nHL$, with the area of the fin edges ignored.

For isothermal fins with $t < S$

$$S_{opt} = 2.714 \left(\frac{L}{Ra^{1/4}} \right)$$

with

$$Ra = \frac{g\beta(T_w - T_\infty L^3}{\nu^2}$$

The corresponding value of the heat transfer coefficient is

$$h = 1.31k/S_{opt}$$

All fluid properties are evaluated at the film temperature.