

Basic Concepts of Thermodynamics



Reading

1-1 → 1-11

Problems

1-33, 1-36, 1-37, 1-40, 1-49, 1-53

Thermal Sciences

The thermal sciences involve the *storage*, *transfer* and *conversion* of energy.

IMPORTANT QUESTIONS ABOUT ENERGY:

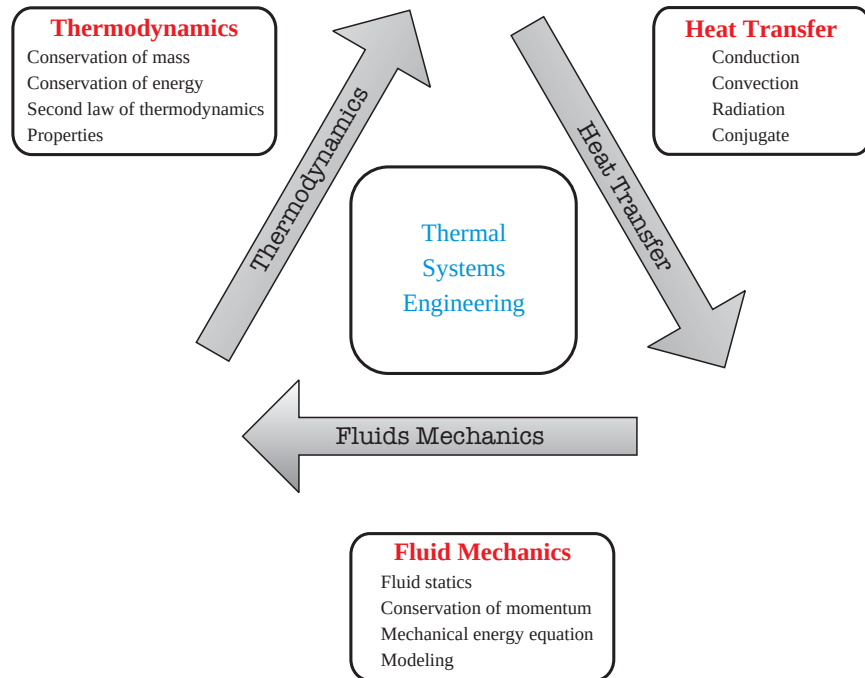
- how is energy stored in matter?
 - kinetic energy (flywheels, windmills) and gravitational potential (hydroelectric dams)
- how is energy exchanged between a system and its surroundings?
 - work, heat transfer
- how is energy transformed or converted?
 - chemical → electrical or mechanical (fuel cells and automotive engines)
 - phonons → electric (photovoltaics) or thermal (solar panels)
 - nuclear → electric (fission reactors)

We will study the basic laws and principles that govern the three disciplines of thermal sciences, namely, thermodynamics, heat transfer and fluid mechanics.

Thermodynamics: the study of energy, energy transformations and its relation to matter.

The analysis of thermal systems is achieved through the application of the governing conservation equations, namely *Conservation of Mass*, *Conservation of Energy* (1st law of thermodynamics), the 2nd law of thermodynamics and the property relations.

Heat Transfer: the study of energy in transit including the relationship between energy, matter, space and time. The three principal modes of heat transfer examined are conduction, convection and radiation, where all three modes are affected by the thermophysical properties, geometrical constraints and the temperatures associated with the heat sources and sinks used to drive heat transfer.



Fluid Mechanics: the study of fluids at rest or in motion. While this course will not deal extensively with fluid mechanics we will be influenced by the governing equations for fluid flow, namely *Conservation of Momentum* and *Conservation of Mass*.

Thermodynamics

Whenever there is an interaction between energy and matter, thermodynamics is involved.

The following common thermodynamic systems are just a few examples of forms of energy conversion in everyday life.

Given Energy Form	→ Thermodynamic System	→ Desired Energy Form
<i>Chemical, Solar, Nuclear, Kinetic (Wind), Potential (Hydroelectric)</i>	→ <i>PowerPlant</i>	→ <i>Electricity</i>
<i>Chemical (gas)</i>	→ <i>Vehicle</i>	→ <i>Motion</i>
<i>Chemical (food)</i>	→ <i>Human</i>	→ <i>Motion</i>

Instead of tracking the movement of matter and energy on a particle by particle basis (**microscopic** approach), classical thermodynamics uses a **macroscopic** approach where we use the conservation equations (energy and mass) to track movement on an average over a fixed domain.

Dimensions and Units

SI: International System

- SI is the preferred because it is logical (base 10) and needs no correction factors
- unit convention:

Parameter	Units	Symbol
length, L	meters	m
mass, m	kilograms	kg
time, t	seconds	s
temperature, T	kelvin	K
velocity, $\mathcal{V}$	meter per second, $\equiv L/t$	m/s
acceleration, a	meter per second squared $\equiv L/t^2$	m/s^2
force, F	newton, $\equiv m \cdot L/t^2$	N
energy, E	joule $\equiv m \cdot L^2/t^2$	J

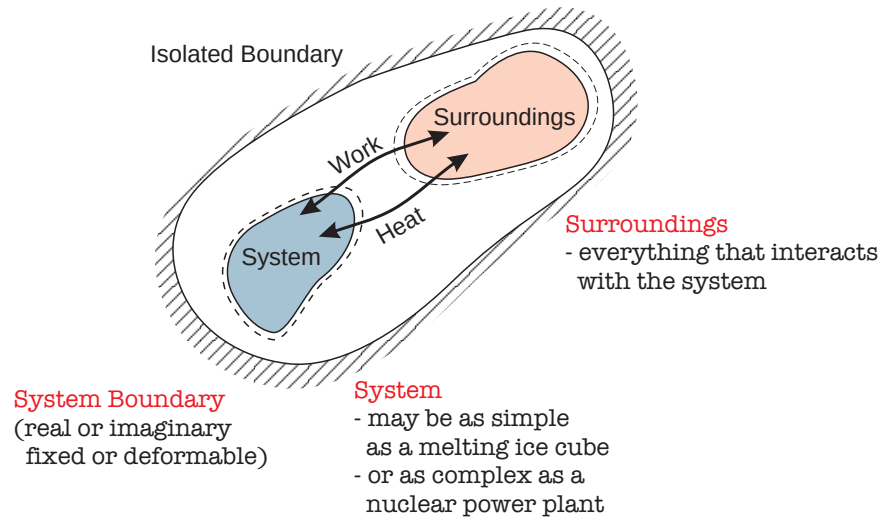
Note:

$$\begin{aligned}
 1 \, N &= 1 \, kg \cdot m/s^2 \\
 1 \, J &= 1 \, N \cdot m = 1 \, kg \cdot m^2/s^2 \\
 g_c &= 1 \, kg \cdot m/N \cdot s^2
 \end{aligned}$$

Thermodynamic Systems

SYSTEM:

- thermodynamic systems are classified as either *open* or *closed*
- Closed System:** composed of a control (or fixed) mass where heat and work can cross the boundary but no mass crosses the boundary.
- Open System:** composed of a control volume (or region in space) where heat, work, and mass can cross the boundary or the control surface



WORK & HEAT TRANSFER:

- thermodynamics deals with the properties of matter as a system interacts with its surroundings through work and heat transfer
- work and heat transfer are NOT properties → they are the forms that energy takes to cross the system boundary

Thermodynamic Properties of Systems

Basic Definitions

Thermodynamic Property: Any observable or measurable characteristic of a system.
Any mathematical combination of the measurable characteristics of a system

Intensive Properties: Properties which are independent of the size (or mass) of the system

- they are **not** additive $\Rightarrow X_{A+B} \neq X_A + X_B$
- examples include: pressure, temperature, and density

Extensive Properties: Properties which are dependent of the size (or mass) of the system

- they are additive $\Rightarrow X_{A+B} = X_A + X_B$
- examples include: volume, energy, entropy and surface area

Intensive Properties		
Pressure	P	N/m^2 or Pa
Temperature	T	K
Extensive Properties		
Volume	V	m^3
Mass	m	kg
Other Properties (Intensive and Extensive)		
Specific Volume ^A	$V/m = v$	m^3/kg
Density	$\rho = 1/v = m/V$	kg/m^3
Total Energy ^B	$E/m = e$	J/kg
Internal Energy ^B	$U/m = u$	J/kg
Enthalpy ^B	$H/m = h = u + Pv$	J/kg
Entropy ^B	$S/m = s$	$J/kg \cdot K$
Specific Heat	Cp, Cv	$J/kg \cdot K$
Ideal Gas Constant	$R = Cp - Cv$	$J/kg \cdot K$

Notes: ^A The term “specific” denotes that the property is independent of mass.

There are two exceptions:

- 1) specific gravity (or relative density) is given where:

$$\rho_s = \rho/\rho_{H_2O} \text{ and } \rho_{H_2O} \text{ is } 1000 \text{ kg/m}^3.$$

- 2) specific weight is weight per unit volume
(i.e. $W/V = mg/V = \rho g = \gamma$)

^B The given internal energy, enthalpy and entropy are specific properties.
The term “specific” is dropped.

Measurable Properties

- P, V, T , and m are important because they are measurable quantities. Many other thermodynamic quantities can only be calculated and used in calculations when they are related to P, V, T , and m
 - Pressure (P) and Temperature (T) are easily measured intensive properties.
Note: They are not always independent of one another.
 - Volume (V) and mass (m) are easily measured extensive properties

Pressure

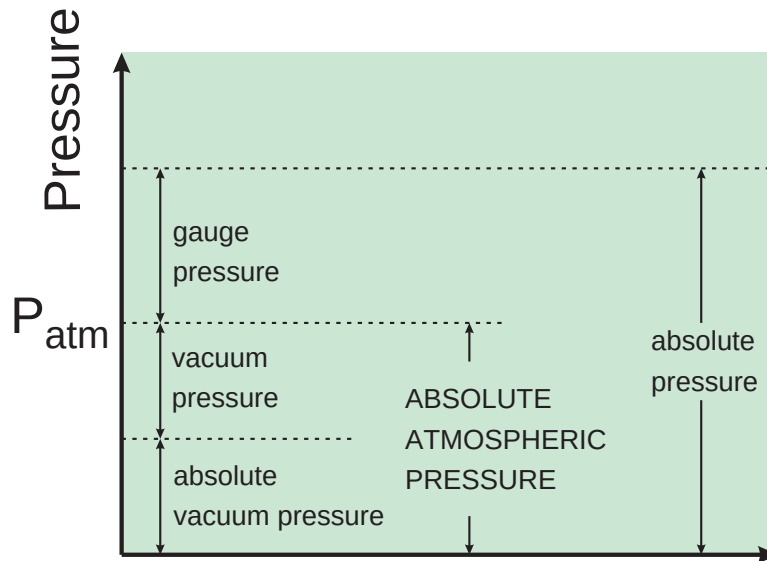
- gauge pressure = absolute pressure - atmospheric pressure

$$P_{gauge} = P_{abs} - P_{atm} \quad \text{when} \quad P_{abs} > P_{atm}$$

- vacuum pressure

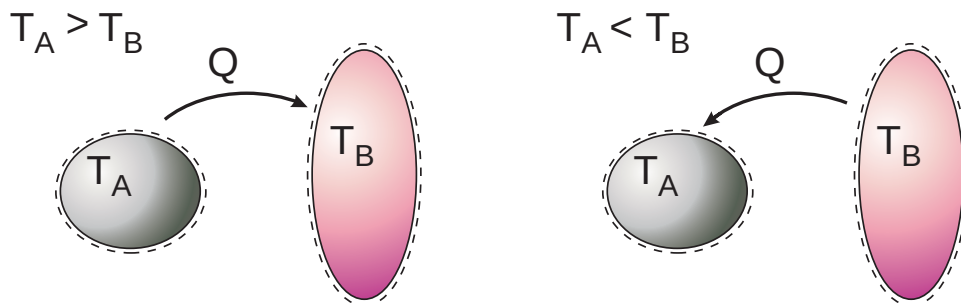
$$P_{vac} = P_{atm} - P_{abs} \quad \text{when} \quad P_{abs} < P_{atm}$$

- in thermodynamics calculations, always absolute pressure



Temperature

- temperature is a pointer for the direction of energy transfer as heat

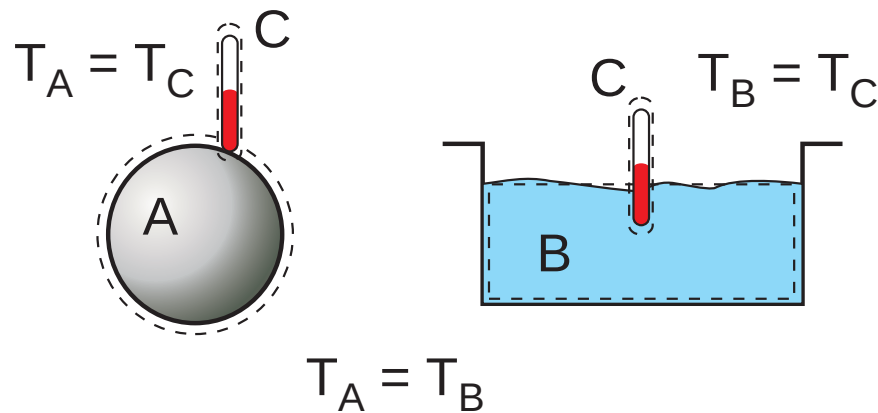


It follows that temperature is a property two systems have in common when they are in thermal equilibrium with each other

Thermal Equilibrium: $T_A = T_B$ and $Q = 0$

0th Law of Thermodynamics: if system C is in thermal equilibrium with system A, and also with system B, then $T_A = T_B = T_C$

- the 0th law makes a thermometer possible

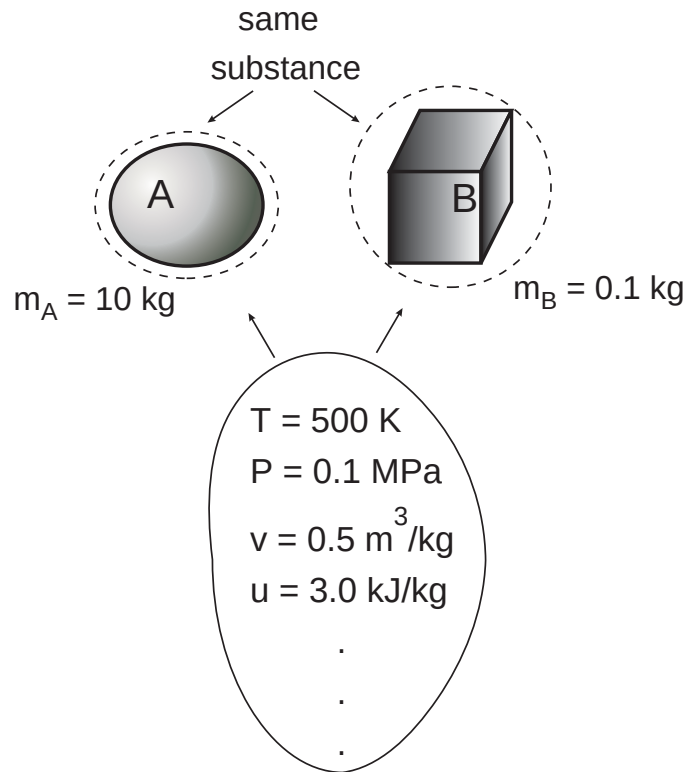


State and Equilibrium

- the state of a system is its condition as described by a set of relevant energy related properties.
- thermodynamic equilibrium refers to a condition of equilibrium with respect to all possible changes in a given system.

State Postulate

- when the intensive properties of a system are specified, the thermodynamic state of the system is known intensively
- how long does the list of intensive properties have to be in order to describe the intensive state of the system?



- System A and B have the same intensive state, but totally different extensive states.

State Postulate (for a simple compressible system): The state of a simple compressible system is completely specified by 2 independent and intensive properties.

- note: a simple compressible system experiences negligible electrical, magnetic, gravitational, motion, and surface tension effects, and only PdV work is done
- if the system is not simple, for each additional effect, one extra property has to be known to fix the state. (i.e. if gravitational effects are important, the elevation must be specified and two independent and intensive properties)

Thermodynamic Processes

Process: Any change that a system undergoes from one equilibrium state (initial state) to another (end state)

Quasi-Equilibrium Process: System remains infinitesimally close to an equilibrium state at all times

Cyclic Process: A process for which the end state is the same as the initial state

Isothermal Process: A process in which the temperature remains constant

Isobaric Process: A process in which the pressure remains constant

Isochoric (Isometric) Process: A process in which the volume remains constant

Isentropic Process: A process in which the entropy remains constant

Isenthalpic Process: A process in which the enthalpy remains constant

Adiabatic Process: A process in which no heat transfer occurs

Reversible Process: An imaginary process in which the system and surroundings can be returned to its original condition after the process has been executed

Irreversible Process: A process for which it's impossible to retain the initial state of the system without having a net effect on the surroundings

Energy

Process: Stored (static) Energy: Energy forms which can be stored in the matter

Transit (dynamic) Energy: Energy forms which are defined at the system boundary

Internal Energy: All the microscopic forms of energy

Sensible Energy (heat): Part of the internal energy due to the motion of molecules

Latent Energy (heat): Part of the internal energy due to molecular binding forces

Kinetic Energy: All the macroscopic forms of the stored energy which are related to the motion of the matter

Potential Energy: All the macroscopic forms of the stored energy which are related to the position of the matter

Total Energy: Sum of all forms of stored energy

Heat (transfer): All transit energy forms that appear because of a temperature difference

Work (transfer): All transit energy forms that appear because of driving forces other than a temperature difference

Transit Energy

Heat

- transit form of energy that occurs when there is ΔT (a temperature gradient)
- heat flow can be slowed (but not stopped) using insulation
- notation - Q (kJ), q (kJ/kg), $\dot{Q}$ (kW), $\dot{q}$ (kW/kg)
- thermodynamics does not specify how q is calculated when ΔT is known (this will be discussed when you take “Heat Transfer”)

Work

- transit form of energy that occur due to all other driving forces
- notation - W (kJ), w (kJ/kg), $\dot{W}$ (kW), $\dot{w}$ (kW/kg)
- we will consider different work transfer modes and the method of calculating W

Properties of Pure Substances



Reading

2-1 $\rightarrow$ 2-8

Problems

2-29, 2-31, 2-32, 2-39, 2-43, 2-47, 2-62

Pure Substances

- materials can be classified from different points of view
- a *Pure Substance* is the most common material model used in thermodynamics.
 - it has a fixed chemical composition throughout (chemically uniform)
 - a homogeneous mixture of various chemical elements or compounds can also be considered as a pure substance (uniform chemical composition)
 - a pure substance is not necessarily physically uniform (different phases)

Phases of Pure Substances

- 3 principal phases:

solid $\rightarrow$ liquid $\rightarrow$ gas

- a phase is characterized as having a distinct molecular arrangement that is homogeneous throughout and separated from the others by easily identifiable boundary surfaces.

Solids:

- strong molecular bonds
- molecules form a fixed (but vibrating) structure (lattice)
- the velocity of the molecules = $f(T)$

Liquids:

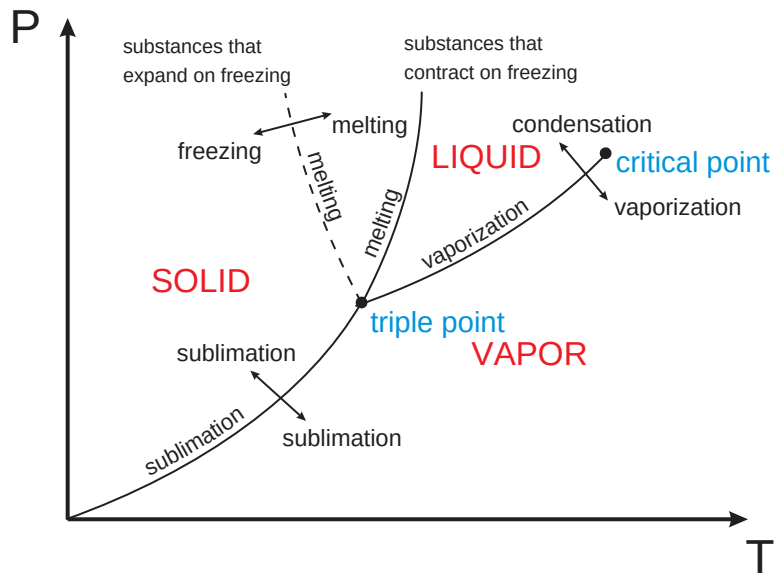
- molecules are no longer in a fixed position relative to one another
- chunks of molecules float about each other

Gases:

- there is no molecular order
- intermolecular forces ≈ 0

Behavior of Pure Substances (Phase Change Processes)

- phase change processes:
 - the following diagram show the typical behaviors of pure substances

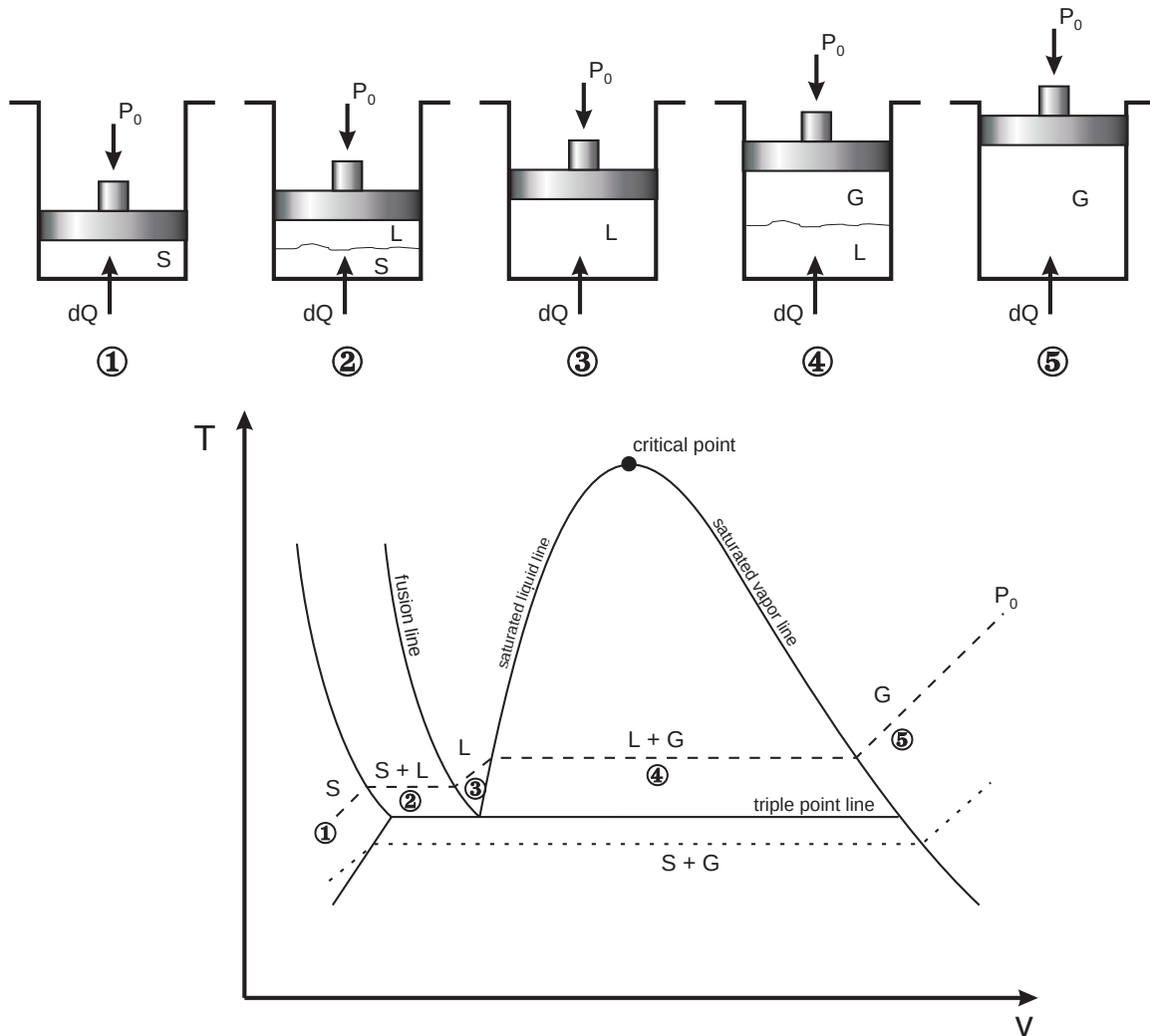


- this is called a “phase diagram” because all three phases are separated from one another by three lines
- the “critical point” is the point at which the liquid and vapor phases are not distinguishable (we will talk about this more)
- the “triple point” is the point at which the liquid, solid, and vapor phases can exist together

	Critical Point		Triple Point	
	$P \text{ (atm)}$	$T \text{ (K / } ^\circ\text{C)}$	$P \text{ (atm)}$	$T \text{ (K / } ^\circ\text{C)}$
H_2O	218	647.30/(374.14)	0.006	273.17 (0.01)
O_2	50.136	154.80/(−118.36)	0.0015	54.16/(−219.00)

$T - v$ Diagram for a Simple Compressible Substance

- consider an experiment in which a substance starts as a solid and is heated up at constant pressure until it all becomes as gas

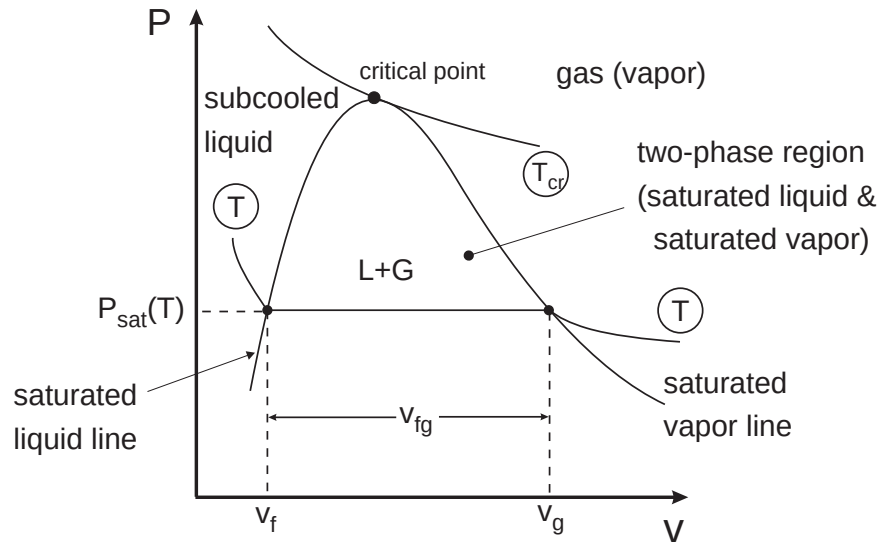


- depending on the prevailing pressure, the matter will pass through various phase transformations. At P_0 :

1. solid
2. mixed phase of liquid and solid
3. sub-cooled or compressed liquid
4. wet vapor (saturated liquid-vapor mixture)
5. superheat vapor

The Vapor Dome

- the general shape of a $P - v$ diagram for a pure substance is very similar to that of a $T - v$ diagram



- the subscripts denote: **f** - saturated liquid (fluid) and **g** - saturated vapor (gas)
- we will express values of properties for conditions on the vapor dome as:

specific volume: v_f , v_g and $v_{fg} = v_f - v_g$

internal energy: u_f , u_g and $u_{fg} = u_f - u_g$

specific enthalpy: h_f , h_g and $h_{fg} = h_f - h_g$

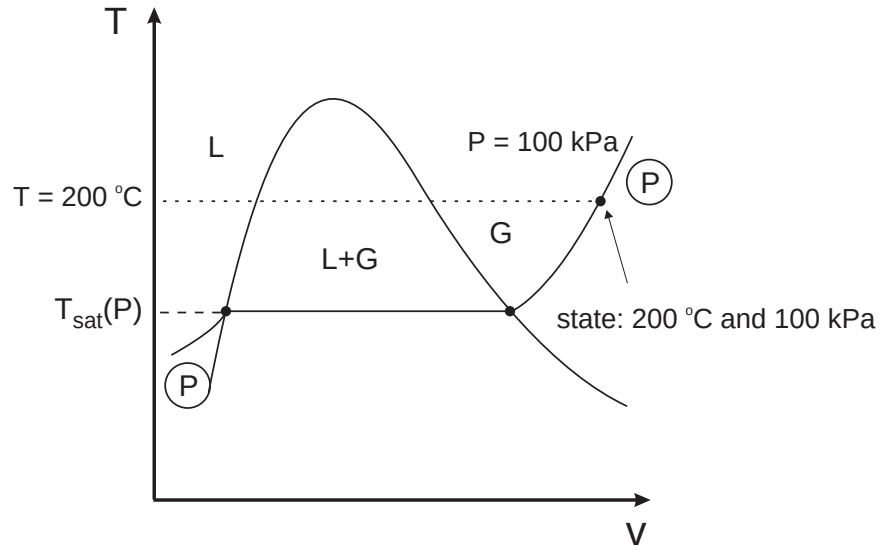
specific entropy: s_f , s_g and $s_{fg} = s_f - s_g$

Properties of Saturated Mixtures

- P and T are not independent in the wet vapor region. To fix the state we require (T, x) , (T, v) , $(P, x) \dots$
- all the calculations done in the vapor dome can be performed using Tables.
 - in Table A-4, the properties are listed under Temperature
 - in Table A-5, the properties are listed under Pressure
- use if given mixture:
 - is saturated, 2) has a quality, 3) liquid & vapor are present

Properties of Superheated Vapor

- superheated means $T > T_{sat}$ at the prevailing P , eg. water at **200 °C** and **100 kPa** has a saturation temperature of $T_{sat}(P) = 99.63\text{ °C}$.
- To fix the state we require $(T, x), (T, v), (P, x), (P, T) \dots$



Properties of Sub-cooled Liquid

- sub-cooled liquid means $T < T_{sat}$ at the prevailing P , eg. water at **20 °C** and **100 kPa** has a saturation temperature of $T_{sat}(P) = 99.63\text{ °C}$.
- To fix the state we require $(T, x), (T, v), (P, x), (P, T) \dots$

- can be treated as incompressible (independent of P)

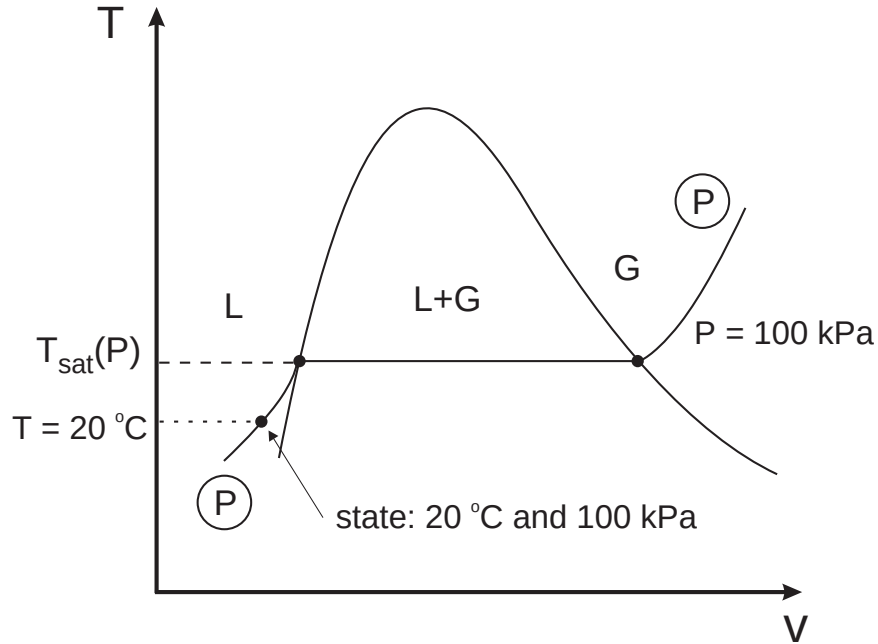
$$v = v(T, P) \Rightarrow v \approx v(T) = v_f(T)$$

$$u = u(T, P) \Rightarrow u \approx u(T) = u_f(T)$$

$$s = s(T, P) \Rightarrow s \approx s(T) = s_f(T)$$

- for enthalpy:

$$\begin{aligned} h(T, P) &= u(T, P) + v(T, P)P \approx u_f(T) + v_f(T)P \\ &= \underbrace{u_f(T) + v_f(T)P_{sat}(T)}_{h_f(T)} + v_f(T)[P - P_{sat}(T)] \end{aligned}$$



therefore we can write

$$h(T, P) = h_f(T) + v_f(T)[P - P_{sat}(T)]$$

Using the “Equation of State” to Calculate the Properties of Gaseous Pure Substances

Thermodynamic relations are generally given in three different forms:

Tables: water (Tables A-4 → A-6, R134a (Tables A-8 → A-10))

Graphs: water (Figure A-7), R134a (Figure A-11)

Equations: air (Table A-12)

- since most gases are highly superheated at atmospheric conditions. This means we have to use the **superheat tables** – but this can be very **inconvenient**
- *a better alternative for gases:* use the Equation of State which is a functional relationship between ***P***, ***v***, and ***T*** (3 measurable properties)
- experience shows that **real gases** obey the following equation closely:

$$Pv = ZRT \quad (T \text{ and } P \text{ are in absolute terms})$$

- this equation is used to find the third quantity when two others are known
- Z is the compressibility factor
- R is the gas constant and is known for different gases ($R = R_u/\tilde{M}$)
 - * $R_u = 8.314 \text{ kJ/kmol} \cdot K$
 - * $\tilde{M}$ is the molar mass (kg/kmol)
- Note: $R_{air} = 0.287 \text{ kJ/kg} \cdot K$, $R_{H_2} = 4.124 \text{ kJ/kg} \cdot K$
- what is the compressibility factor (Z)?
 - if we “reduce” the properties with respect to the values at the critical point, i.e.

$$\text{reduced pressure} = P_r = \frac{P}{P_c} \quad P_c = \text{critical pressure}$$

$$\text{reduced temperature} = T_r = \frac{T}{T_c} \quad T_c = \text{critical temperature}$$

- for all gases a single plot relating Z, P_r, T_r is obtained

Reference Values for u, h, s

- values of enthalpy, h and entropy, s listed in the tables are with respect to a datum where we arbitrarily assign the zero value. For instance:

Table A-4, A-5 & A-6: saturated and superheated water - the reference for both h_f and s_f is taken as 0°C . This is shown as follows:

$$u_f(@T = 0^\circ\text{C}) = 0 \text{ kJ/kg}$$

$$h_f(@T = 0^\circ\text{C}) = 0 \text{ kJ/kg}$$

$$s_f(@T = 0^\circ\text{C}) = 0 \text{ kJ/kg} \cdot K$$

Table A-9 & A-10: saturated and superheated R134a - the reference for both h_f and s_f is taken as -40°C . This is shown as follows:

$$h_f(@T = -40^\circ\text{C}) = 0 \text{ kJ/kg}$$

$$h_f(@T = -40^\circ\text{C}) = 0 \text{ kJ/kg}$$

$$s_f(@T = -40^\circ\text{C}) = 0 \text{ kJ/kg} \cdot K$$

Others: sometimes tables will use 0°C as the reference for all tables. While this standardizes the reference, it tends to lead to larger values of enthalpy and entropy.

Calculation of the Stored Energy

- for most of our 1st law analyses, we need to calculate ΔE

$$\Delta E = \Delta U + \Delta KE + \Delta PE$$

- for stationary systems, $\Delta KE = \Delta PE = 0$
- in general: $\Delta KE = \frac{1}{2}m(\mathcal{V}_2^2 - \mathcal{V}_1^2)$ and $\Delta PE = mg(z_2 - z_1)$
- how do we calculate ΔU ?
 1. one can often find u_1 and u_2 in the thermodynamic tables (like those examined for the states of water).
 2. we can also explicitly relate ΔU to ΔT (as a mathematical expression) by using the thermodynamic properties C_p and C_v .

Specific Heats: Ideal Gases

- for an ideal gas

$$C_v = \frac{du}{dT} \Rightarrow C_v = C_v(T) \text{ only}$$

$$C_p = \frac{dh}{dT} \Rightarrow C_p = C_p(T) \text{ only}$$

From the equation for enthalpy,

$$h(T) = u(T) + p(T)v(T)$$

If we differentiate with respect to T

$$R = \frac{dh}{dT} - \frac{du}{dT}$$

$$\boxed{R = C_p - C_v}$$

- the calculation of Δu and Δh for an ideal gas is given as

$$\Delta u = u_2 - u_1 = \int_1^2 C_v(t) dT \quad (kJ/kg)$$

$$\Delta h = h_2 - h_1 = \int_1^2 C_p(t) dT \quad (kJ/kg)$$

Specific Heats: Solids and Liquids

- solids and liquids are incompressible. (i.e., $\rho = \textit{constant}$).
- for solids and liquids it can be shown (mathematically) that for incompressible materials:

$$C_p = C_v = C \quad \text{and} \quad dQ = mC dT$$

- Table A-3 gives C for incompressible materials
- as with ideal gases, for incompressible materials $C = C(T)$

$$\Delta u = \int_1^2 C(T) dT \approx C_{avg} \Delta T$$

$$\Delta h = \Delta u + \Delta P v = \Delta u + v \Delta p \approx C_{avg} \Delta T + v \Delta P$$

– for solids:

$$\Delta h = \Delta u = C_{avg} \Delta T \quad (v \Delta P \text{ is negligible})$$

– for liquids:

$$\text{If } P = \textit{const}, \Delta h = C_{avg} \Delta T$$

If $T = \textit{const}$, $\Delta h = v \Delta P$ (we already used this to do more accurate calculations in the subcooled region).

First Law of Thermodynamics



Reading

3-1 $\rightarrow$ 3-8

4-1 $\rightarrow$ 4-3

Problems

3-35, 3-39, 3-64, 3-68, 3-72, 3-78, 3-116

4-19, 4-29, 4-38, 4-45, 4-54

Control Mass (Closed System)

A thermodynamic analysis of a system can be performed on a fixed amount of matter known as a *control mass* or over a region of space, known as a *control volume*. In this section we will examine the case of a control surface that is closed to mass flow, so that no mass can escape or enter the defined control region. Since mass is fixed, the system is considered to be a control mass.

The two underlying conservation equations for performing a thermodynamic analysis on a *Closed System* or a *Control Mass*, are:

1. Conservation of Mass, and
2. Conservation of Energy

Conservation of Mass

Conservation of Mass, which states that mass cannot be created or destroyed, is implicitly satisfied by the definition of a control mass.

Conservation of Energy

The first law of thermodynamics states:

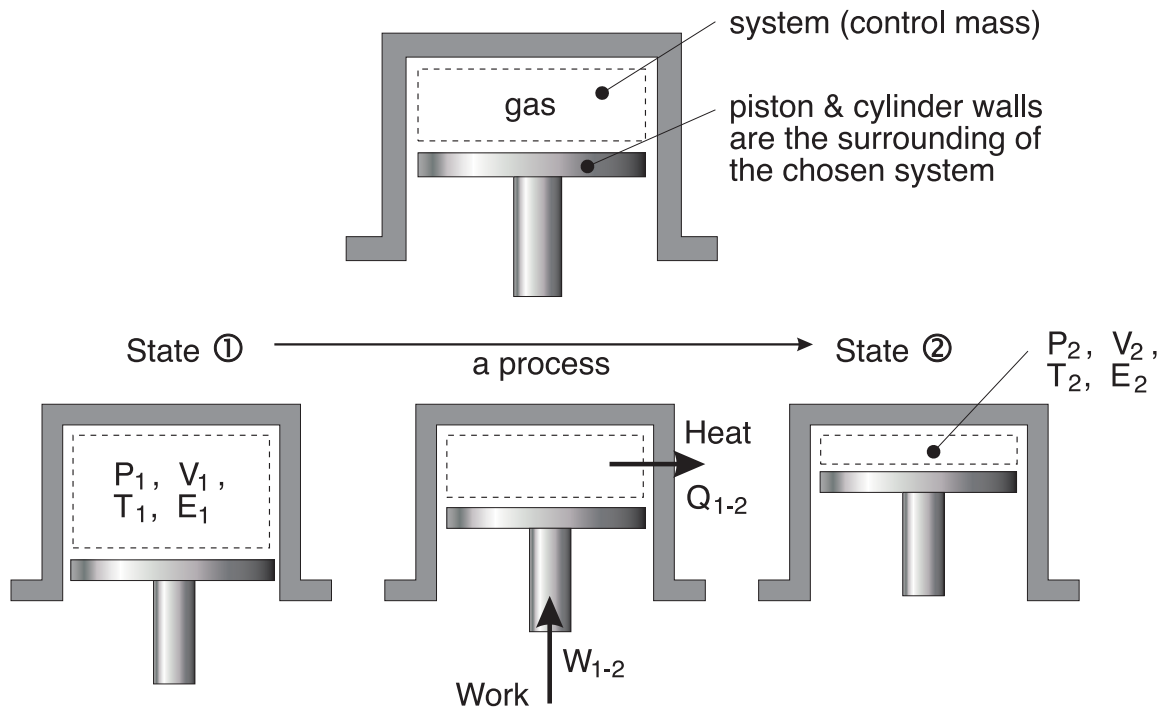
Energy cannot be created or destroyed it can only change forms.

- energy transformation is accomplished through energy transfer as work and/or heat. Work and heat are the forms that energy can take in order to be transferred across the system boundary.

- the first law leads to the principle of Conservation of Energy where we can stipulate the energy content of an *isolated* system is constant.

$$\text{energy entering} - \text{energy leaving} = \text{change of energy within the system}$$

Example: A Gas Compressor



Performing a 1st law energy balance:

$$\left\{ \begin{array}{c} \text{Initial} \\ \text{Energy} \\ E_1 \end{array} \right\} + \left\{ \begin{array}{c} \text{Energy gain } W_{1-2} \\ \text{Energy loss } Q_{1-2} \end{array} \right\} = \left\{ \begin{array}{c} \text{Final} \\ \text{Energy} \\ E_2 \end{array} \right\}$$

$$E_1 + W_{1-2} - Q_{1-2} = E_2$$

or

$$\Delta E = Q - W$$

A first law balance for a control mass can also be written in differential form as follows:

$$dE = \delta Q - \delta W$$

The differential form of the energy balance can be written as a rate equation by dividing through by dt , a differential time, and then letting $dt \rightarrow 0$ in the limit to give

$$\frac{dE}{dt} = \frac{\delta Q}{dt} - \frac{\delta W}{dt} \Rightarrow \frac{dE}{dt} = \dot{Q} = \dot{W}$$

where

$$\frac{dE}{dt} = \text{rate of energy increase within the system,} \equiv \frac{dU}{dt} + \frac{dKE}{dt} + \frac{dPE}{dt}$$

$$\dot{Q} = \text{rate of heat transfer}$$

$$\dot{W} = \text{rate of work done,} \equiv \text{power}$$

Forms of Energy Transfer

Work Versus Heat

- Work is macroscopically organized energy transfer.
- Heat is microscopically disorganized energy transfer.

Heat Energy

- heat is defined as a form of energy that is transferred solely due to a temperature difference (without mass transfer)
- *Note:* It is the thermal (internal) energy that can be stored in a system. Heat is a form of energy in transition and as a result can only be identified at the system boundary.
- Notation:
 - Q (kJ) amount of heat transfer
 - $\dot{Q}$ (kW) rate of heat transfer (power)
 - q (kJ/kg) - heat transfer per unit mass
 - $\dot{q}$ (kW/kg) - power per unit mass

- heat transfer is a directional (or vector) quantity with magnitude, direction and point of action
- modes of heat transfer:
 - **conduction:** diffusion of heat in a stationary medium (Chapters 8 & 9)
 - **convection:** it is common to include convective heat transfer in traditional heat transfer analysis. However, it is considered mass transfer in thermodynamics. (Chapters 10 & 11)
 - **radiation:** heat transfer by photons or electromagnetic waves (Chapter 12)

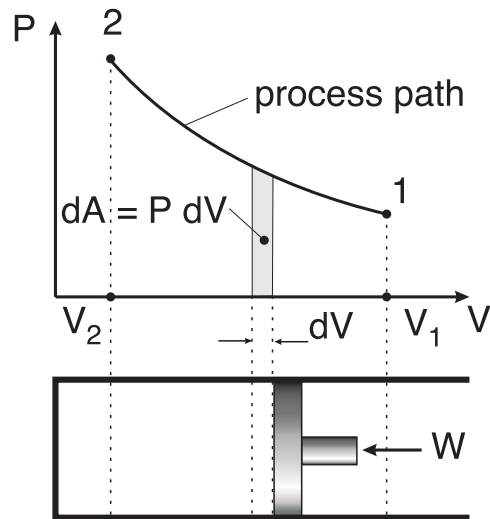
Work Energy

- work is a form of energy in transit. One should not attribute work to a system.
- work (like heat) is a “path function” (magnitude depends on the process path)
 - i.e Δw is incorrect, use

$$\int_1^2 \delta w = w_{12}$$

- Notation:
 - W (kJ) amount of work transfer
 - $\dot{W}$ (kW) power
 - w (kJ/kg) - work per unit mass
 - $\dot{w}$ (kW/kg) - power per unit mass
- work transfer mechanisms in general, are a force acting over a distance

Moving Boundary Work



- consider compression in a piston/cylinder, where A is the piston cross sectional area (frictionless)
- the area under the process curve on a $P-V$ diagram is proportional to $\int_1^2 P dV$
- the work is:
 - $+ve$ for compression
 - $-ve$ for expansion
- sometimes called $P dV$ work or compression /expansion work

$$W_{12} = -\int_1^2 F ds = -\int_1^2 P \cdot A ds = -\int_1^2 P dV$$

- a decrease in the volume, $dV \rightarrow -ve$ results in work addition ($+ve$) on the system
- *polytropic processes*: where $PV^n = C$
- examples of polytropic processes include:

Isobaric process: if $n = 0$ then $P = C$ and we have a constant pressure process

Isothermal process: if $n = 1$ then from the ideal gas equation $PV = RT$ and PV is only a function of temperature

Isometric process: if $n \rightarrow \infty$ then $P^{1/n}V = C^{1/n}$ and we have a constant volume process

Isentropic process: if $n = k = C_p/C_v$ then we have an isentropic process

Gravitational Work

- if dz is a component parallel to gravity

$$W_{12} = -\int_1^2 F ds = -\int_1^2 mg ds = -\int_1^2 mg dz$$

- work is added to the system when gravity assists, i.e. $dz \rightarrow -ve$

- integrating from state **1** to state **2** gives

$$W_{12} = -mg(z_2 - z_1)$$

Acceleration Work

- if the system is accelerating, the work associated with the change of the velocity can be calculated as follows:

$$W_{12} = -\int_1^2 F ds = -\int_1^2 ma ds = -\int_1^2 m \frac{d\mathcal{V}}{dt} ds = -\int_1^2 m d\mathcal{V} \frac{ds}{dt}$$

and we can then write

$$W_{12} = -\int_1^2 m\mathcal{V} d\mathcal{V} = m \left(\frac{\mathcal{V}_1}{2} - \frac{\mathcal{V}_2}{2} \right)$$

Charge Transfer Work (Electrical Work)

- charge is a conserved quantity, like mass and energy
- current, I is the rate of charge transfer

$$I \equiv \frac{dq^+}{dt}, \quad \frac{\text{coulomb}}{\text{sec}} = \text{Ampere}$$

where

$$q^+ = -Ne$$

with N being the number of electrons and e the charge of the electron.

- the electrical work done is given as

$$\delta W_e = (\epsilon_1 - \epsilon_2) dq^+ = \epsilon dq^+$$

where ϵ is the electrical potential difference with units $\text{volt} = \frac{\text{Joule}}{\text{Coulomb}}$

- the electrical work done per unit time is power

$$\dot{W}_e = \text{Power} = \frac{\delta W_e}{dt} = \epsilon I \quad (\text{W})$$

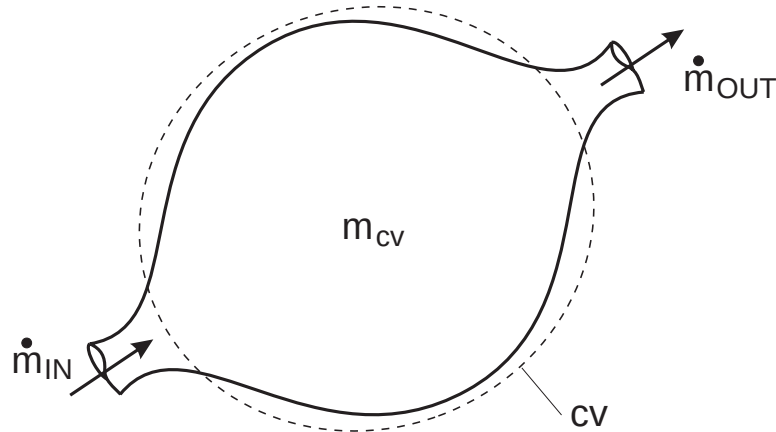
$$\begin{aligned} W_e &= \int_1^2 \epsilon I dt \\ &= \epsilon I \Delta t \end{aligned}$$

Control Volume (Open System)

The major difference between a *Control Mass* and a *Control Volume* is that mass crosses the system boundary of a control volume. We will define a region in space that may be moving or changing shape. As the process proceeds we will monitor the volume as mass, heat and work cross the system boundary. The control volume approach will be used for many engineering problems where a mass flow rate is present, such as:

- turbines, pumps and compressors
- heat exchangers
- nozzles and diffusers, etc.

CONSERVATION OF MASS:



$$\left\{ \begin{array}{l} \text{rate of increase} \\ \text{of mass within} \\ \text{the CV} \end{array} \right\} = \left\{ \begin{array}{l} \text{net rate of} \\ \text{mass flow} \\ IN \end{array} \right\} - \left\{ \begin{array}{l} \text{net rate of} \\ \text{mass flow} \\ OUT \end{array} \right\}$$

$$\frac{d}{dt}(m_{CV}) = \dot{m}_{IN} - \dot{m}_{OUT}$$

where:

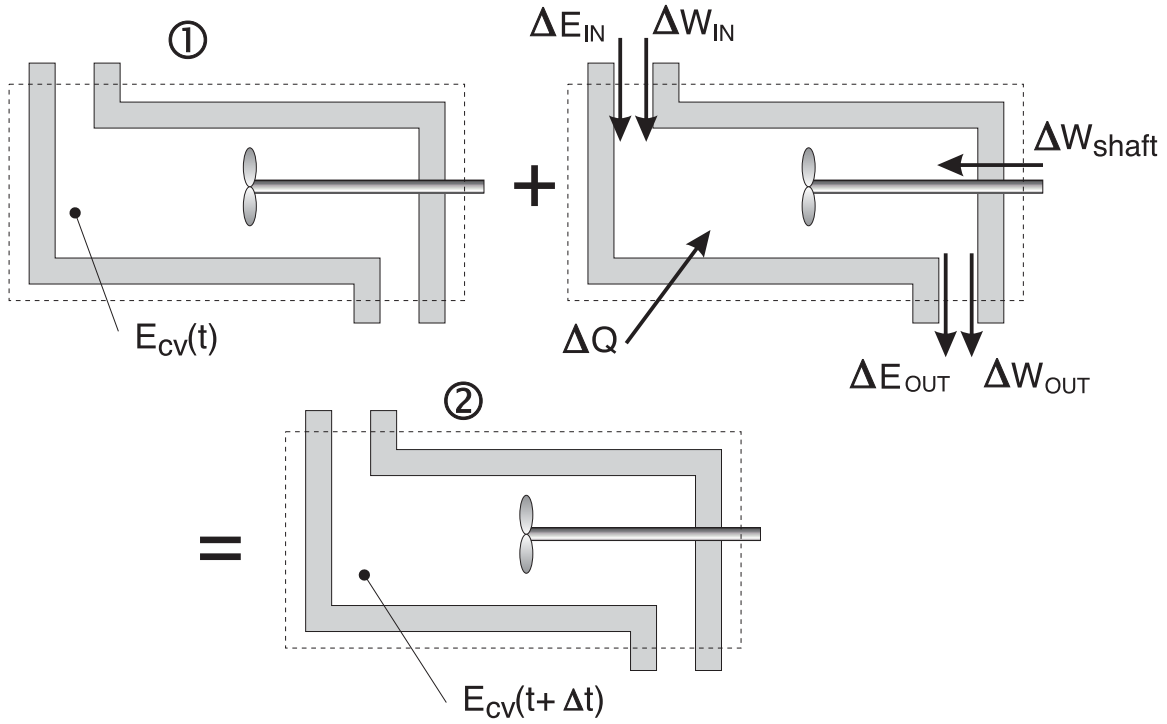
$$m_{CV} = \int_V \rho \, dV$$

$$\dot{m}_{IN} = (\rho \, \mathcal{V} \, A)_{IN}$$

$$\dot{m}_{OUT} = (\rho \, \mathcal{V} \, A)_{OUT}$$

with $\mathcal{V}$ = average velocity

CONSERVATION OF ENERGY:



The 1st law states:

$$E_{CV}(t) + \Delta Q + \Delta W_{shaft} + (\Delta E_{IN} - \Delta E_{OUT}) + (\Delta W_{IN} - \Delta W_{OUT}) = E_{CV}(t + \Delta t) \quad (1)$$

where:

$$\Delta E_{IN} = e_{IN} \Delta m_{IN}$$

$$\Delta E_{OUT} = e_{OUT} \Delta m_{OUT}$$

$$\Delta W = \text{flow work}$$

$$e = \frac{E}{m} = \underbrace{u}_{\text{internal}} + \underbrace{\frac{v^2}{2}}_{\text{kinetic}} + \underbrace{gz}_{\text{potential}}$$

Some Practical Assumptions for Control Volumes

Steady State Process: The properties of the material inside the control volume do not change with time.

Steady Flow Process: The properties of the material crossing the control surface do not change with time.

The steadiness refers to variation with respect to time

- if the process is not steady, it is *unsteady* or *transient*
- often steady flow implies both *steady flow* and *steady state*

Uniform State Process: The properties of the material inside the control volume are uniform and may change with time. For example



Uniform Flow Process: The properties of the material crossing the control surface are spatially uniform and may change with time.

Uniformity is a concept related to the spatial distribution. If the flow field in a process is not uniform, it is *distributed*.

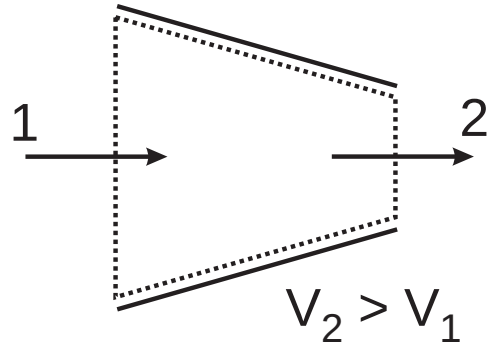
Steady-State, Steady-Flow Process

Idealizations:

- the control volume does not move relative to the reference frame
- the state of the mass at each point within the control volume does not change with time
- the flows in and out of the control volume are steady, i.e. there is no mass accumulation within the control volume
- the rates at which work and heat cross the control volume boundary remain constant

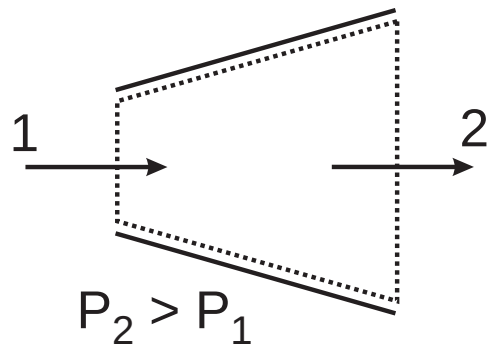
Nozzles:

- used to increase the velocity of a fluid (sub-sonic flow)
- common assumptions and idealizations
 - $\dot{Q} = 0$ (no time for heat transfer)
 - $\dot{W} = 0$
 - $\Delta PE = 0$
 - steady flow, steady state, uniform flow



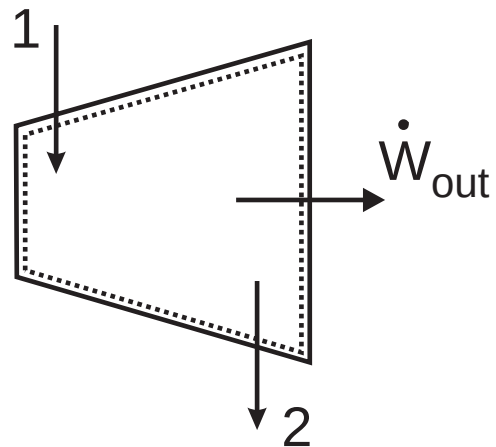
Diffusers:

- used to increase the pressure of a fluid (sub-sonic flow)
- common assumptions and idealizations
 - $\dot{Q} = 0$
 - $\dot{W} = 0$
 - $\Delta PE = 0$
 - steady flow, steady state, uniform flow



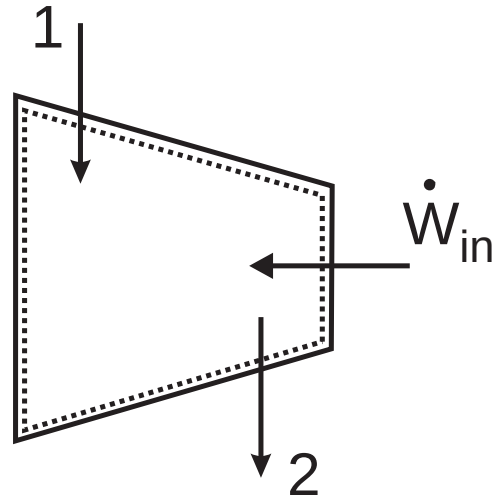
Turbines (steam, gas, hydraulic):

- used to produce power (output = shaft power)
- common assumptions and idealizations
 - $\dot{Q} = 0$
 - $\Delta KE = 0$
 - $\Delta PE = 0$
 - steady flow, steady state, uniform flow



Compressors (pumps, fans):

- used to increase the pressure of the fluid using power input
- common assumptions and idealizations
 - $\dot{Q} = 0$
 - $\Delta KE = 0$
 - $\Delta PE = 0$
 - steady flow, steady state, uniform flow



Throttling Valves:

- used to drop the pressure and/or temperature (converts u to Pv)
- common assumptions and idealizations
 - $\dot{Q} = 0$
 - $\dot{W} = 0$
 - $\Delta KE = 0$
 - $\Delta PE = 0$
 - steady flow, steady state, uniform flow



a) an adjustable valve



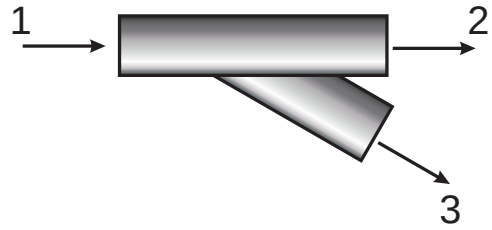
b) a porous plug



c) a capillary tube

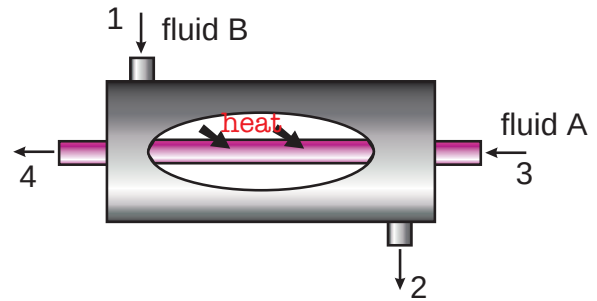
Mixing Chambers (direct contact heat exchangers):

- used to mix two or more fluids
- common assumptions and idealizations
 - $\dot{Q} = 0$
 - $\dot{W} = 0$
 - $\Delta KE = 0$
 - $\Delta PE = 0$
 - steady flow, steady state, uniform flow



Heat Exchangers (boilers, heat exchangers, condensers):

- used to exchange heat between two fluid streams without mixing
- common assumptions and idealizations
 - $\dot{W} = 0$
 - $\Delta KE = 0$
 - $\Delta PE = 0$
 - steady flow, steady state, uniform flow

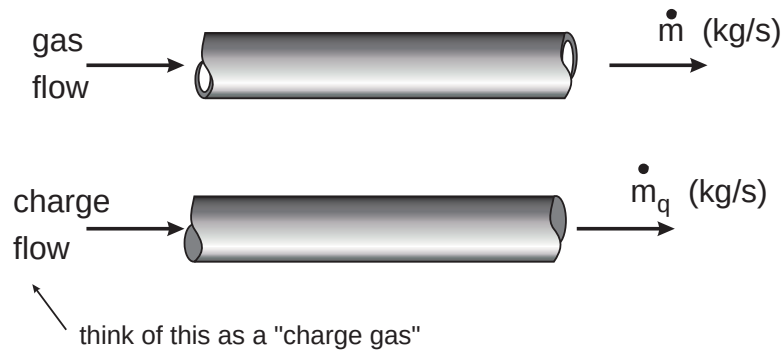


Control Volume Analysis for Electrical Devices

We recall from the 1st law for a control volume

$$\frac{dE_{cv}}{dt} = \dot{Q} - \dot{W} + \dot{m}_{in}(e + Pv)_{in} - \dot{m}_{out}(e + Pv)_{out}$$

We can consider the following analogy



Let

- N = number of charged particles
- q^+ = +ve charge on each particle
- Nq^+ = total charge
- m_q = mass of each charged particle
- Nm_q = total mass of the “charge gas”
- $\dot{Nm}_q$ = flow rate of the “charge gas”
- ϵ = electrical potential

$$\dot{m}(e + Pv) = (\dot{Nm}_q) \left(\underbrace{u + Pv + \frac{v^2}{2} + gz}_{\text{negligible}} + \underbrace{\frac{(Nq^+)\epsilon}{(Nm_q)}}_{\substack{\text{electrical potential} \\ \text{per unit mass}}} \right)$$

$$\dot{m}(e + Pv) = (\dot{Nm}_q) \left(\frac{(Nq^+)\epsilon}{(Nm_q)} \right)$$

$$\begin{aligned}
&= \dot{N}q^+\epsilon \\
&= I\epsilon
\end{aligned}$$

where $I = \text{current} \equiv \dot{N}q^+ = -\dot{N}e$

The first law takes the form

$$\frac{dE_{cv}}{dt} = \dot{Q} - \dot{W} + (\dot{N}q^+\epsilon)_{in} - (\dot{N}q^+\epsilon)_{out}$$

Transient Processes

- examples of transient processes include filling, discharging, startup, shut down
- common assumptions and idealizations:
 - depends on the problem
 - uniform state, uniform flow often but not necessarily
- general mass and energy balances:

Mass

$$\frac{dm}{dt} = \sum \dot{m}_{in} - \sum \dot{m}_{out}$$

$$\Delta m = \sum m_{in} - \sum m_{out}$$

Energy

$$\frac{dE}{dt} = (\dot{m}_{in}\theta_{in} + \dot{Q}_{in} + \dot{W}_{in}) - (\dot{m}_{out}\theta_{out} + \dot{Q}_{out} + \dot{W}_{out})$$

$$\dot{m}\theta = \frac{d}{dt} \int_m \left(h + \frac{\mathcal{V}^2}{2} + gz \right) dm = \int_A \left(h + \frac{\mathcal{V}^2}{2} + gz \right) \rho \mathcal{V} dA$$

$$E = me = \int_m \left(u + \frac{\mathcal{V}^2}{2} + gz \right) dm = \int_V \left(u + \frac{\mathcal{V}^2}{2} + gz \right) \rho dV$$

This is usually taken out of rate form

$$\begin{aligned} Q_{in} - W_{out} + \sum_{in} \int_1^2 \left(h + \frac{\mathcal{V}^2}{2} + gz \right) dm - \sum_{out} \int_1^2 \left(h + \frac{\mathcal{V}^2}{2} + gz \right) dm \\ = \Delta \left(m \left(u + \frac{\mathcal{V}^2}{2} + gz \right) \right) \end{aligned}$$

Steady conditions at the inlet

$$\begin{aligned} Q_{in} - W_{out} + \sum_{in} m \left(h + \frac{\mathcal{V}^2}{2} + gz \right) dm - \sum_{out} m \left(h + \frac{\mathcal{V}^2}{2} + gz \right) dm \\ = \Delta \left(m \left(u + \frac{\mathcal{V}^2}{2} + gz \right) \right) \end{aligned}$$

Entropy and the Second Law of Thermodynamics



Reading

5-1 $\rightarrow$ 5-3, 5.6, 5.7
6-1 $\rightarrow$ 6-11

Problems

5-25, 5-71, 5-74
6-31, 6-33, 6-42, 6-57, 6-59, 6-65, 6-72, 6-77

Why do we need another law in thermodynamics?

Answer: While the 1st law allowed us to determine the quantity of energy transfer in a process it does not provide any information about the direction of energy transfer nor the quality of the energy transferred in the process. In addition, we can not determine from the 1st law alone whether the process is possible or not. The second law will provide answers to these unanswered questions.

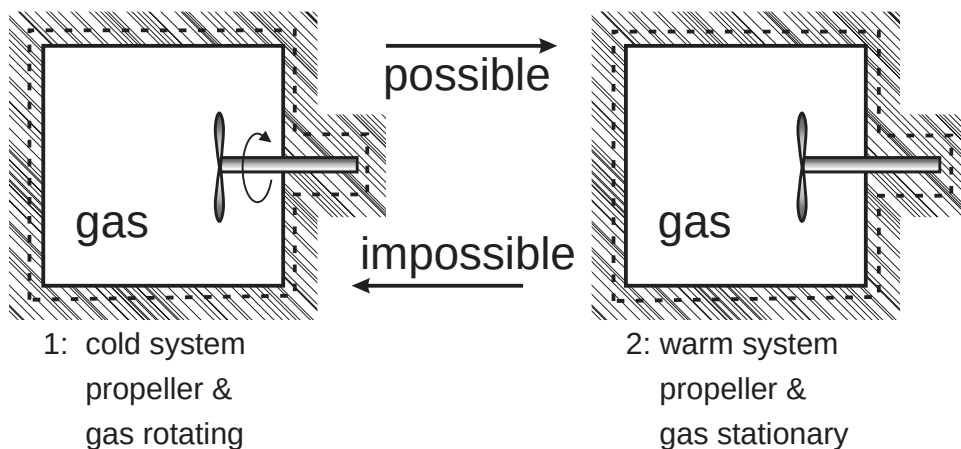
A process will not occur unless it satisfies both the first and the second laws of thermodynamics.

1. Direction:

Consider an isolated system where $Q = W = 0$

From the 1st law we know

$$\underbrace{(U + KE)_1}_{E_1} = \underbrace{U_2}_{E_2} \quad \left\{ \begin{array}{l} \text{same both ways} \\ 1 \rightarrow 2 \\ 2 \rightarrow 1 \end{array} \right.$$



The physical interpretation of this is:

State 1: Most of the energy is in a highly organized form residing in the macroscopic KE of the propeller and the rotating gas.

State 2: All of the energy is now in a disorganized form residing in the microscopic E , i.e. U of the propeller and the gas.

- The process $1 \rightarrow 2$ has resulted in a higher state of molecular chaos. **ENTROPY** is the thermodynamic property that describes the degree of molecular disorder in matter. Hence, $S_2 > S_1$. Entropy can be considered a quantitative index that describes the quality of energy.
- The process $2 \rightarrow 1$ is impossible because it would require disorganized KE to produce macroscopically organized KE . That is $S_2 < S_1$ which is impossible for an isolated system.
- A thermodynamic process is only possible if it satisfies both the 1st and 2nd laws simultaneously.

2. Quality of Energy:

A heat engine produces reversible work as it transfers heat from a high temperature reservoir at T_H to a low temperature reservoir at T_L . If we fix the low temperature reservoir at $T_L = 300\text{ K}$, we can determine the relationship between the efficiency of the heat engine,

$$\eta = 1 - \frac{Q_L}{Q_H} = 1 - \frac{T_L}{T_H}$$

as the temperature of the high temperature reservoir changes. In effect we are determining the quality of the energy transferred at high temperature versus that transferred at low temperature.

Second Law of Thermodynamics

The second law of thermodynamics states:

The entropy of an isolated system can never decrease. When an isolated system reaches equilibrium, its entropy attains the maximum value possible under the constraints of the system

Definition

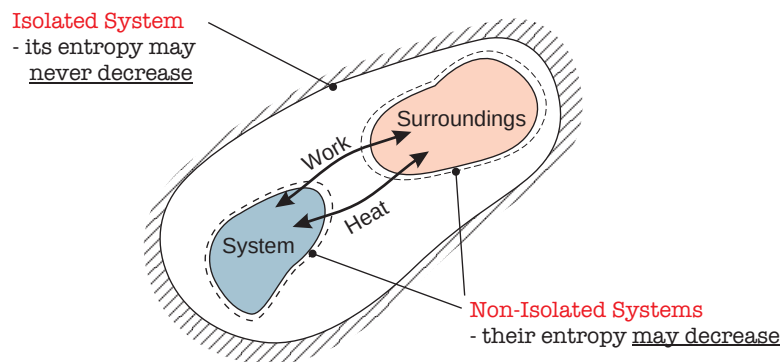
Unlike mass and energy, entropy can be produced but it can never be destroyed. That is, the entropy of a system plus its surroundings (i.e. an isolated system) can never decrease (2nd law).

$$\mathcal{P}_m = m_2 - m_1 = 0 \quad (\text{conservation of mass})$$

$$\mathcal{P}_E = E_2 - E_1 = 0 \quad (\text{conservation of energy}) \rightarrow \text{1st law}$$

$$\mathcal{P}_S = S_{gen} = S_2 - S_1 \geq 0 \rightarrow \text{2nd law}$$

- we have conservation of mass and energy, but not entropy. Entropy is not conserved.
- the 2nd law dictates why processes occur in a specific direction i.e., S_{gen} cannot be $-ve$



The second law states, for an isolated system:

$$(\Delta S)_{system} + (\Delta S)_{surr.} \geq 0$$

where $\Delta \equiv final - initial$

Entropy

1. Like mass and energy, every system has entropy.

Entropy is a measure of the degree of microscopic disorder and represents our uncertainty about the microscopic state.

2. **Reference:** In a perfect crystal of a pure substance at $T = 0\text{ K}$, the molecules are completely motionless and are stacked precisely in accordance with the crystal structure. Since entropy is a measure of microscopic disorder, then in this case $S = 0$. That is, there is no uncertainty about the microscopic state.
3. **Relationship to Work:** For a given system, an increase in the microscopic disorder (that is an increase in entropy) results in a loss of ability to do useful work. Work can completely (and easily) be transformed into thermal energy and then exchanged as heat.
4. **Work:** Energy transfer by work is microscopically organized and therefore entropy-free. Work can completely (and easily) be transformed into thermal energy and then exchanged as heat.
5. **Heat:** Energy transfer as heat takes place as work at the microscopic level but in a random, disorganized way. This type of energy transfer carries with it some chaos and thus results in entropy flow in or out of the system. Heat transfer from a heat source cannot be used to exchange the same amount of energy as work.
6. **Reversibility:** According to the 2nd law, no real process can be reversed without leaving any trace on the surroundings. In fact, the 2nd law says that it is impossible to do things perfectly (it is ideal but impossible). It is our job (as engineers) to make the process as close as possible to ideal, i.e. no loss and perfect recovery. A reversible process is the theoretical limit for an irreversible (real) process.

Gibb's Equation

From a 1st law energy balance when KE and PE are neglected

$$\text{Energy Input} = \text{Energy Output} + \text{Increase in Energy Storage}$$

$$\underbrace{\delta Q}_{\text{amount}} = \delta W + \underbrace{dU}_{\text{differential}} \quad (1)$$

We know that the differential form of entropy is

$$dS = \frac{\delta Q}{T} \quad (2)$$

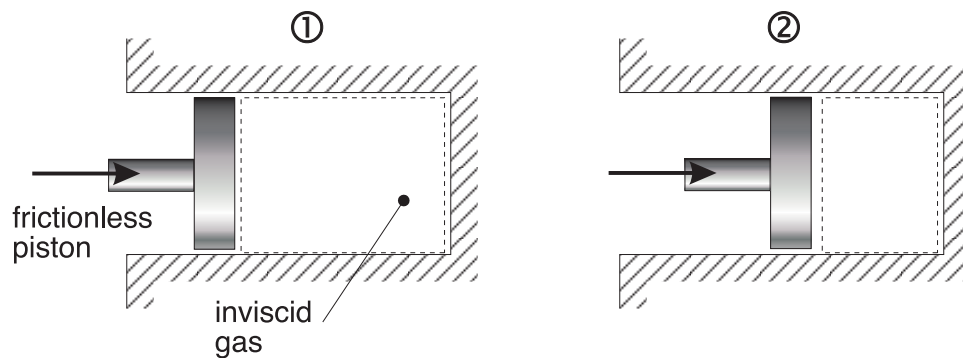
$$\delta W = PdV \quad (3)$$

Combining Eqs. 1, 2 and 3

$$dS = \frac{dU}{T} + \frac{PdV}{T} \Rightarrow \underbrace{ds = \frac{du}{T} + \frac{Pdv}{T}}_{\text{per unit mass}}$$

Reversible Process

Example: Slow adiabatic compression of a gas



Idealization - which makes
the process completely
reversible

A process $1 \rightarrow 2$ is said to be reversible if the reverse process $2 \rightarrow 1$ restores the system to its original state without leaving any change in either the system or its surroundings.

$$\underbrace{\text{Reversible}}_{S_{gen}=0} + \underbrace{\text{Adiabatic Process}}_{Q=0} \Rightarrow \underbrace{\text{Isentropic Process}}_{S_1=S_2}$$

The 2nd law states:

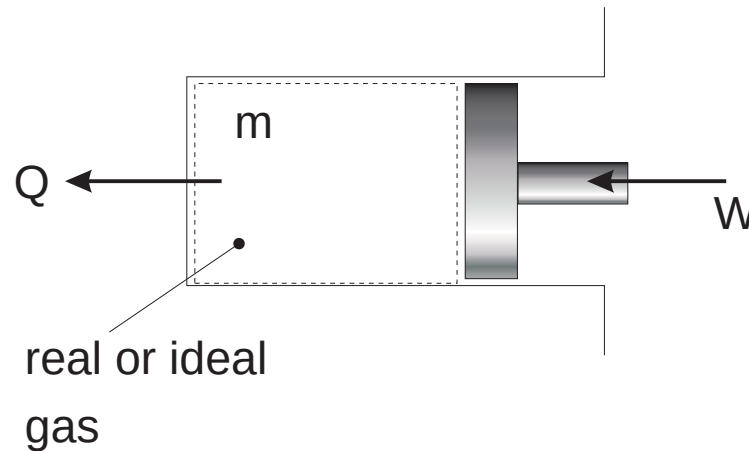
$$(\Delta S)_{system} + (\Delta S)_{surr} = S_{gen} \geq 0$$

where:

> 0 irreversible (real world)

$= 0$ reversible (frictionless, perfectly elastic, inviscid fluid)

Reversible Compression and Expansion



From the 1st law

increase in energy storage = energy input

$$dU = \delta W - \delta Q$$

$$mdu = \delta W - \delta Q \quad (1)$$

From the 2nd law for a control mass

$$dS = -\frac{\delta Q}{T} + S_{gen}$$

where $\mathbf{S}_{gen} = \mathbf{0}$ for a reversible process.

Therefore

$$m ds = -\frac{\delta Q}{T} \quad (2)$$

Combining (1) and (2) through the $-\delta Q$ term gives

$$m du = \delta W + m T ds \quad (3)$$

From Gibb's Eq.

$$\begin{aligned} T ds &= du + P dv \\ m du &= m T ds - m P dv \end{aligned} \quad (4)$$

Combining Eqs. (3) and (4)

$$\begin{aligned} m T ds - m P dv &= \delta W + m T ds \\ \delta W &= -m P dv \end{aligned}$$

Integrating gives

$$W_{1-2} = -m \int_1^2 P dv$$

or on a per unit mass basis

$$\boxed{w_{1-2} = \frac{W_{1-2}}{m} = -\int_1^2 P dv}$$

Calculation of ΔS in Processes

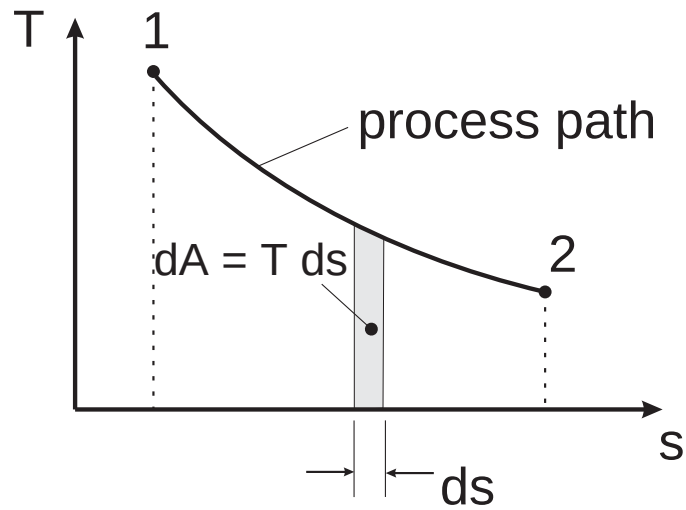
For a simple compressible system, Gibbs's equations are given as

$$Tds = du + Pdv$$

$$Tds = dh - vdP$$

- $T - s$ and $h - s$ (Mollier) diagrams are very useful

$$q_{int,rev} = \int_1^2 T ds \quad \text{and} \quad q_{int,rev,isothermal} = T\Delta s$$

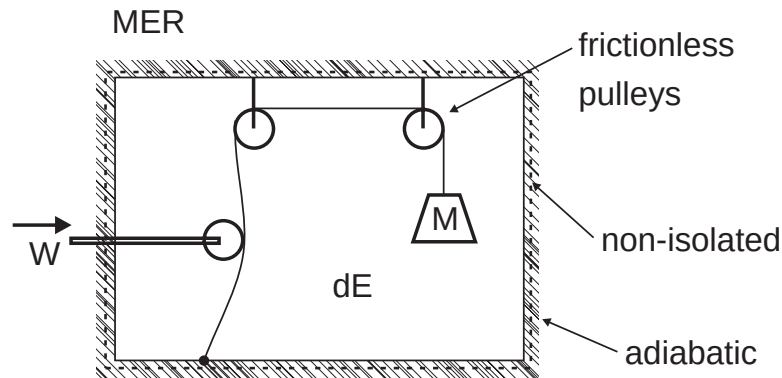


System Level Analysis

Energy Reservoirs

Idealized Mechanical Energy Reservoir (MER)

This is a source or sink for organized energy.

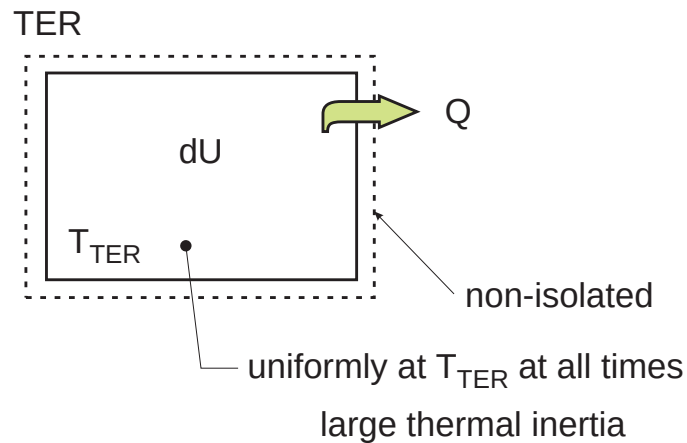


The system shown above is open to work transfer only, therefore it is entropy free.

$$dS = 0 \quad \text{for a MER}$$

Idealized Thermal Energy Reservoir (TER)

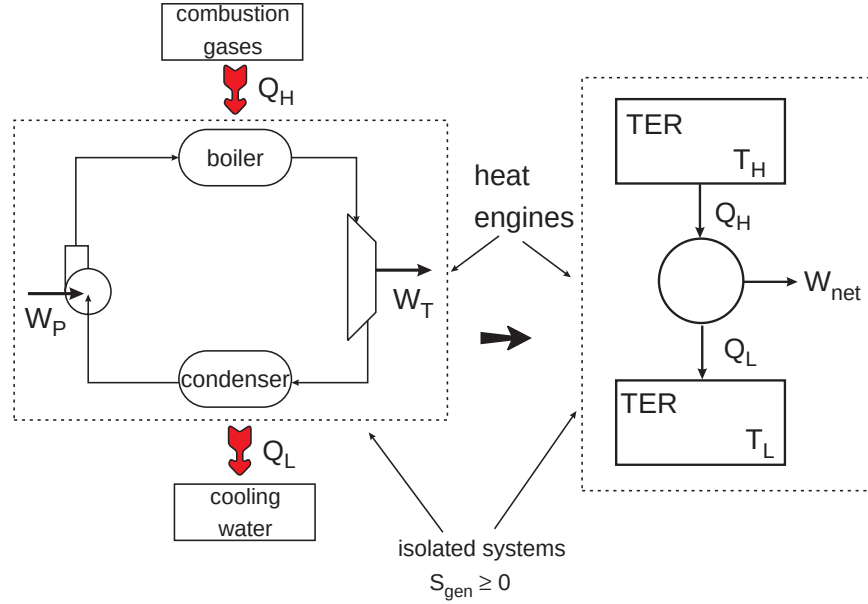
This is a source or sink for disorganized energy.



Examples include; oceans, rivers, lakes, the atmosphere, etc.

Reversible Heat Engine

A heat engine is a device in which a working substance (control mass) undergoes a cyclic process while operating between two temperature reservoirs (TER).



The engine efficiency is defined as the benefit over the cost

$$\eta = \frac{\text{benefit}}{\text{cost}} = \frac{W_{net}}{Q_H} = 1 - \frac{T_L}{T_H} - \frac{T_L S_{gen}}{Q_H}$$

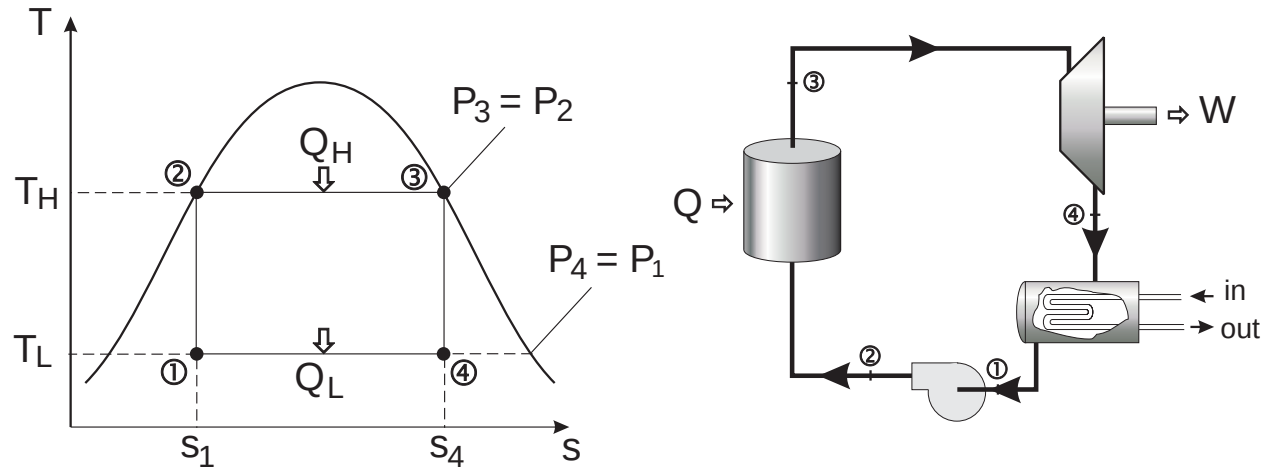
For a perfect (reversible) system where $S_{gen} = 0$

$$\eta = 1 - \frac{T_L}{T_H} \quad \Leftarrow \quad \text{Carnot efficiency}$$

It is clear that even for a “perfect” the efficiency cannot be **100%**

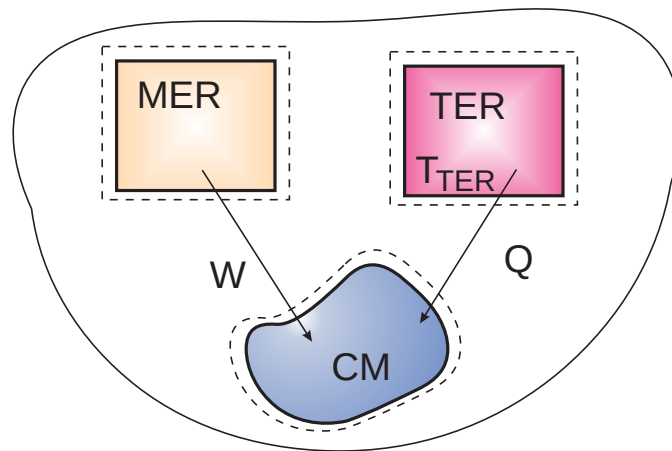
The Carnot Cycle

An ideal theoretical cycle that is the most efficient conceivable.



The cycle is totally reversible. The reversed Carnot cycle is called the Carnot refrigeration cycle.

2nd Law Analysis for a Closed System (Control Mass)



We can first perform a 1st law energy balance on the system shown above.

$$dU = \delta Q + (-\delta W) \quad (1)$$

where the work term is $-\delta W$ because work is assumed to be work outflow and in our case it is directed inward.

For a simple compressible system

$$\delta W = P dV \quad (2)$$

From Gibb's equation we know

$$T_{TER} dS = dU + P dV \quad (3)$$

Combining (1), (2) and (3) we get

$$T_{TER} dS = \delta Q$$

where

$$\text{net in-flow} \quad dS = \frac{\delta Q}{T_{TER}}$$

$$\text{net out-flow} \quad dS = -\frac{\delta Q}{T_{TER}}$$

Therefore

$$\underbrace{(dS)_{CM}}_{\equiv \text{storage}} = \underbrace{\frac{dQ}{T_{TER}}}_{\equiv \text{entropy flow}} + \underbrace{d\mathcal{P}_S}_{\equiv \text{production}}$$

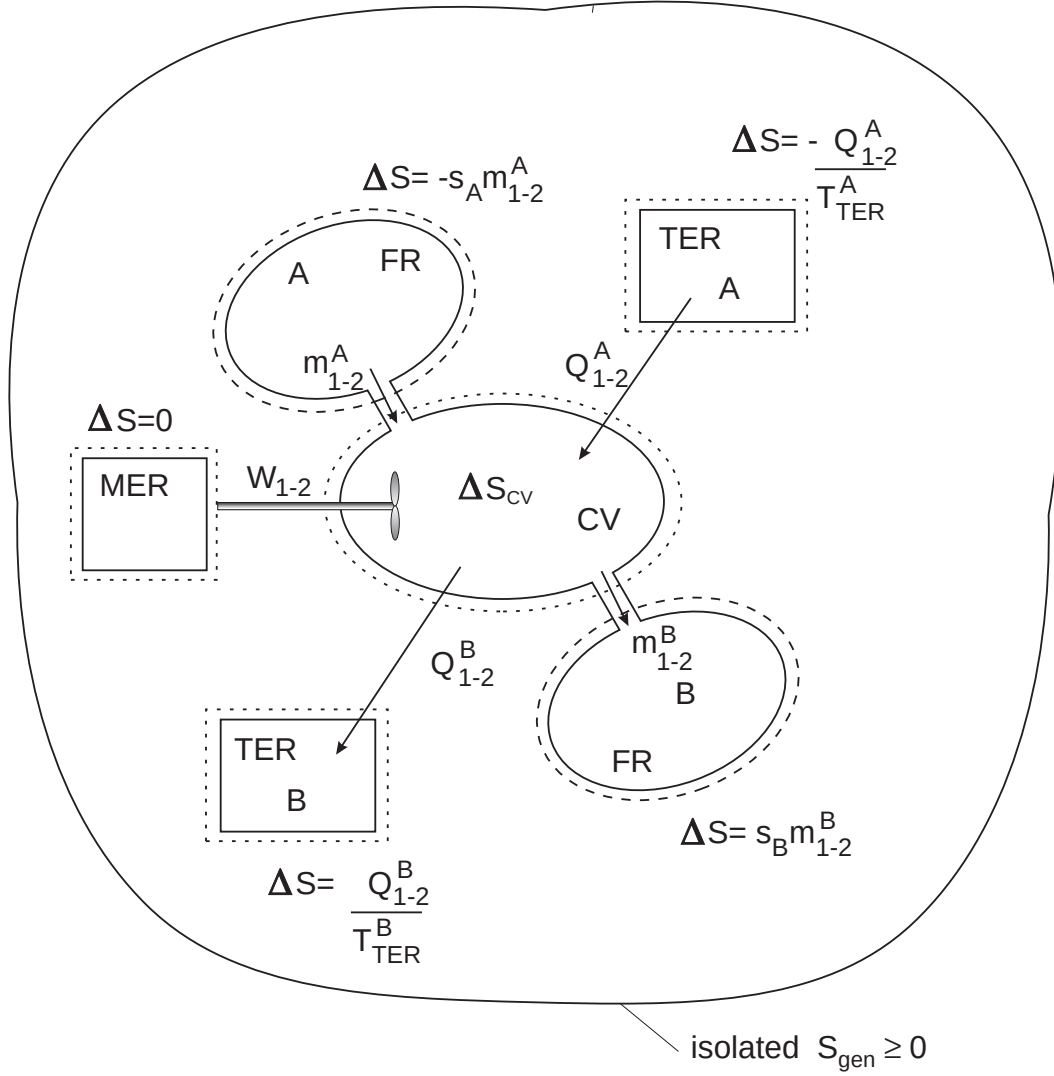
Integrating gives

$$(S_2 - S_1)_{CM} = \frac{Q_{1-2}}{T_{TER}} + \underbrace{S_{gen}}_{\geq 0}$$

where

$$\frac{Q_{1-2}}{T_{TER}} \text{ - the entropy associated with heat transfer across a finite temperature difference, i.e. } T > 0$$

2nd Law Analysis for Open Systems (Control Volume)



where:

- FR - fluid reservoir
- TER - thermal energy reservoir
- MER - mechanical energy reservoir

For the isolated system going through a process from $1 \rightarrow 2$

$$\delta S_{gen} = (\Delta S)_{sys} + (\Delta S)_{sur}$$

$$\delta S_{gen} = \underbrace{\Delta S_{CV}}_{system} + \underbrace{\left(-s_A m_{1-2}^A + s_B m_{1-2}^B - \frac{\delta Q_{1-2}^A}{T_{TER}^A} + \frac{\delta Q_{1-2}^B}{T_{TER}^B} \right)}_{surroundings}$$

or as a rate equation

$$\dot{S}_{gen} = \left(\frac{dS}{dt} \right)_{CV} + \left(s\dot{m} + \frac{\dot{Q}}{T_{TER}} \right)_{OUT} - \left(s\dot{m} + \frac{\dot{Q}}{T_{TER}} \right)_{IN}$$

This can be thought of as

$$\textit{generation} = \textit{accumulation} + \textit{OUT} - \textit{IN}$$

Conduction Heat Transfer



Reading

8-1 → 8-9

9-1 → 9-2

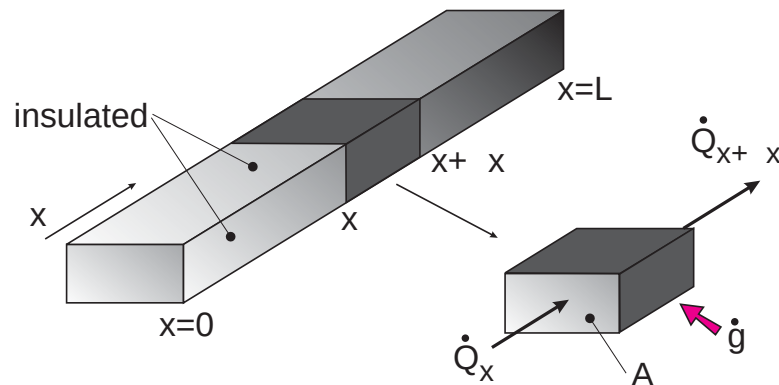
Problems

8-35, 8-63, 8-64, 8-69, 8-75, 8-80, 8-84, 8-89, 8-100, 8-103, 8-112, 8-126, 8-141, 8-150, 8-177

9-15, 9-20, 9-21, 9-36, 9-41, 9-42

Fourier Law of Heat Conduction

Consider a long solid bar where heat travels in a one-dimensional manner in the positive x direction as shown below.



$$\frac{\partial E}{\partial t} = \dot{Q}_x - \dot{Q}_{x+\Delta x} - \dot{W}$$

If the volume of the element is given as $V = A \cdot \Delta x$, then the mass of the element is

$$m = \rho \cdot A \cdot \Delta x$$

$$E = m \cdot u = (\rho \cdot A \cdot \Delta x) \cdot u$$

For an incompressible substance the internal energy is $du = C dT$ and we can write

$$\frac{\partial E}{\partial t} = \rho C A \Delta x \frac{\partial T}{\partial t}$$

We will assume that any electrical resistance heat addition to the body is denoted at $-\dot{W}$. Therefore

$$-\dot{W} = \dot{g} \cdot V = \dot{g} \cdot A \cdot \Delta x \quad \Leftarrow \quad \dot{g} = \text{volumetric heat addition, } (\text{W}/\text{m}^3)$$

We can also write the heat flow along the x -direction as a product of the temperature difference (the driving force for heat transfer) and a proportionality constant, based on experimental observation.

$$\dot{Q}_x = \frac{kA}{\Delta x}(T_x - T_{x+\Delta x})$$

where k is the thermal conductivity of the material. In the limit as $\Delta x \rightarrow 0$

$$\dot{Q}_x = -kA \frac{\partial T}{\partial x}$$

This is *Fourier's law of heat conduction*.

$$\dot{Q}_{x+\Delta x} = \dot{Q} + \frac{\partial \dot{Q}_x}{\partial x} \Delta x$$

when combined with Fourier's equation gives

$$\dot{Q}_{x+\Delta x} = \underbrace{-kA \frac{\partial T}{\partial x}}_{\dot{Q}_x} - \frac{\partial}{\partial x} \left(kA \frac{\partial T}{\partial x} \right) \Delta x$$

Noting that

$$\dot{Q}_x - \dot{Q}_{x+\Delta x} = \frac{\partial E}{\partial t} + \dot{W} = \rho C A \Delta x \frac{\partial T}{\partial t} + A \Delta x \dot{g}$$

By removing the common factor of $A \Delta x$ we can then write the general 1-D conduction equation as

$$\underbrace{\frac{\partial}{\partial x} \left(k \frac{\partial T}{\partial x} \right)}_{\text{longitudinal conduction}} + \underbrace{\dot{g}}_{\substack{\text{internal} \\ \text{heat} \\ \text{generation}}} = \underbrace{\rho C \frac{\partial T}{\partial t}}_{\substack{\text{thermal} \\ \text{inertia}}}$$

Thermal Resistance Networks

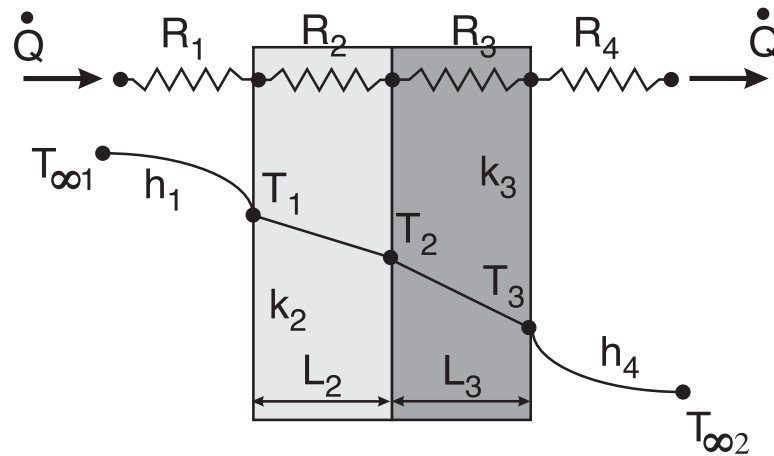
Resistances in Series

The heat transfer across the fluid/solid interface is based on *Newton's law of cooling*

$$\dot{Q} = hA(T_{in} - T_{out}) = \frac{T_{in} - T_{out}}{R_{conv}} \quad \text{where} \quad R_{conv} = \frac{1}{hA}$$

The heat flow through a solid material of conductivity, k is

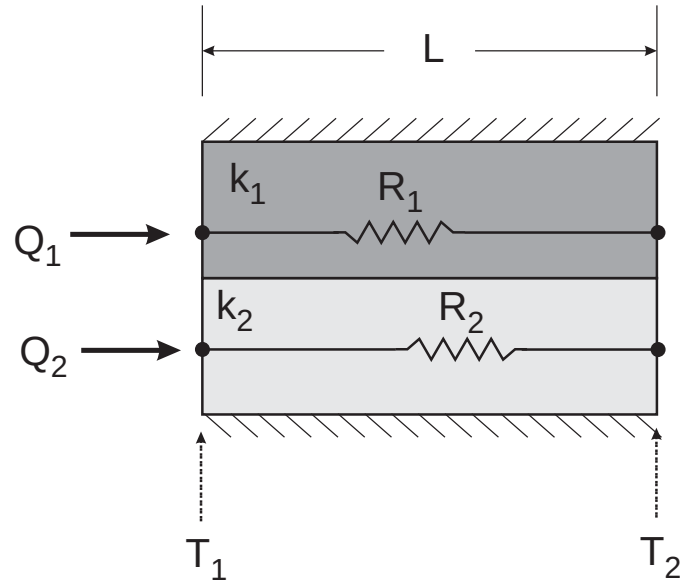
$$\dot{Q} = \frac{kA}{L}(T_{in} - T_{out}) = \frac{T_{in} - T_{out}}{R_{cond}} \quad \text{where} \quad R_{cond} = \frac{L}{kA}$$



The total heat flow across the system can be written as

$$\dot{Q} = \frac{T_{\infty_1} - T_{\infty_2}}{R_{total}} \quad \text{where} \quad R_{total} = \sum_{i=1}^4 R_i$$

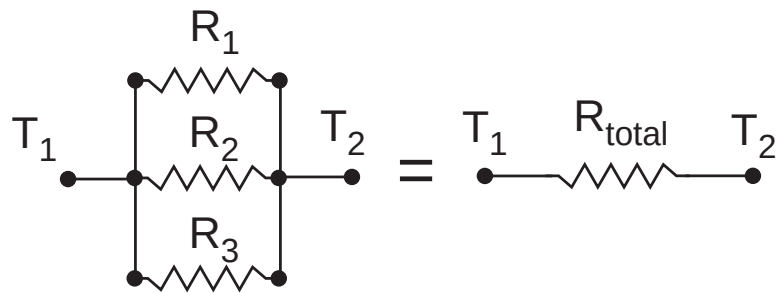
Resistances in Parallel



For systems of parallel flow paths as shown above, we can use the **1st** law to preserve the total energy

$$\dot{Q} = \dot{Q}_1 + \dot{Q}_2$$

In general, for parallel networks we can use a parallel resistor network as follows:



where

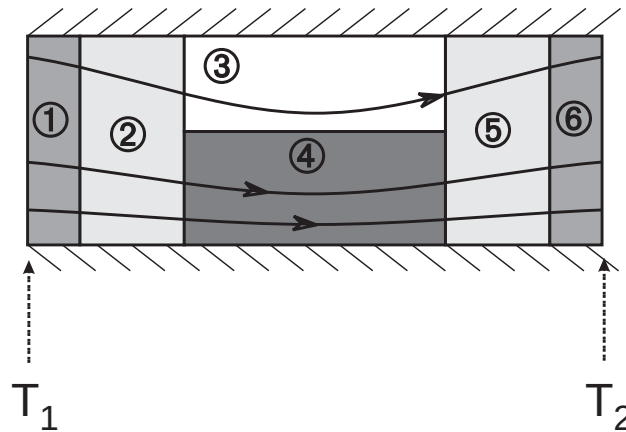
$$\frac{1}{R_{total}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \dots$$

and

$$\dot{Q} = \frac{T_1 - T_2}{R_{total}}$$

Flow in a Composite Wall

For a composite system where $k_3 \ll k_4$ the heat flow lines would look like

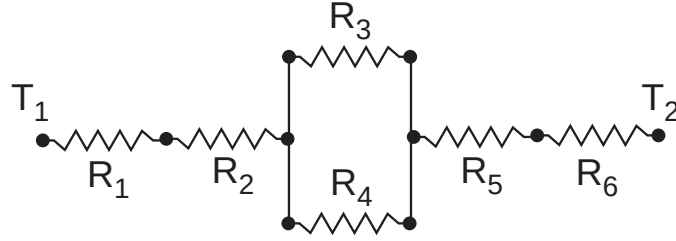


This problem cannot be easily solved using only a series network or a parallel network *but* we can use these two approaches as bounds on the problem and hopefully find an average of the two limits that will provide a reasonable compromise.

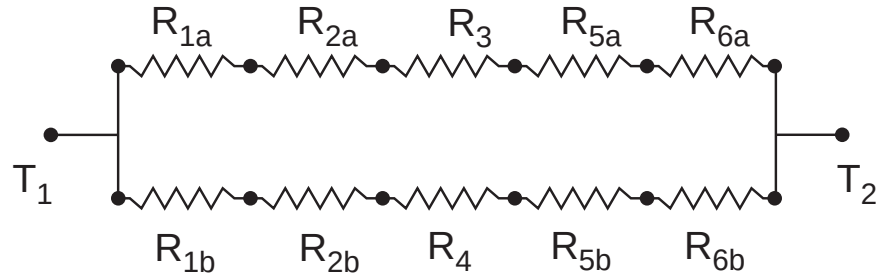
a) Upper bound on heat transfer: If we assume that heat flows in a 1-D manner within each section we can use a combined series/parallel resistor network as follows:

Since there is no turning in the heat flow path, the thermal resistance will be minimized and we will have an upper bound on the heat transfer.

b) Lower bound on heat transfer: The lower bound on heat transfer (upper bound on resistance) is achieved by forcing heat flow through an upper and a lower path as shown below.

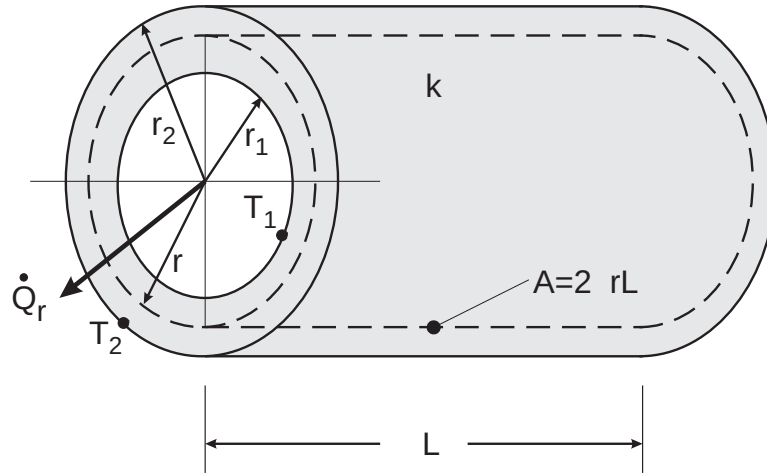


1a	2a	3		5a	6a
1b	2b	4		5b	6b



- equivalent to inserting a thin insulator along the dotted line
 - forcing heat flow through the upper and lower paths
 - prevents heat flow from finding a path of least resistance
 - leads to a lower bound on heat transfer
- since the solid resistance in each element is $\mathbf{R} = \frac{L}{kA}$ the area used in the upper and lower sections will be adjusted to reflect the cross sectional flow area of that section

Cylindrical Systems



Steady, 1D heat flow from T_1 to T_2 in a cylindrical systems occurs in a radial direction where the lines of constant temperature (isotherms) are concentric circles, as shown by the dotted line in the figure above and $T = T(r)$.

Performing a 1st law energy balance on a *control mass* from the annular ring of the cylindrical cylinder

$$\frac{dE_{C.M.}}{dt} = \dot{E}_{in} - \dot{E}_{out} + \dot{E}_{gen}$$

Therefore

$$\dot{Q}_r - \left[\dot{Q}_r + \left(\frac{d\dot{Q}_r}{dr} \right) dr \right] = 0 \quad \Rightarrow \quad \frac{d\dot{Q}_r}{dr} = 0$$

Integrating this result we obtain

$$\dot{Q}_r = \text{constant}$$

Using Fourier's law of heat conduction

$$\dot{Q}_r = -k A \frac{dT}{dr} = \text{constant}$$

where the cross sectional area for heat flow is given as the product of the circumference of the circular ring and the length of the cylinder

$$A = 2\pi r\mathcal{L}$$

Separating the above equation gives

$$dT = -\frac{\dot{Q}}{2\pi k\mathcal{L}} \frac{dr}{r}$$

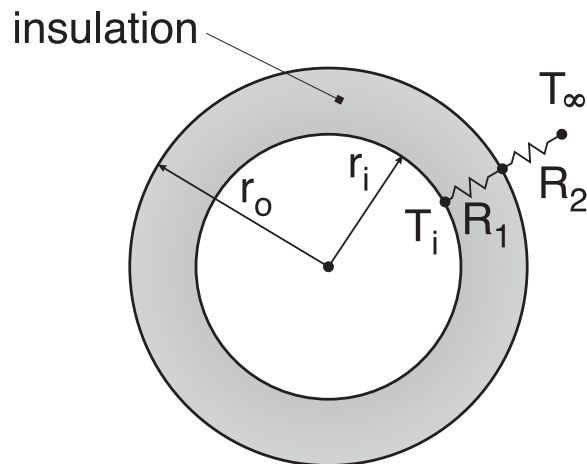
Integrating from $T = T_1$ @ $r = r_1$ to $T = T_2$ @ $r = r_2$

$$T_2 - T_1 = -\frac{\dot{Q}_r}{2\pi k\mathcal{L}}(\ln r_2 - \ln r_1) = -\frac{\dot{Q}_r}{2\pi k\mathcal{L}} \ln \frac{r_2}{r_1}$$

Therefore we can write

$$\dot{Q}_r = \frac{T_1 - T_2}{\left(\frac{\ln(r_2/r_1)}{2\pi k\mathcal{L}}\right)} \quad \text{where} \quad R = \left(\frac{\ln(r_2/r_1)}{2\pi k\mathcal{L}}\right)$$

Critical Thickness of Insulation



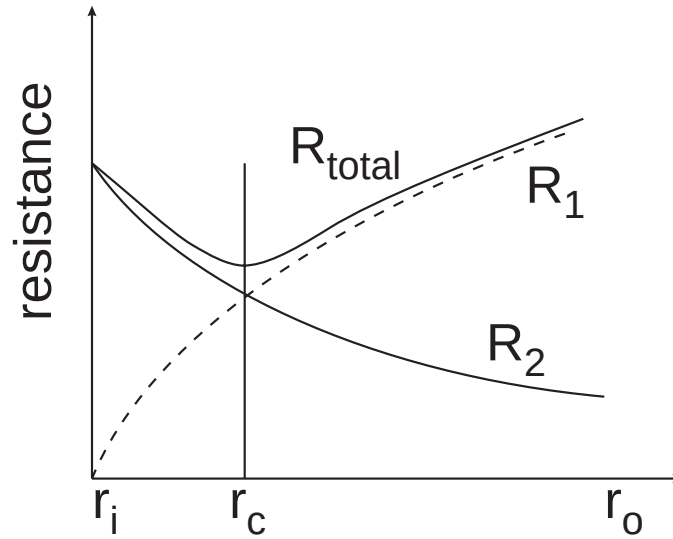
Could there be a situation in which adding insulation increases the overall heat transfer?

To find the critical radius, r_c , where adding more insulation begins to decrease heat transfer, set

$$\frac{dR_{total}}{dr_o} = 0$$

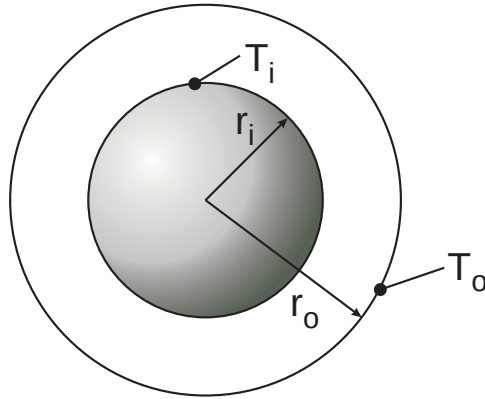
$$\frac{dR_{total}}{dr_o} = \frac{1}{2\pi k r_o \mathcal{L}} - \frac{1}{h 2\pi r_o^2 \mathcal{L}} = 0$$

$$r_c = \frac{k}{h}$$



There is always a value of r_c , but there is a minimum in heat transfer only if $r_c > r_i$

Spherical Systems



For steady, 1D heat flow in spherical geometries we can use a **1st** law energy balance similar to the analysis for cylindrical systems to obtain

$$\dot{Q} = \left(-k \frac{dT}{dr} \right) \underbrace{4\pi r^2}_A = \text{constant}$$

Then integrating from the inner to the outer radius

$$\begin{aligned} \int_{T_i}^{T_o} dT &= - \int_{r_i}^{r_o} \frac{\dot{Q}}{4\pi r^2 k} dr \\ T_o - T_i &= \frac{\dot{Q}}{4\pi k} \left[\frac{1}{r_o} - \frac{1}{r_i} \right] \end{aligned}$$

We can write the heat transfer as

$$\dot{Q} = \frac{4\pi k r_i r_o}{(r_o - r_i)} (T_i - T_o) = \frac{(T_i - T_o)}{R} \quad \text{where} \quad R = \frac{r_o - r_i}{4\pi k r_i r_o}$$

Heat Generation in a Solid

Heat can be generated within a solid as a result of:

- resistance heating in wires
- chemical reactions
- nuclear reactions

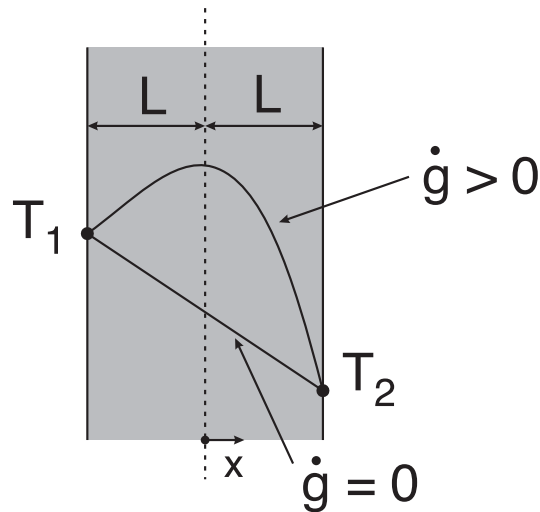
A volumetric heat generation terms will be defined as follows:

$$\dot{g} = \frac{\dot{E}_g}{V} \quad (W/m^3)$$

for heat generation in wires, we will define $\dot{g}$ as

$$\dot{g} = \frac{I^2 R_e}{\pi r_o^2 \mathcal{L}}$$

Slab System



Assume steady, 1D conduction with constant $\dot{g}$ in the slab and constant k .

We can write Fourier's law of conduction for a steady system as

$$k \frac{d^2 T}{dx^2} + \dot{g} = 0$$

with boundary conditions

$$T = T_1 \quad @ \quad x = -L$$

$$T = T_2 \quad @ \quad x = +L$$

Integrating Fourier's equation twice gives

$$T = -\frac{\dot{q}x^2}{2k} + C_1x + C_2$$

From the boundary conditions we can find

$$C_2 = \frac{(T_1 - T_2)}{2} + \frac{\dot{q}L^2}{2k}$$

$$C_1 = -\left(\frac{T_1 - T_2}{2L}\right)$$

Therefore

$$T = \frac{T_1 + T_2}{2} - \left(\frac{T_1 - T_2}{2}\right) \frac{x}{L} + \frac{\dot{q}L^2}{2k} \left(1 - \left(\frac{x}{L}\right)^2\right)$$

Case 1: if $T_1 = T_2 = T_3$

$$T = T_s + \frac{\dot{q}L^2}{2k} \left(1 - \left(\frac{x}{L}\right)^2\right)$$

This is a concave-down parabola. Why must the shape be of this form?

Case 2: If there is also heat loss from the outside edges to T_∞

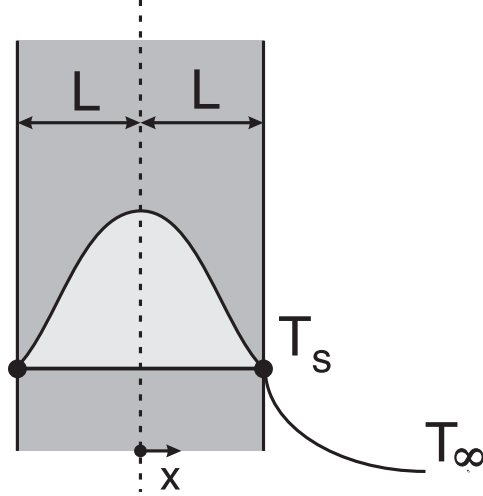
$$\dot{q} = -\left[k\frac{\partial T}{\partial x}\right]_{x=L^-} = -k\left\{\frac{\dot{q}L^2}{2k}\left(-\frac{2x}{L^2}\right)\right\}_{x=L^-} = +\dot{q}L$$

but Newton's law of cooling tells us

$$\dot{q} = h(T_s - T_\infty)$$

Therefore

$$T_s = T_\infty + \frac{\dot{q}L}{h}$$



Cylindrical System

The general conduction equation for a cylindrical system is given as

$$\frac{1}{r} \frac{d}{dr} \left(rk \frac{dT}{dr} \right) + \dot{q} = 0$$

Integrating once gives

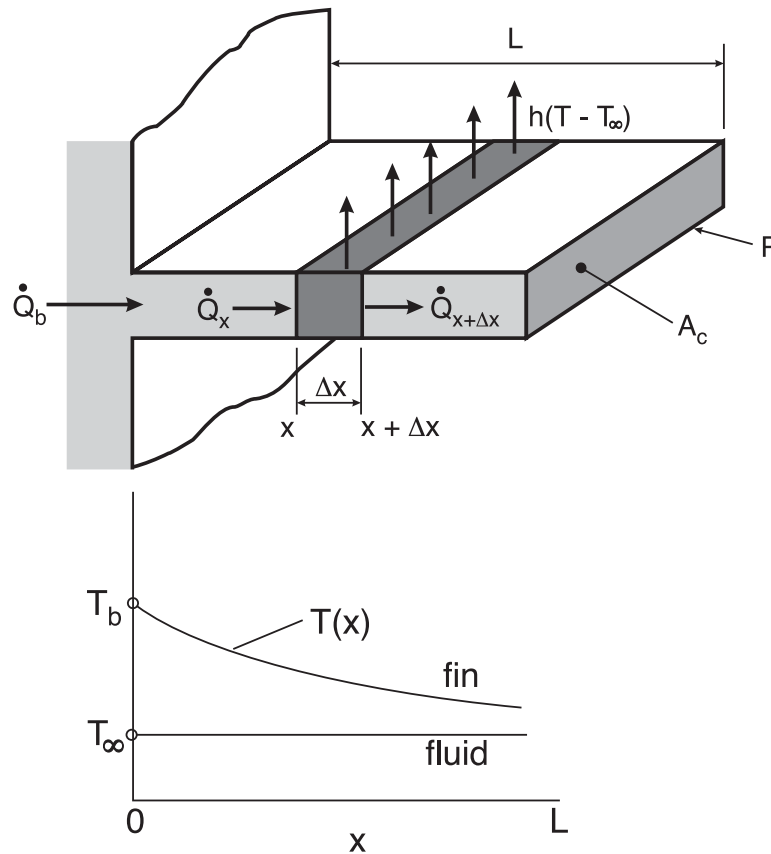
$$-2\pi r \mathcal{L} k \frac{dT}{dr} = (\pi r^2 \mathcal{L}) \dot{q}$$

While integrating a second time and noting the boundary condition $T = T_s$ @ $r = r_0$, gives

$$T = T_s + \frac{\dot{q} r_0^2}{4k} \left(1 - \left(\frac{r}{r_0} \right)^2 \right)$$

Heat Transfer from Finned Surfaces

- heat transfer is improved by $h \uparrow$ and $A \uparrow$
- it is difficult to obtain a significant increase in h
- an increase in A is easily achieved through extended surfaces or fins



We will assume a constant cross sectional area and that the temperature inside the fin is *only* a function of x .

Just as in the development for Fourier's law of conduction, we can establish a **1st** law balance over the thin slice of the fin between x and $x + \Delta x$ such that

$$\dot{Q}_x - \dot{Q}_{x+\Delta x} - P\Delta x h(T - T_\infty) = 0$$

From Fourier's law we know

$$\dot{Q}_x - \dot{Q}_{x+\Delta x} = kA_c \frac{d^2 T}{dx^2} \Delta x$$

Therefore the conduction equation for a fin with constant cross section is

$$\underbrace{kA_c \frac{d^2 T}{dx^2}}_{\text{longitudinal conduction}} - \underbrace{hP(T - T_\infty)}_{\text{lateral convection}} = 0$$

The temperature difference between the fin and the surroundings (temperature excess) is usually expressed as

$$\theta = T(x) - T_\infty$$

which allows the 1-D fin equation to be written as

$$\frac{d^2 \theta}{dx^2} - m^2 \theta = 0$$

where the fin parameter m is

$$m = \left(\frac{hP}{kA_c} \right)^{1/2}$$

and the boundary conditions are

$$\begin{aligned} \theta &= \theta_b \quad @ \quad x = 0 \\ \theta &\rightarrow 0 \quad \text{as} \quad x \rightarrow \infty \end{aligned}$$

The solution to the differential equation for θ is

$$\theta(x) = C_1 \sinh(mx) + C_2 \cosh(mx)$$

substituting the boundary conditions to find the constants of integration

$$\theta = \theta_b \frac{\cosh[m(L - x)]}{\cosh(mL)}$$

The heat transfer flowing through the base of the fin can be determined as

$$\begin{aligned} Q_b &= A_c \left(-k \frac{dT}{dx} \right)_{@x=0} \\ &= \theta_b (kA_c hP)^{1/2} \tanh(mL) \end{aligned}$$

Fin Efficiency and Effectiveness

The dimensionless parameter that compares the actual heat transfer from the fin to the ideal heat transfer from the fin is the *fin efficiency*

$$\eta = \frac{\text{actual heat transfer rate}}{\text{maximum heat transfer rate when the entire fin is at } T_b} = \frac{\dot{Q}_b}{hPL_c\theta_b}$$

If the fin has a constant cross section then

$$\eta = \frac{\tanh(mL)}{mL}$$

An alternative figure of merit is the *fin effectiveness* given as

$$\epsilon_{fin} = \frac{\text{total fin heat transfer}}{\text{the heat transfer that would have occurred through the base area in the absence of the fin}} = \frac{\dot{Q}_b}{hA_c\theta_b}$$

Transient Heat Conduction

Various applications of transient heat conduction include:

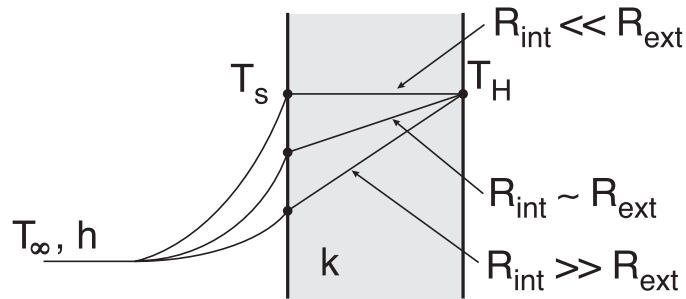
- surface hardness control
- stress relieving
- processes requiring chemical reaction (i.e. curing of rubber)
- wire drawing and coating
- welding

Performing a 1st law energy balance on a plane wall gives

$$\dot{E}_{in} - \dot{E}_{out} \Rightarrow \dot{Q}_{cond} = \frac{T_H - T_s}{L/(k \cdot A)} = \dot{Q}_{conv} = \frac{T_s - T_\infty}{1/(h \cdot A)}$$

where

$$\begin{aligned} \frac{T_H - T_s}{T_s - T_\infty} &= \frac{L/(k \cdot A)}{1/(h \cdot A)} = \frac{\text{internal resistance to H.T.}}{\text{external resistance to H.T.}} \\ &= \frac{hL}{k} = Bi \equiv \text{Biot number} \end{aligned}$$



$R_{int} \ll R_{ext}$: the Biot number is small and we can conclude

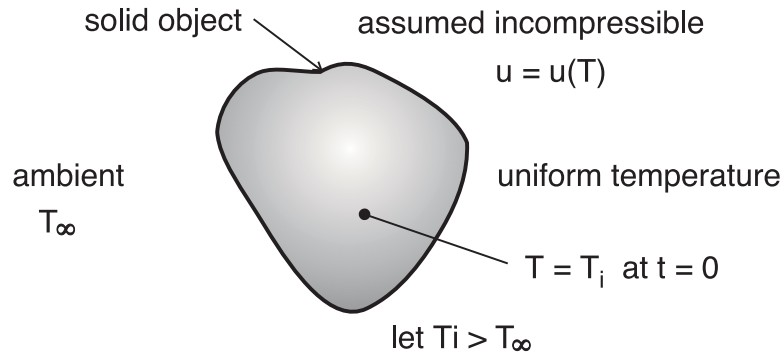
$$T_H - T_s \ll T_s - T_\infty \quad \text{and in the limit} \quad T_H \approx T_s$$

$$R_{ext} \ll R_{int}:$$

$R_{int} \ll R_{ext}$: the Biot number is large and we can conclude

$$T_s - T_\infty \ll T_H - T_s \quad \text{and in the limit} \quad T_s \approx T_\infty$$

Transient Conduction Analysis

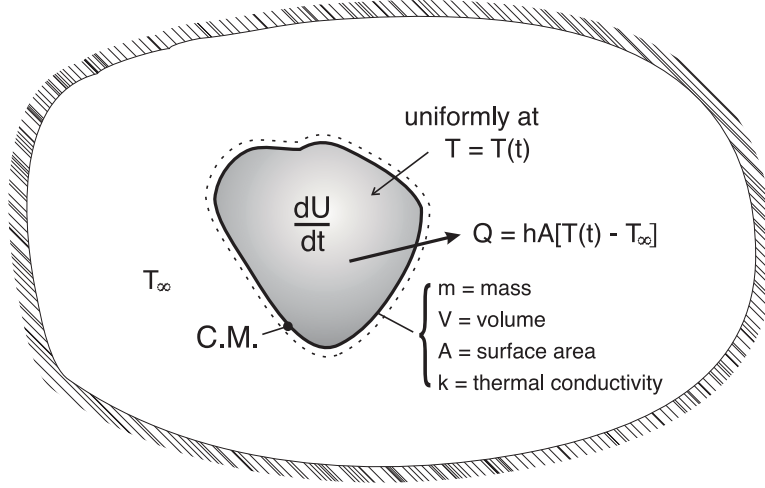


For the 3-D body of volume V and surface area A , we can use a lumped system analysis if

$$Bi = \frac{hV}{kA} < 0.1 \quad \Leftarrow \text{results in an error of less than } 5\%$$

The characteristic length for the 3-D object is given as $\mathcal{L} = V/A$. Other characteristic lengths for conventional bodies include:

Slab	$\frac{V}{A_s} = \frac{WH2L}{2WH} = L$
Rod	$\frac{V}{A_s} = \frac{\pi r_o^2 L}{2\pi r_o L} = \frac{r_o}{2}$
Sphere	$\frac{V}{A_s} = \frac{4/3\pi r_o^3}{4\pi r_o^2} = \frac{r_o}{3}$



At $t > 0$, $T = T(x, y, z, t)$, however, when $Bi < 0.1$ then we can assume $T \approx T(t)$.

Performing a 1st law energy balance on the control volume shown below

$$\frac{dE_{C.M.}}{dt} = \dot{E}_{in} - \dot{E}_{out} + \dot{E}_g^0$$

If we assume PE and KE to be negligible then

$$\frac{dU}{dt} = -\dot{Q} \quad \Leftrightarrow \quad \frac{dU}{dt} < 0 \text{ implies } U \text{ is decreasing}$$

For an incompressible substance specific heat is constant and we can write

$$\underbrace{mC}_{\equiv C_{th}} \frac{dT}{dt} = - \underbrace{Ah}_{1/R_{th}} (T - T_{\infty})$$

where

$$C_{th} = \text{lumped capacitance}$$

It should be clearly noted that we have neglected the spatial dependence of the temperature within the object. This type of an approach is only valid for $Bi = \frac{hV}{kA} < 0.1$

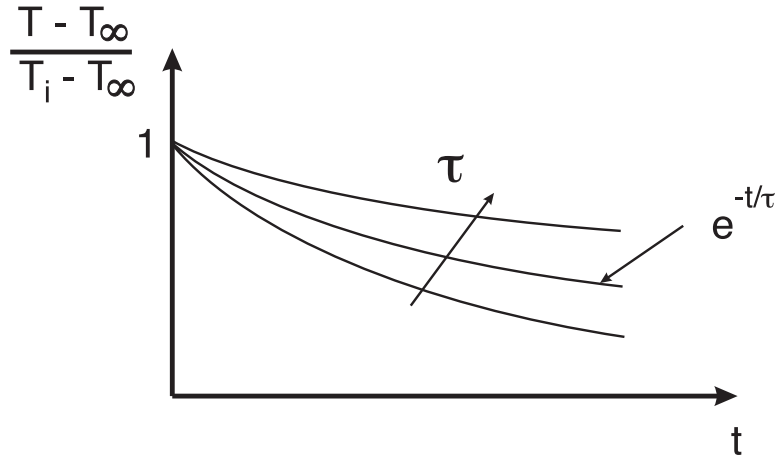
$$C_{th} \frac{dT}{dt} = -\frac{1}{R_{th}} (T - T_{\infty})$$

We can integrate and apply the initial condition, $T = T_i$ @ $t = 0$ to obtain

$$\frac{T(t) - T_{\infty}}{T_i - T_{\infty}} = e^{-t/(R_{th} \cdot C_{th})} = e^{-t/\tau}$$

where

$$\tau = R_{th} \cdot C_{th} = \text{thermal time constant} = \frac{mC}{Ah}$$



The total heat transfer rate can be determined by integrating $\dot{Q}$ with respect to time.

$$\begin{aligned} \dot{Q}_{total} &= \int_0^{t^*} \dot{Q} dt = \int_0^{t^*} hA(T - T_{\infty}) dt \\ &= ha(T_i - T_{\infty}) \int_0^{t^*} e^{-t/\tau} dt \\ &= hA(T_i - T_{\infty})(\tau)[1 - e^{-t^*/\tau}] \end{aligned}$$

Therefore

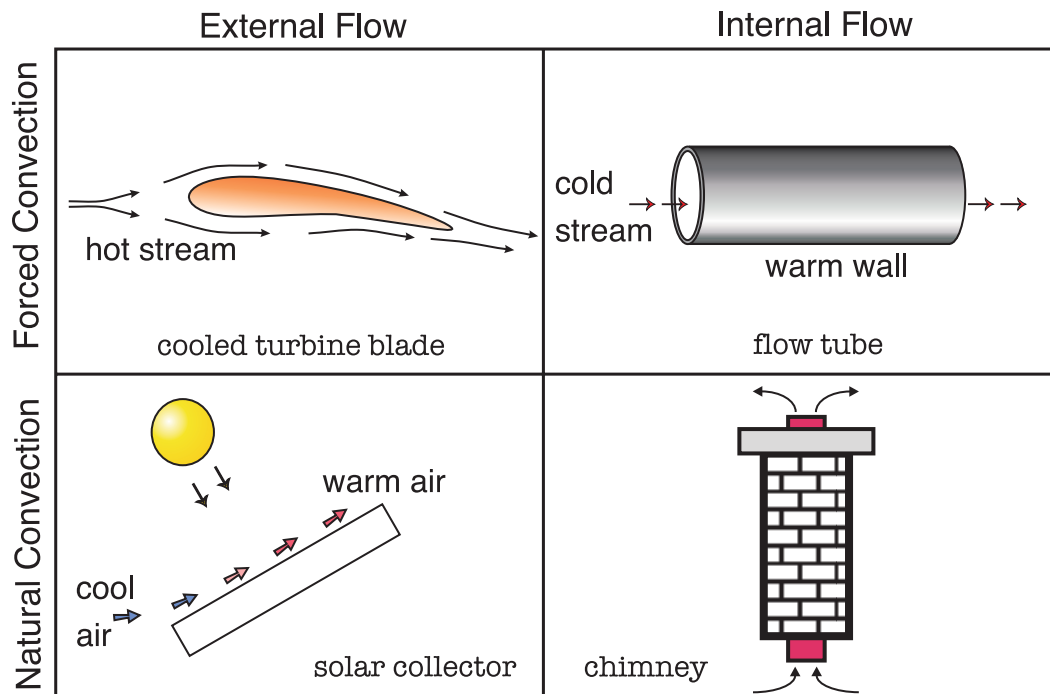
$$\dot{Q}_{total} = \dot{m}C(T_i - T_{\infty})[1 - e^{-t^*/\tau}]$$

Convection Heat Transfer

	Reading	Problems
	10-1 → 10-7 11-1 → 11-2, 11-4	10-22, 10-31, 10-33, 10-43, 10-49, 10-51, 10-55, 10-58, 10-83, 10-108, 11-18, 11-33, 11-55, 11-56

Introduction

- *convection heat transfer* is the transport mechanism made possible through the *motion of fluid*
- the types of flow arrangements and heat transfer configurations associated with convection are extremely diverse including forced, natural and mixed convection for both internal and external flow geometries
- in addition to these convection mechanisms, flow can be laminar or turbulent and fluids can be single or multi-phase as in the case of boiling



- the controlling equation for convection is *Newton's Law of Cooling*

$$\dot{Q}_{conv} = \frac{\Delta T}{R_{conv}} = hA(T_w - T_\infty) \quad \Rightarrow \quad R_{conv} = \frac{1}{hA}$$

where

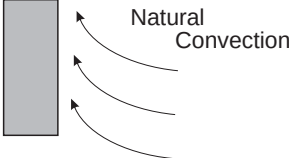
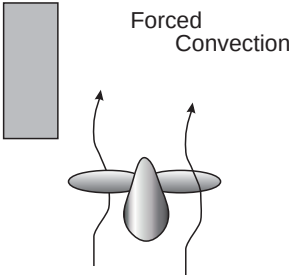
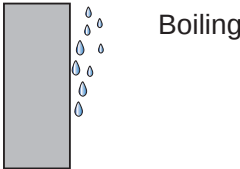
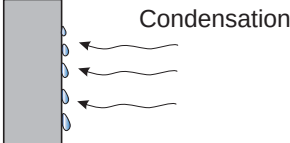
A = total convective area, m^2

h = heat transfer coefficient, $W/(m^2 \cdot K)$

T_w = surface temperature, $^{\circ}C$

T_{∞} = fluid temperature, $^{\circ}C$

The following table gives the range of heat transfer coefficient expected for different convection mechanisms and fluid types.

Process	h [$W/(m^2 \cdot K)$]
 Natural Convection	• gases 3 - 20 • water 60 - 900
 Forced Convection	• gases 30 - 300 • oils 60 - 1 800 • water 100 - 1 500
 Boiling	• water 3 000 - 100 000
 Condensation	• steam 3 000 - 100 000

Dimensionless Groups

In the study and analysis of convection processes it is common practice reduce the total number of functional variables by forming dimensionless groups consisting of relevant thermophysical properties, geometry, boundary and flow conditions.

Prandtl number: $Pr = \nu/\alpha$ where $0 < Pr < \infty$ ($Pr \rightarrow 0$ for liquid metals and $Pr \rightarrow \infty$ for viscous oils). A measure of ratio between the diffusion of momentum to the diffusion of heat.

Oils	$Pr \approx 10^3$
Water	$Pr \approx 5$
Air	$Pr \approx 0.7$
Liquid Metals	$Pr \approx 10^{-2}$

Reynolds number: $Re = \rho U \mathcal{L} / \mu \equiv U \mathcal{L} / \nu$ (forced convection). A measure of the balance between the inertial forces and the viscous forces.

Peclet number: $Pe = U \mathcal{L} / \alpha \equiv Re Pr$

Grashof number: $Gr = g \beta (T_w - T_f) \mathcal{L}^3 / \nu^2$ (natural convection)

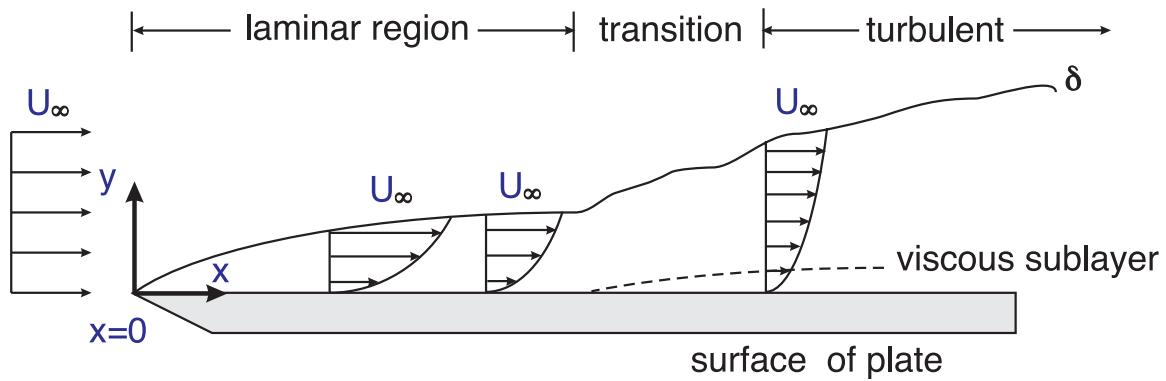
Rayleigh number: $Ra = g \beta (T_w - T_f) \mathcal{L}^3 / (\alpha \cdot \nu) \equiv Gr Pr$

Nusselt number: $Nu = h \mathcal{L} / k_f$ This can be considered as the dimensionless heat transfer coefficient.

Stanton number: $St = h / (U \rho C_p) \equiv Nu / (Re Pr)$

Forced Convection

The simplest forced convection configuration to consider is the flow of mass and heat near a flat plate as shown below.

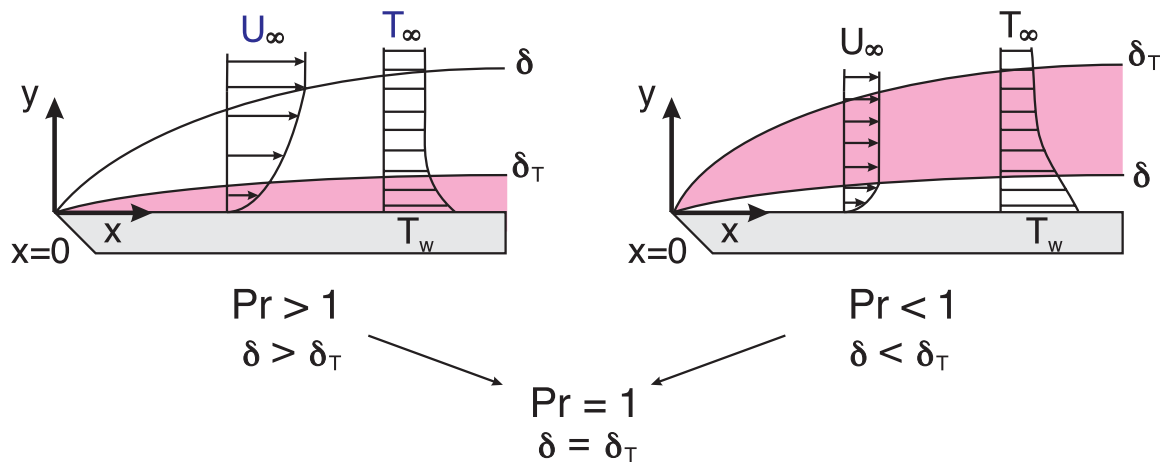


- as Reynolds number increases the flow has a tendency to become more chaotic resulting in disordered motion known as turbulent flow
 - transition from laminar to turbulent is called the critical Reynolds number, Re_{cr}

$$Re_{cr} = \frac{U_\infty x_{cr}}{\nu}$$

- for flow over a flat plate $Re_{cr} \approx 500,000$

Boundary Layers



Velocity Boundary Layer

- the region of fluid flow over the plate where viscous effects dominate is called the *velocity* or *hydrodynamic* boundary layer
- the velocity at the surface of the plate, $y = 0$, is set to zero, $U_{@y=0} = 0 \text{ m/s}$ because of the *no slip condition* at the wall
- the velocity of the fluid progressively increases away from the wall until we reach approximately $0.99 U_\infty$ which is denoted as the δ , the *velocity boundary layer thickness*. Note: 99% is an arbitrarily selected value.
- the region beyond the velocity boundary layer is denoted as the *inviscid flow* region, where frictional effects are negligible and the velocity remains relatively constant at U_∞

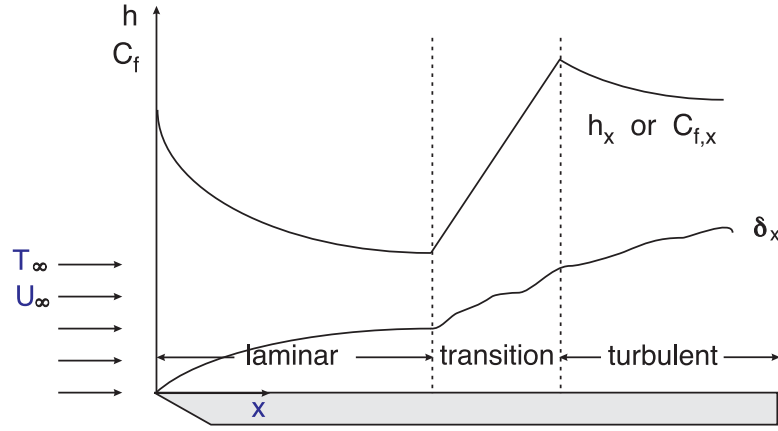
Thermal Boundary Layer

- the thermal boundary layer is arbitrarily selected as the locus of points where

$$\frac{T - T_w}{T_\infty - T_w} = 0.99$$

- for *low Prandtl number* fluids, i.e. liquid metals, momentum diffuses much slower than heat flow (remember $Pr = \nu/\alpha$) and the velocity boundary layer is fully contained within the thermal boundary layer
- conversely, for *high Prandtl number* fluids, i.e. oils, heat diffuses slower than the momentum and the thermal boundary layer is contained within the velocity boundary layer

Flow Over Plates



1. Laminar Boundary Layer Flow, Isothermal (UWT)

The local values of the skin friction and the Nusselt number are given as

$$C_{f,x} = \frac{0.664}{Re_x^{1/2}}$$

$$Nu_x = 0.332 Re_x^{1/2} Pr^{1/3} \Rightarrow \text{local, laminar, UWT, } Pr \geq 0.6$$

where x is the distance from the leading edge of the plate.

$$Nu_L = \frac{h_L L}{k_f} = 0.664 Re_L^{1/2} Pr^{1/3} \Rightarrow \text{average, laminar, UWT, } Pr \geq 0.6$$

For low Prandtl numbers, i.e. liquid metals

$$Nu_x = 0.565 Re_x^{1/2} Pr^{1/2} \Rightarrow \text{local, laminar, UWT, } Pr \leq 0.6$$

2. Turbulent Boundary Layer Flow, Isothermal (UWT)

The local skin friction is given as

$$\boxed{C_{f,x} = \frac{\tau_0}{(1/2)\rho U_\infty^2} = \frac{0.0592}{Re_x^{0.2}}} \Rightarrow \text{local, turbulent, UWT, } Pr \geq 0.6$$

As mentioned previously, we know that

$$Pr^{2/3} St = \frac{C_{f,x}}{2}$$

Therefore

$$Pr^{2/3} \frac{Nu_x}{Re_x Pr} = \frac{1}{2} \left(\frac{0.0592}{Re_x^{0.2}} \right)$$

and

$$\boxed{Nu_x = 0.0296 Re_x^{0.8} Pr^{1/3}} \Rightarrow \begin{array}{l} \text{local, turbulent, UWT,} \\ 0.6 < Pr < 100, \quad Re_x > 500,000 \end{array}$$

$$\boxed{Nu_L = 0.037 Re_L^{0.8} Pr^{1/3}} \Rightarrow \begin{array}{l} \text{average, turbulent, UWT,} \\ 0.6 < Pr < 100, \quad Re_x > 500,000 \end{array}$$

3. Combined Laminar and Turbulent Boundary Layer Flow, Isothermal (UWT)

When $(T_w - T_\infty)$ constant

$$h_L = \frac{1}{L} \int_0^L h dx = \frac{1}{L} \left\{ \int_0^{x_{cr}} h_x^{lam} dx + \int_0^{x_{cr}} h_x^{tur} dx \right\}$$

$$\boxed{Nu_L = \frac{h_L L}{k} = (0.037 Re_L^{0.8} - 871) Pr^{1/3}} \Rightarrow \begin{array}{l} \text{average, turbulent, UWT,} \\ 0.6 < Pr < 100, \quad Re_L > 500,000 \end{array}$$

4. Laminar Boundary Layer Flow, Isoflux (UWF)

$$\boxed{Nu_x = 0.453 Re_x^{1/2} Pr^{1/3}} \Rightarrow \text{local, laminar, UWF, } Pr \geq 0.6$$

5. Turbulent Boundary Layer Flow, Isoflux (UWF)

$$\boxed{Nu_x = 0.0308 Re_x^{4/5} Pr^{1/3}} \Rightarrow \text{local, turbulent, UWF, } Pr \geq 0.6$$

Flow Over Cylinders and Spheres

1. Boundary Layer Flow Over Circular Cylinders, Isothermal (UWT)

The Churchill-Berstein (1977) correlation for the average Nusselt number for long ($L/D > 100$) cylinders is

$$\boxed{Nu_D = S_D^* + f(Pr) Re_D^{1/2} \left[1 + \left(\frac{Re_D}{28200} \right)^{5/8} \right]^{4/5}} \Rightarrow \begin{array}{l} \text{average, UWT, } Re < 10^7 \\ 0 \leq Pr \leq \infty, Re \cdot Pr > 0.2 \end{array}$$

where S_D^* is the diffusive term associated with $Re_D \rightarrow 0$ and is given as

$$S_D^* = \frac{4}{\pi} \left(\frac{1 + 0.869(L/D)^{0.76}}{0.5 + L/D} \right), \quad 0 \leq \frac{L}{D} \leq 8$$

$$S_D^* = \frac{4}{\sqrt{\pi}} \left(\frac{1}{\sqrt{1 + 0.5D/L}} \right) \left(\frac{1}{\ln(2L/D)} \right), \quad 0 \leq \frac{L}{D} \leq 8$$

and the Prandtl number function is

$$f(Pr) = \frac{0.62 Pr^{1/3}}{[1 + (0.4/Pr)^{2/3}]^{1/4}}$$

All fluid properties are evaluated at $T_f = (T_w + T_\infty)/2$.

2. Boundary Layer Flow Over Non-Circular Cylinders, Isothermal (UWT)

The empirical formulations of Zhukauskas and Jakob given in Table 10-3 are commonly used, where

$$\boxed{Nu_D \approx \frac{\bar{h}D}{k} = C Re_D^m Pr^{1/3}} \Rightarrow \text{see Table 10-3 for conditions}$$

3. Boundary Layer Flow Over a Sphere, Isothermal (UWT)

For flow over an isothermal sphere of diameter D

$$\boxed{Nu_D = S_D^* + 0.4 Re_D^{1/2} + 0.06 Re_D^{2/3} Pr^{0.4} \left(\frac{\mu_\infty}{\mu_w} \right)^{1/4}} \Rightarrow \begin{array}{l} \text{average, UWT,} \\ 0.7 \leq Pr \leq 380 \\ 3.5 < Re_D < 80,000 \\ 1.0 < \mu_\infty/\mu_w < 3.2 \end{array}$$

where the diffusive term at $Re_D \rightarrow 0$ is

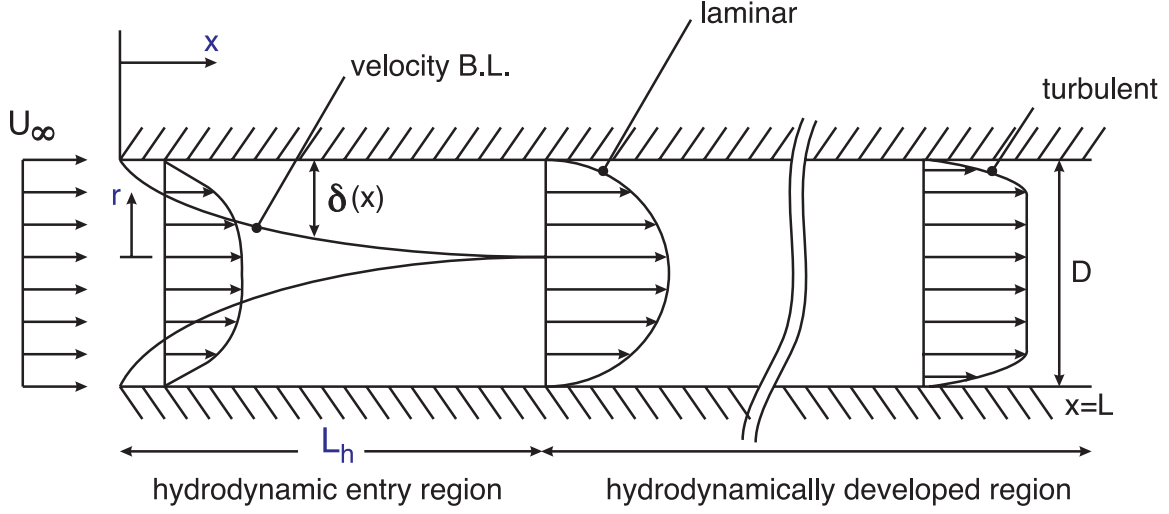
$$S_D^* = 2$$

and the dynamic viscosity of the fluid in the bulk flow, μ_∞ is based on T_∞ and the dynamic viscosity of the fluid at the surface, μ_w , is based on T_w . All other properties are based on T_f .

Internal Flow

Lets consider fluid flow in a duct bounded by a wall that is at a different temperature than the fluid. For simplicity we will examine a round tube of diameter D as shown below

The velocity profile across the tube changes from $U = 0$ at the wall to a maximum value along the center line. The average velocity, obtained by integrating this velocity profile, is



called the *mean velocity* and is given as

$$U_m = \frac{1}{A_c} \int_{A_c} u \, dA = \frac{\dot{m}}{\rho_m A_c}$$

where the area of the tube is given as $A_c = \pi D^2/4$ and the fluid density, ρ_m is evaluated at T_m .

The Reynolds number is given as

$$Re_D = \frac{U_m D}{\nu}$$

For flow in a tube:

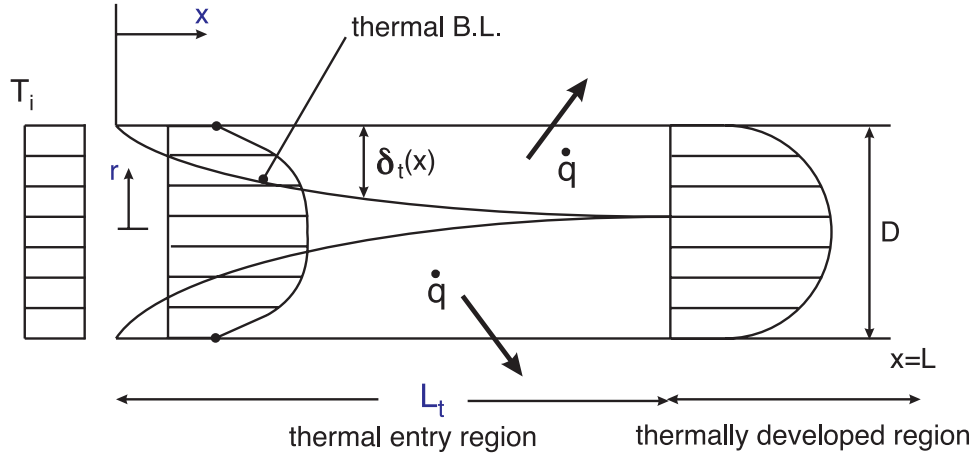
$Re_D < 2300$	laminar flow
$2300 < Re_D < 4000$	transition to turbulent flow
$Re_D > 4000$	turbulent flow

Hydrodynamic (Velocity) Boundary Layer

- the hydrodynamic boundary layer thickness can be approximated as

$$\delta(x) \approx 5x \left(\frac{U_m x}{\nu} \right)^{-1/2} = \frac{5x}{\sqrt{Re_x}}$$

Thermal Boundary Layer



- the thermal entry length can be approximated as

$$L_t \approx 0.05 Re_D Pr D \quad (\text{laminar flow})$$

- for turbulent flow $L_h \approx L_t \approx 10D$

1. Laminar Flow in Circular Tubes, Isothermal (UWT) and Isoflux (UWF)

For laminar flow where $Re_D \leq 2300$

$$\boxed{Nu_D = 3.66} \Rightarrow \text{fully developed, laminar, UWT, } L > L_t \text{ \& } L_h$$

$$\boxed{Nu_D = 4.36} \Rightarrow \text{fully developed, laminar, UWF, } L > L_t \text{ \& } L_h$$

$$\boxed{Nu_D = 1.86 \left(\frac{Re_D Pr D}{L} \right)^{1/3} \left(\frac{\mu_b}{\mu_w} \right)^{0.4}} \Rightarrow \begin{array}{l} \text{developing laminar flow, UWT,} \\ Pr > 0.5 \\ L < L_h \text{ or } L < L_t \end{array}$$

For non-circular tubes the hydraulic diameter, $D_h = 4A_c/P$ can be used in conjunction

with Table 10-4 to determine the Reynolds number and in turn the Nusselt number.

In all cases the fluid properties are evaluated at the mean fluid temperature given as

$$T_{mean} = \frac{1}{2} (T_{m,in} + T_{m,out})$$

except for μ_w which is evaluated at the wall temperature, T_w .

2. Turbulent Flow in Circular Tubes, Isothermal (UWT) and Isoflux (UWF)

For turbulent flow where $Re_D \geq 2300$ the Dittus-Bouler equation (Eq. 10-76) can be used

$$\begin{aligned} & \text{turbulent flow, UWT or UWF,} \\ & \mathbf{0.7 \leq Pr \leq 160} \\ & \mathbf{Re_D > 2,300} \\ & \mathbf{n = 0.4} \text{ heating} \\ \boxed{Nu_D = 0.023 Re_D^{0.8} Pr^n} \Rightarrow & \mathbf{n = 0.3} \text{ cooling} \end{aligned}$$

For non-circular tubes, again we can use the hydraulic diameter, $D_h = 4A_c/P$ to determine both the Reynolds and the Nusselt numbers.

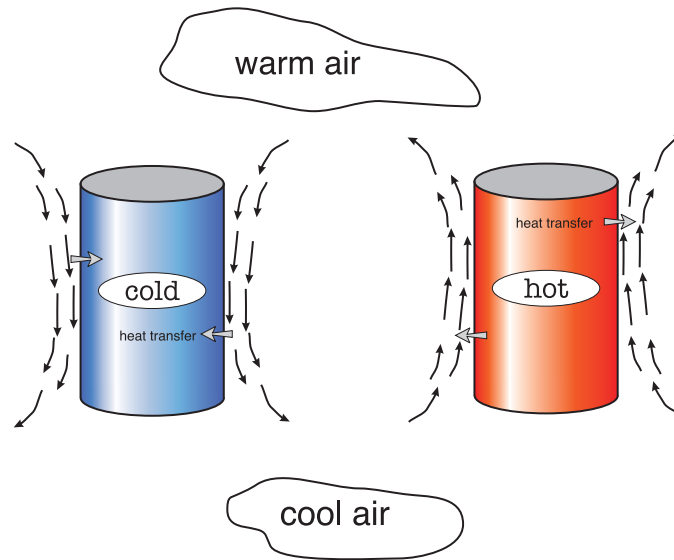
In all cases the fluid properties are evaluated at the mean fluid temperature given as

$$T_{mean} = \frac{1}{2} (T_{m,in} + T_{m,out})$$

Natural Convection

What Drives Natural Convection?

- fluid flow is driven by the effects of buoyancy
- therefore a fluid layer adjacent to a surface will become lighter if heated and heavier if cooled by the surface



In natural convection, we do not have a Reynolds number but we have an analogous dimensionless group called the *Grashof number*

$$Gr = \frac{\text{buoyancy force}}{\text{viscous force}} = \frac{g\beta(T_w - T_\infty)\mathcal{L}^3}{\nu^2}$$

where

g = gravitational acceleration, m/s^2

β = volumetric expansion coefficient, $\beta \equiv 1/T$ (T is ambient temp. in K)

T_w = wall temperature, K

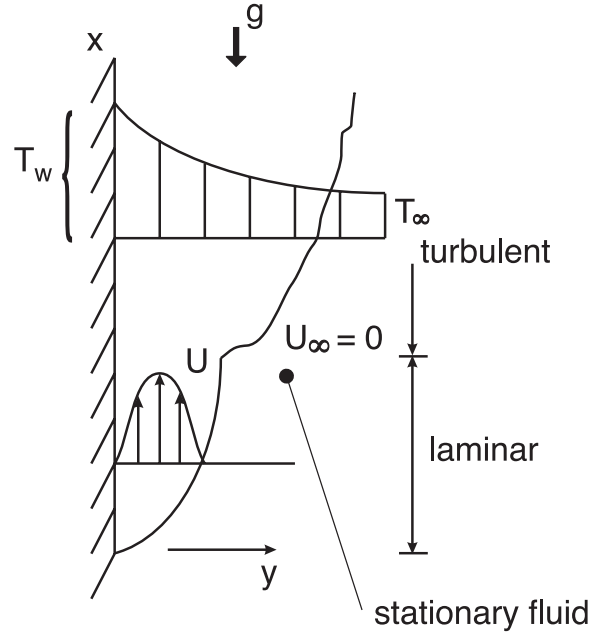
T_∞ = ambient temperature, K

$\mathcal{L}$ = characteristic length, m

ν = kinematic viscosity, m^2/s

Natural Convection Over Surfaces

- the velocity and temperature profiles within a boundary layer formed on a vertical plate in a stationary fluid looks as follows:



- note that unlike forced convection, the velocity at the edge of the boundary layer goes to zero

Natural Convection Heat Transfer Correlations

The general form of the Nusselt number for natural convection is as follows:

$$Nu = f(Gr, Pr) \equiv C Ra^m Pr^n \quad \text{where } Ra = Gr \cdot Pr$$

1. Laminar Flow Over a Vertical Plate, Isothermal (UWT)

The general form of the Nusselt number is given as

$$Nu_{\mathcal{L}} = \frac{h\mathcal{L}}{k_f} = C \left(\underbrace{\frac{g\beta(T_w - T_\infty)\mathcal{L}^3}{\nu^2}}_{\equiv Gr} \right)^{1/4} \left(\underbrace{\frac{\nu}{\alpha}}_{\equiv Pr} \right)^{1/4} = C \underbrace{Gr_{\mathcal{L}}^{1/4} Pr^{1/4}}_{Ra^{1/4}}$$

where

$$Ra_{\mathcal{L}} = Gr_{\mathcal{L}} Pr = \frac{g\beta(T_w - T_{\infty})\mathcal{L}^3}{\alpha\nu}$$

for gases $\beta = 1/T_{\infty}$, $(1/K)$.

The correlation equation for UWT with a vertical plate and laminar flow is

$$\boxed{Nu_{\mathcal{L}} = \frac{0.67 Ra_{\mathcal{L}}^{1/4}}{[1 + (0.5/Pr)^{9/16}]^{4/9}}} \Rightarrow \begin{array}{l} \text{average, UWT,} \\ 0 \leq Pr \leq \infty \\ 10^4 < Gr_{\mathcal{L}} < 10^8 \end{array}$$

2. Laminar Flow Over a Long Horizontal Circular Cylinder, Isothermal (UWT)

The general boundary layer correlation is

$$Nu_D = \frac{hD}{k_f} = C \left(\underbrace{\frac{g\beta(T_w - T_{\infty})D^3}{\nu^2}}_{\equiv Gr} \right)^{1/4} \left(\underbrace{\frac{\nu}{\alpha}}_{\equiv Pr} \right)^{1/4} = C \underbrace{Gr_D^{1/4} Pr^{1/4}}_{Ra_D^{1/4}}$$

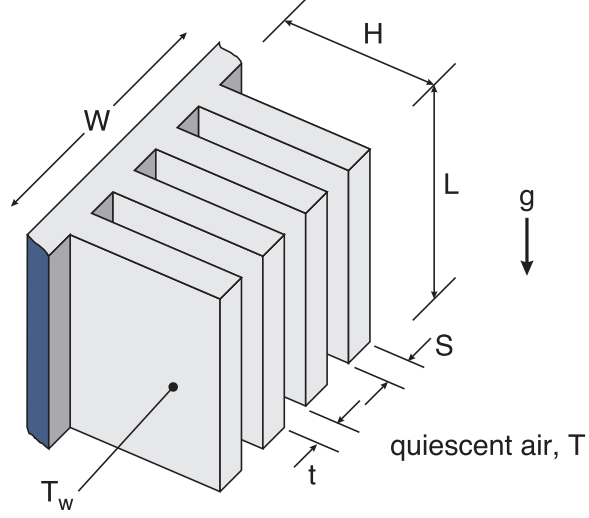
where

$$Ra_D = Gr_D Pr = \frac{g\beta(T_w - T_{\infty})\mathcal{L}^3}{\alpha\nu}$$

Natural Convection From Plate Fin Heat Sinks

Plate fin heat sinks are often used in natural convection to increase the heat transfer surface area and in turn reduce the boundary layer resistance

$$R \downarrow = \frac{1}{hA \uparrow}$$



A basic optimization of the fin spacing can be obtained as follows:

$$\dot{Q} = hA(T_w - T_\infty)$$

where the fins are assumed to be isothermal and the surface area is $2nHL$, with the area of the fin edges ignored.

For isothermal fins with $t < S$

$$S_{opt} = 2.714 \left(\frac{L}{Ra^{1/4}} \right)$$

with

$$Ra = \frac{g\beta(T_w - T_\infty)L^3}{\nu^2}$$

The corresponding value of the heat transfer coefficient is

$$h = 1.31k/S_{opt}$$

All fluid properties are evaluated at the film temperature.

Radiation Heat Transfer



Reading

12-1 → 12-8

Problems

12-19, 12-29, 12-33, 12-47, 12-48, 12-58, 12-67,
12-76, 12-79, 12-83, 12-105

Introduction

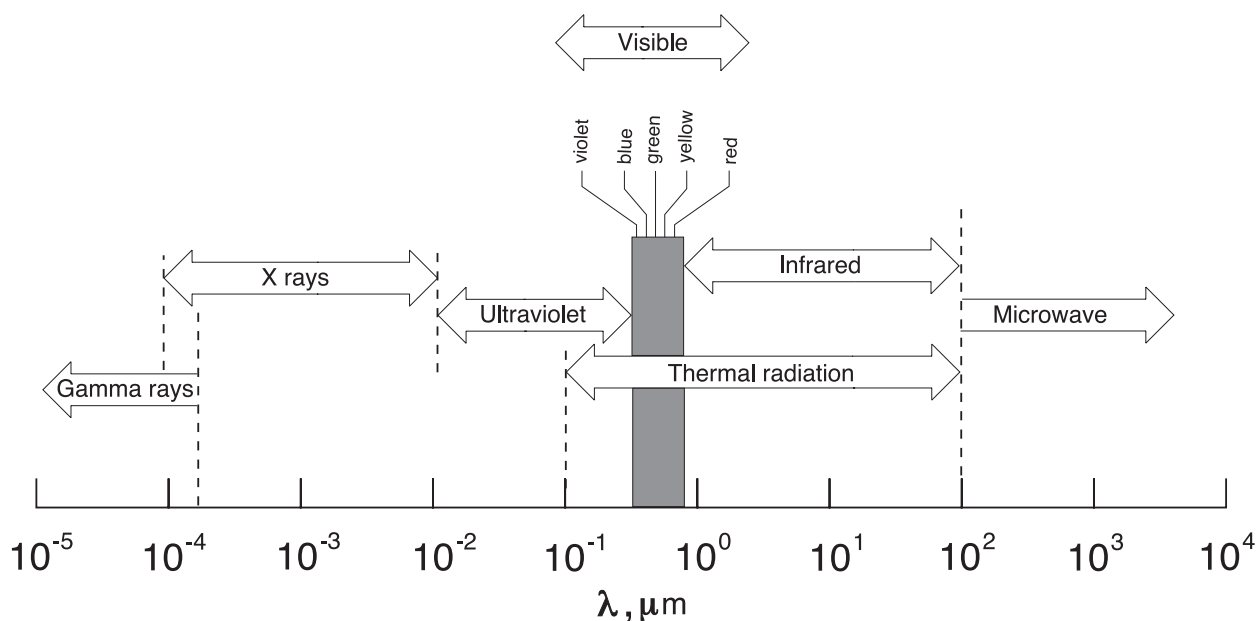
Radiation is a photon emission that occurs when electrons change orbit. *Thermal radiation* occurs when the excitation is caused by heating.

How does the sun's energy travel to earth through a nearly perfect vacuum? Both conduction and convection required particle motion to produce heat transfer. The answer is *radiation*.

It should be readily apparent that radiation heat transfer calculations required several additional considerations in addition to those of conduction and convection, i.e.

- optical aspects: the manner in which an emitting body “sees” its neighbors
- surface conditions

The following figure shows the relatively narrow band occupied by thermal radiation.



Blackbody Radiation

A blackbody is an ideal radiator that

- *absorbs* all incident radiation regardless of wavelength and direction
- at a given temperature and wavelength, no surface can emit more energy than a blackbody
- *emitted* radiation is a function of wavelength and temperature but is independent of direction, i.e. a black body is a diffuse emitter (independent of direction)

Definitions

1. **Blackbody emissive power:** the radiation emitted by a blackbody per unit time and per unit surface area

$$E_b = \sigma T^4 \quad [W/m^2] \quad \Leftarrow \text{Stefan-Boltzmann law}$$

where

$$\sigma = \text{Stefan-Boltzmann constant} = 5.67 \times 10^{-8} W/(m^2 \cdot K^4)$$

and the temperature T is given in K .

2. **Spectral blackbody emissive power:** the amount of radiation energy emitted by a blackbody per unit surface area and per unit wavelength about the wavelength λ . The following relationship between emissive power, temperature and wavelength is known as *Planck's distribution law*

$$E_{b,\lambda} = \frac{C_1}{\lambda^5 [\exp(C_2/\lambda T) - 1]} \quad [W/(m^2 \cdot \mu m)]$$

where

$$C_1 = 2\pi h C_0^2 = 3.743 \times 10^8 [W \cdot \mu m^4/m^2]$$

$$C_2 = h C_0 / K = 1.439 \times 10^4 [\mu \cdot K]$$

$$K = \text{Boltzmann constant} \equiv 1.3805 \times 10^{-23} [J/K]$$

$$h = \text{Plank's constant} \equiv 6.63 \times 10^{-34} [J \cdot s]$$

$$E_{b,\lambda} = \text{energy of radiation in the wavelength band } d\lambda \text{ per unit area and time}$$

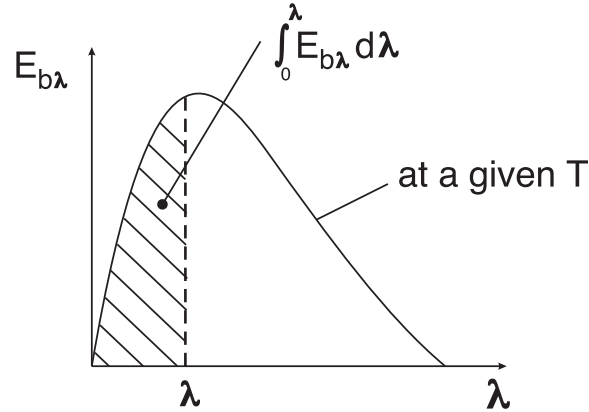
If we integrate the spectral emissive power between $d\lambda = \lambda_2 - \lambda_1$ we will obtain the blackbody emissive power given as $E_b(T) = \sigma T^4$.

The wavelength at which the peak emissive power occurs for a given temperature can be obtained from *Wien's displacement law*

$$(\lambda T)_{max\ power} = 2897.8\ \mu m \cdot K$$

3. **Blackbody radiation function:** the fraction of radiation emitted from a blackbody at temperature, T in the wavelength band $\lambda = 0 \rightarrow \lambda$

$$f_{0 \rightarrow \lambda} = \frac{\int_0^\lambda E_{b,\lambda}(T) d\lambda}{\int_0^\infty E_{b,\lambda}(T) d\lambda} = \frac{\int_0^\lambda \frac{C_1}{\lambda^5 [\exp(C_2/\lambda T) - 1]} d\lambda}{\sigma T^4}$$



$$f_{0 \rightarrow \lambda} = \frac{\int_0^\lambda \frac{C_1 T^5 (1/T) dt}{t^5 [\exp(C_2/t) - 1]}}{\sigma T^4} = \frac{C_1}{\sigma} \int_0^{\lambda T} \frac{dt}{t^5 [\exp(C_2/t) - 1]} = f(\lambda T)$$

$f_{0 \rightarrow \lambda}$ is tabulated as a function λT in Table 12.2

We can easily find the fraction of radiation emitted by a blackbody at temperature T over a discrete wavelength band as

$$f_{\lambda_1 \rightarrow \lambda_2} = f(\lambda_2 T) - f(\lambda_1 T)$$

$$f_{\lambda \rightarrow \infty} = 1 - f_{0 \rightarrow \lambda}$$

Radiation Properties of Real Surfaces

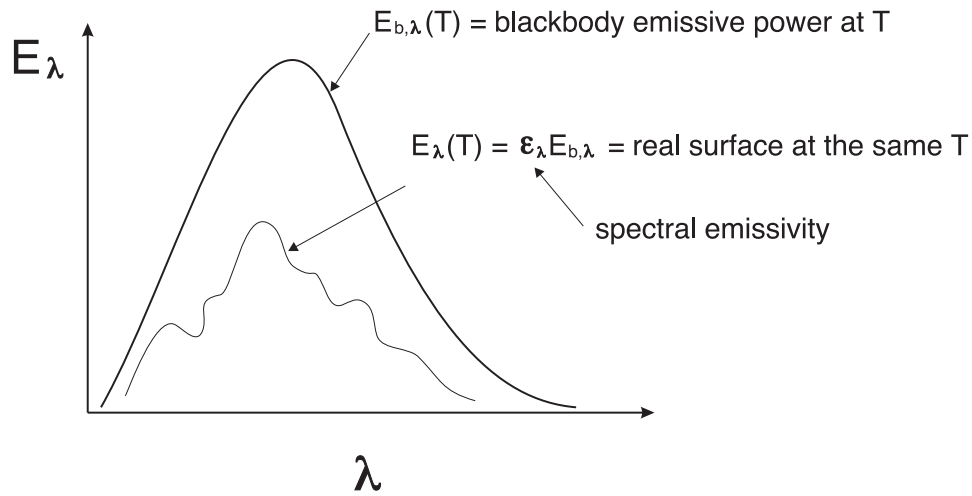
The thermal radiation emitted by a real surface is a function of surface temperature, T , wavelength, λ , direction and surface properties.

$$E_{\lambda} = f(T, \lambda, \text{direction, surface properties}) \Rightarrow \text{spectral emissive power}$$

while for a blackbody, the radiation was only a function of temperature and wavelength

$$E_{b,\lambda} = f(T, \lambda) \rightarrow \text{diffuse emitter} \Rightarrow \text{independent of direction}$$

The following figure gives a comparison of the emissive power profile for a real surface and a blackbody.



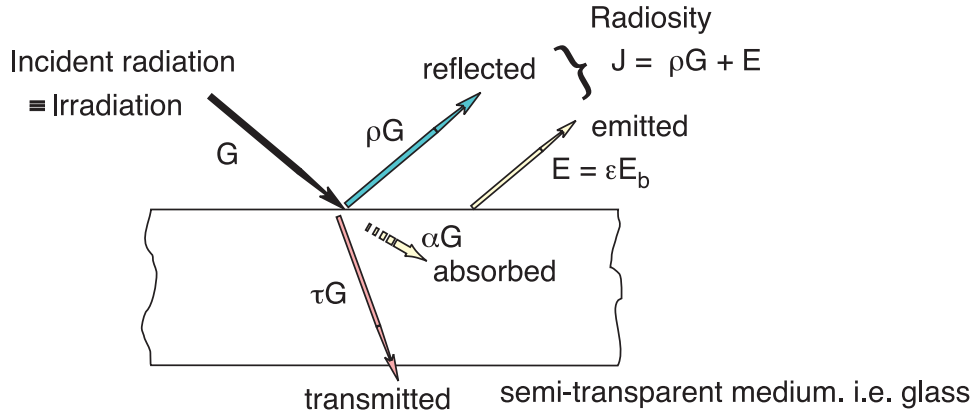
Definitions

1. **Emissivity:** defined as the ratio of radiation emitted by a surface to the radiation emitted by a blackbody at the same surface temperature.

$$\begin{aligned} \epsilon(T) &= \frac{\text{radiation emitted by surface at temperature } T}{\text{radiation emitted by a black surface at } T} \\ &= \frac{\int_0^{\infty} E_{\lambda}(T) d\lambda}{\int_0^{\infty} E_{b\lambda}(T) d\lambda} = \frac{\int_0^{\infty} \epsilon_{\lambda}(T) E_{b\lambda}(T) d\lambda}{E_b(T)} = \frac{E(T)}{\sigma T^4} \end{aligned}$$

where ϵ changes rather quickly with surface temperature.

2. **Diffuse surface:** properties are independent of direction.
3. **Gray surface:** properties are independent of wavelength.
4. **Irradiation, G :** the radiation energy incident on a surface per unit area and per unit time



An energy balance based on incident radiation gives

$$G = \rho G + \alpha G + \tau G$$

If we normalize with respect to the total irradiation

$$\alpha + \rho + \tau = 1$$

In general $\epsilon \neq \alpha$. However, for a diffuse-gray surface (properties are independent of wavelength and direction)

$$\epsilon = \alpha \quad \text{diffuse-gray surface}$$

These unsubscripted values of α , ρ and τ represent the average properties, i.e. due to incident radiation energy from all directions over a hemispherical space and including all wavelengths.

5. **Radiosity, J :** the total radiation energy leaving a surface per unit area and per unit time.

For a surface that is gray and opaque, i.e. $\epsilon = \alpha$ and $\alpha + \rho = 1$, the radiosity is given as

$$\begin{aligned} J &= \text{radiation emitted by the surface} + \text{radiation reflected by the surface} \\ &= \epsilon E_b + \rho G \\ &= \epsilon \sigma T^4 + \rho G \end{aligned}$$

Since $\rho = 0$ for a blackbody, the radiosity of a blackbody is

$$J = \sigma T^4$$

Diffuse-Gray Surfaces, $\epsilon = \alpha$

Kirchhoff's Law

The absorptivity, $\alpha(\lambda, T, \text{direction})$ of a non-black surface is always equal to the emissivity, $\epsilon(\lambda, T, \text{direction})$ of the same surface when the surface is in thermal equilibrium with the radiation that impinges on it.

$$\epsilon(\lambda, T, \phi, \theta) = \alpha(\lambda, T, \phi, \theta)$$

To a lesser degree of certainty we can write a more restrictive form of Kirchhoff's law for diffuse-gray surfaces where

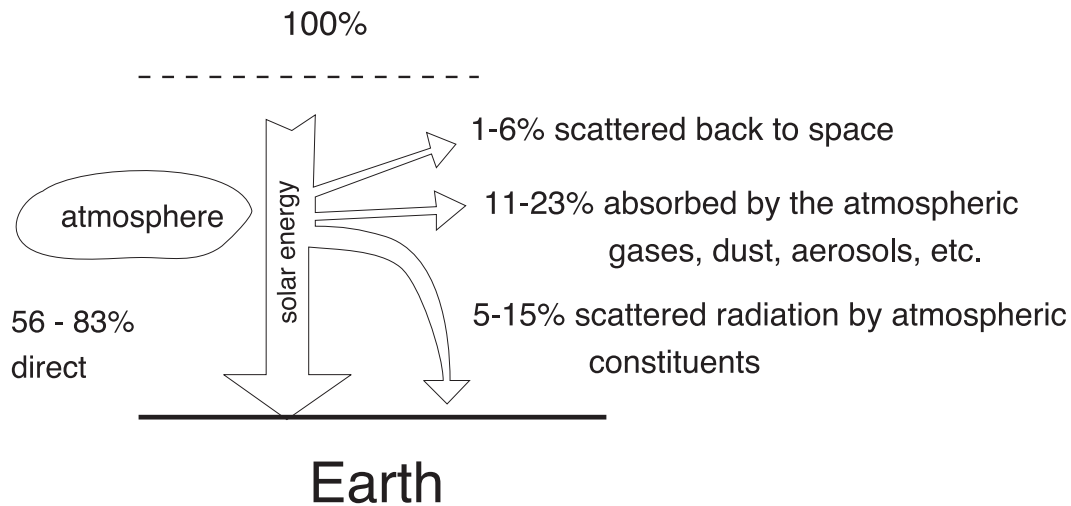
$$\epsilon(T) = \alpha(T)$$

While Kirchhoff's law requires that the radiant source and the surface be in thermal equilibrium, this is seldom the case. The law can still be used but you should proceed with caution when the two temperatures differ by more than **100 K**.

Solar Radiation

The incident radiation energy reaching the earth's atmosphere is known as the solar constant, G_s and has a value of

$$G_s = 1353 \text{ W/m}^2$$



View Factor (Shape Factor, Configuration Factor)

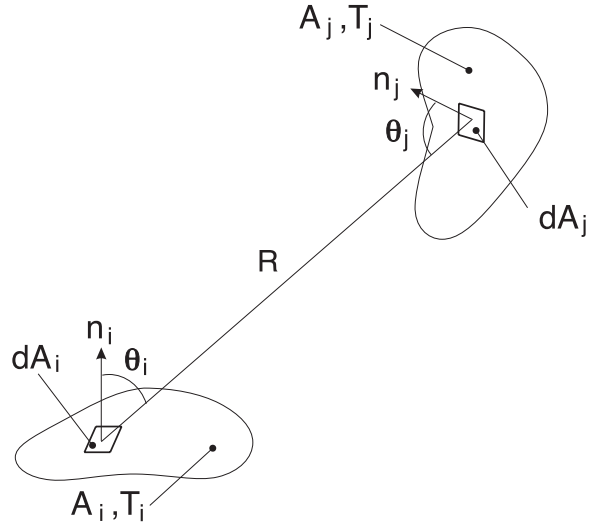
- the radiative exchange between surfaces clearly depends on how well the surfaces “see” one another. This information is provided by using shape factors (or view factors or configuration factors).
- Definition:** The view factor, $F_{i \rightarrow j}$ is defined as the fraction of radiation leaving surface i which is intercepted by surface j . Hence

$$F_{i \rightarrow j} = \frac{\dot{Q}_{i \rightarrow j}}{A_i J_i} = \frac{\text{radiation reaching } j}{\text{radiation reaching } i}$$

- consider 2 surfaces with arbitrary size and configuration with respect to one another, which are diffuse surfaces.

It can be shown that

$$F_{i \rightarrow j} = \frac{1}{A_i} \int_{A_i} \int_{A_j} \frac{\cos \theta_i \cos \theta_j}{\pi R^2} dA_i dA_j$$



This is purely a geometrical property.

It is also found that

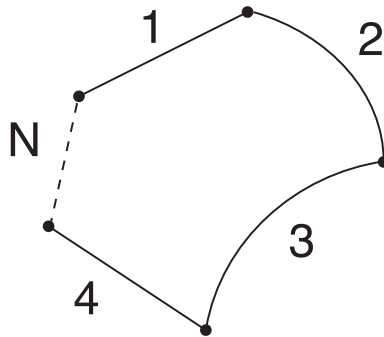
$$F_{j \rightarrow i} = \frac{1}{A_j} \int_{A_i} \int_{A_j} \frac{\cos \theta_i \cos \theta_j}{\pi R^2} dA_i dA_j$$

The last two equations show that

$$A_i F_{i \rightarrow j} = A_j F_{j \rightarrow i}$$

This is called the reciprocity relation.

- consider an enclosure with N surfaces



Since this is an enclosure, the energy leaving a given surface is intercepted by the remaining surfaces in proportion to how well they “see” that surface. For example:

$$A_1 J_1 = \dot{Q}_{1 \rightarrow 1} + \dot{Q}_{1 \rightarrow 2} + \dots + \dot{Q}_{1 \rightarrow N}$$

Therefore

$$1 = \sum_{j=1}^N \left(\frac{\dot{Q}_{i \rightarrow j}}{A_i J_i} \right) = \sum_{j=1}^N F_{i \rightarrow j}$$

Hence

$$\sum_{j=1}^N F_{i \rightarrow j} = 1 \quad ; \quad i = 1, 2, \dots, N$$

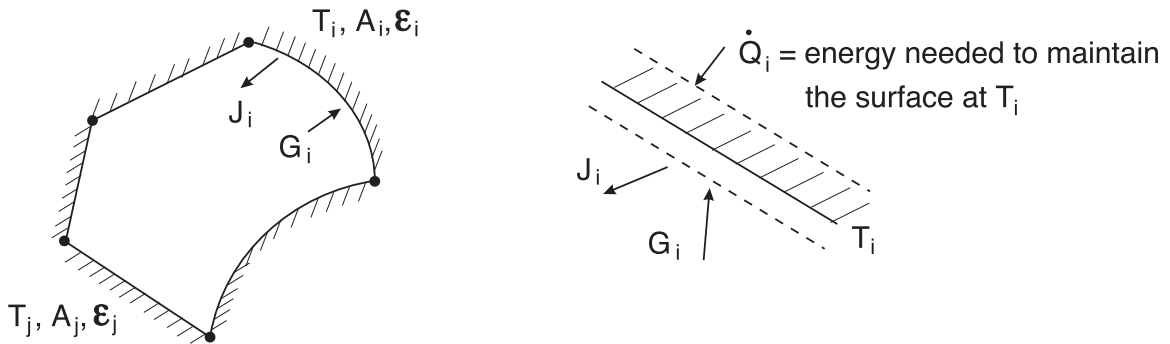
This is called the *summation rule*.

Note that $F_{i \rightarrow i} \neq 0$ for a concave surface. For a plane or convex surface $F_{i \rightarrow i} = 0$.

Radiation Exchange Between Diffuse-Gray Surfaces Forming an Enclosure

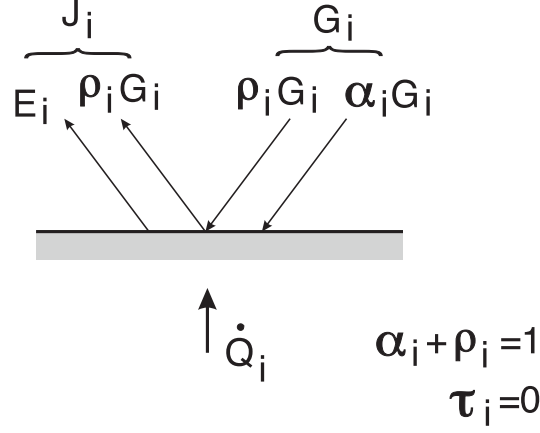
We will assume that:

1. each surface of the enclosure is isothermal
2. radiosity, J_i , and irradiation, G_i are uniform over each surface
3. the surfaces are opaque ($\tau_i = 0$) and diffuse-gray ($\alpha_i = \epsilon_i$)
4. the cavity is filled with a fluid which does not participate in the radiative exchange process



- an energy balance on the *i'th* surface gives:

$$\dot{Q}_i = \dot{q}_i A_i = A_i (J_i - G_i)$$



$$\dot{Q}_i = A_i(E_i - \alpha_i G_i) \quad (1)$$

$$J_i = E_i + \rho_i G_i \quad (2)$$

$$E_i = \epsilon_i E_{b,i} = \epsilon_i \sigma T_i^4 \quad (3)$$

$$\rho_i = 1 - \alpha_i = 1 - \epsilon_i \quad (4) \quad \Rightarrow \text{since } \alpha_i + \rho_i + \cancel{\tau_i}^0 = 1$$

and $\alpha_i = \epsilon_i$

Combining Eqs. 2, 3 and 4 gives

$$J_i = \epsilon_i E_{b,i} + (1 - \epsilon_i) G_i$$

Combining this with Eq. 1 gives

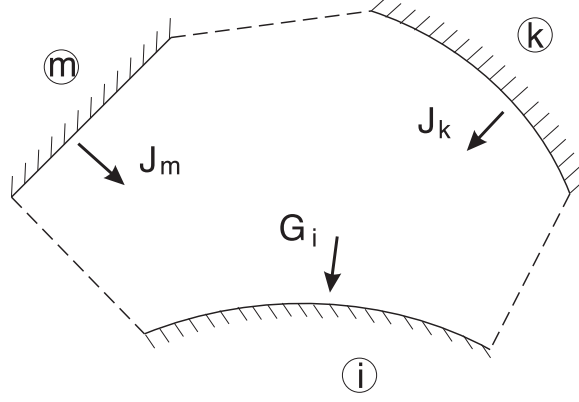
$$Q_i = \frac{E_{b,i} - J_i}{\left(\frac{1 - \epsilon_i}{\epsilon_i A_i} \right)} \equiv \frac{\text{potential difference}}{\text{surface resistance}}$$

This surface resistance represents real surface behavior as opposed to a blackbody surface. Recall that for a blackbody $\epsilon_i = \alpha_i = 1$ and $J_i = E_1 = E_{b,i} = \sigma T_i^4$.

- next consider radiative exchange between the surfaces.

By inspection it is clearly seen that

$$\left\{ \begin{array}{c} \text{irradiation on} \\ \text{surface } i \end{array} \right\} = \left\{ \begin{array}{c} \text{radiation leaving the} \\ \text{remaining surfaces} \end{array} \right\}$$



$$A_i G_i = \sum_{j=1}^N F_{j \rightarrow i} (A_j J_j) = \sum_{j=1}^N A_i F_{i \rightarrow j} J_i$$

Therefore

$$G_i = \sum_{j=1}^N F_{i \rightarrow j} J_j$$

Combining this with Eqs. 2 - 4 gives

$$J_i = \epsilon_i \sigma T_i^4 + (1 - \epsilon_i) \sum_{j=1}^N F_{i \rightarrow j} J_j$$

In addition we can write

$$\dot{Q}_i = A_i J_i - \sum_{j=1}^N A_i F_{i \rightarrow j} J_j$$

Since the summation rule states $\sum_{j=1}^N F_{i \rightarrow j} = 1$, the above equation becomes

$$\begin{aligned} \dot{Q}_i &= A_i \left\{ \sum_{j=1}^N F_{i \rightarrow j} J_i - \sum_{j=1}^N F_{i \rightarrow j} J_j \right\} \\ \dot{Q}_i &= \sum_{j=1}^N A_i F_{i \rightarrow j} (J_i - J_j) = \sum_{j=1}^N \dot{Q}_{i \rightarrow j} \end{aligned}$$

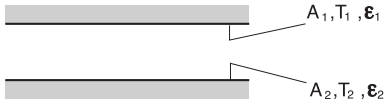
or

$$\dot{Q}_i = \sum_{j=1}^N \frac{J_i - J_j}{\left(\frac{1}{A_i F_{i \rightarrow j}} \right)} \equiv \frac{\text{potential difference}}{\text{configuration resistance}}$$

Some very useful consequences of this equation are given below.

Special Diffuse, gray, two-surface enclosures

Large (infinite) Parallel Plates

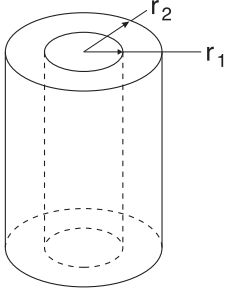


$$A_1 = A_2 = A$$

$$F_{1 \rightarrow 2} = 1$$

$$\dot{Q}_{12} = \frac{A\sigma(T_1^4 - T_2^4)}{\frac{1}{\epsilon_1} + \frac{1}{\epsilon_2} - 1}$$

Long (infinite) Concentric Cylinders

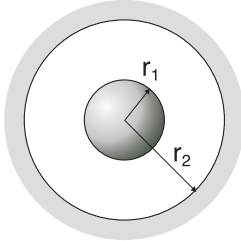


$$\frac{A_1}{A_2} = \frac{r_1}{r_2}$$

$$F_{1 \rightarrow 2} = 1$$

$$\dot{Q}_{12} = \frac{A\sigma(T_1^4 - T_2^4)}{\frac{1}{\epsilon_1} + \frac{1 - \epsilon_2}{\epsilon_2} \left(\frac{r_1}{r_2}\right)}$$

Concentric Spheres

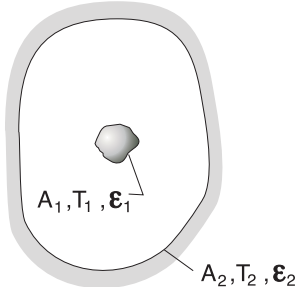


$$\frac{A_1}{A_2} = \frac{r_1^2}{r_2^2}$$

$$F_{1 \rightarrow 2} = 1$$

$$\dot{Q}_{12} = \frac{A\sigma(T_1^4 - T_2^4)}{\frac{1}{\epsilon_1} + \frac{1 - \epsilon_2}{\epsilon_2} \left(\frac{r_1}{r_2}\right)^2}$$

Small Convex Object in a Large Cavity



$$\frac{A_1}{A_2} = 0$$

$$F_{1 \rightarrow 2} = 1$$

$$\dot{Q}_{12} = \sigma A_1 \epsilon_1 (T_1^4 - T_2^4)$$

Cooling of Electronic Equipment



Reading

14-3, 14-6 $\rightarrow$ 14-9

Problems

14-52, 14-55, 14-64

Introduction

Why should we worry about the thermal behavior of electronic equipment?

- a standard Intel Pentium 4 chip is:
 - $10.4 \text{ mm} \times 10.4 \text{ mm}$
 - contains millions of electronic components
 - dissipates approximately **75** watts of power
 - is restricted to a maximum temperature of **85 °C**

This does not seem like a big deal. A **100 W** light bulb dissipates more power than this.

Power vs. Heat Flux

100 W light bulb



Power = 100 W

Area = 100 cm^2

Flux = 1.0 W/cm^2

Pentium 4 Processor



Power = 75.3 W

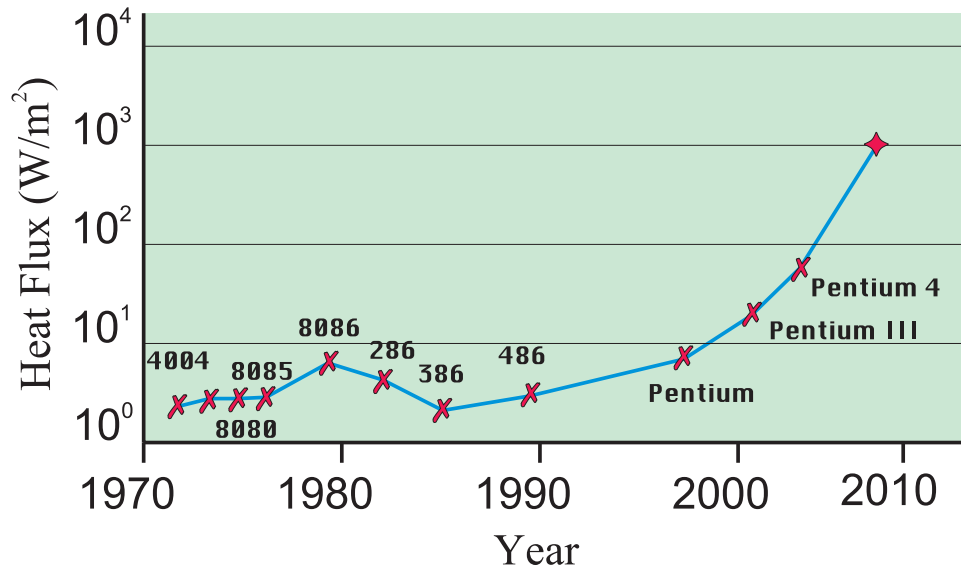
Area = 1.08 cm^2

Flux = 69.7 W/cm^2

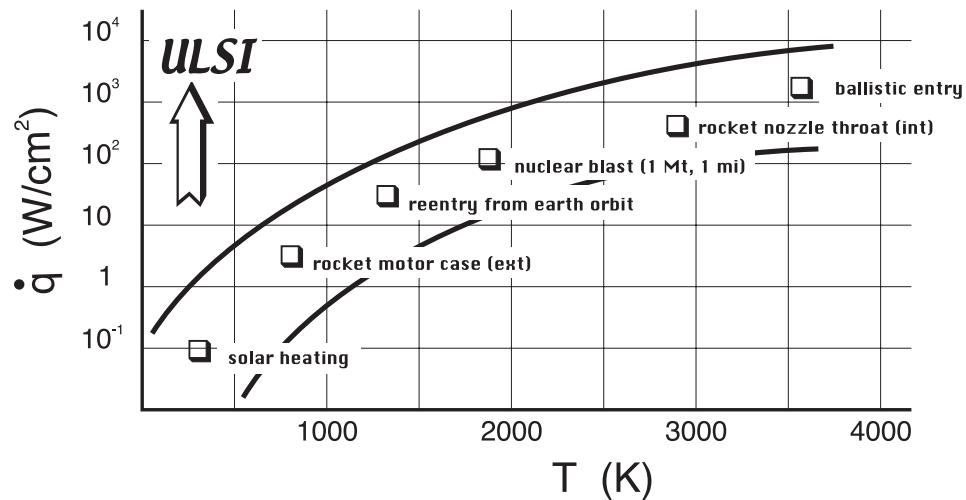
70 times larger!

- the heat flux at the surface on a chip is exceedingly high

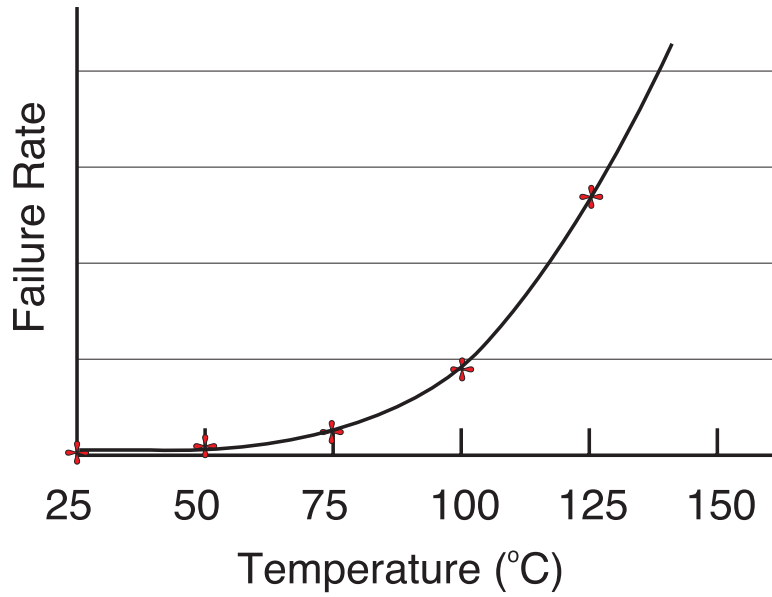
- this is primarily because of the miniaturization of electronic devices in order to maximize signal processing speed
- if we track the evolution of Intel processors over the course of the past several decades we see a pattern which is expected to continue for the next several years



- the following figure puts the magnitude of this heat flux into perspective



- the problem is further compounded by the fact that a maximum operating temperature of approximately $85 - 100\text{ }^{\circ}\text{C}$ is necessary in order to ensure long term reliable operation
- failure rate increases dramatically as operating temperatures rise above $100\text{ }^{\circ}\text{C}$



- electronic cooling presents a significant challenge to design engineers
- is the problem expected to get better anytime soon? not likely. By 2007, Intel expects to have **1 billion transistors/chip!** with power dissipation of more than **100 W** and heat fluxes of order **1000 W/m²**.

Therefore we must deal with the problem with engineering solutions that necessitate a good understanding of all forms of heat transfer.