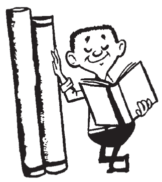
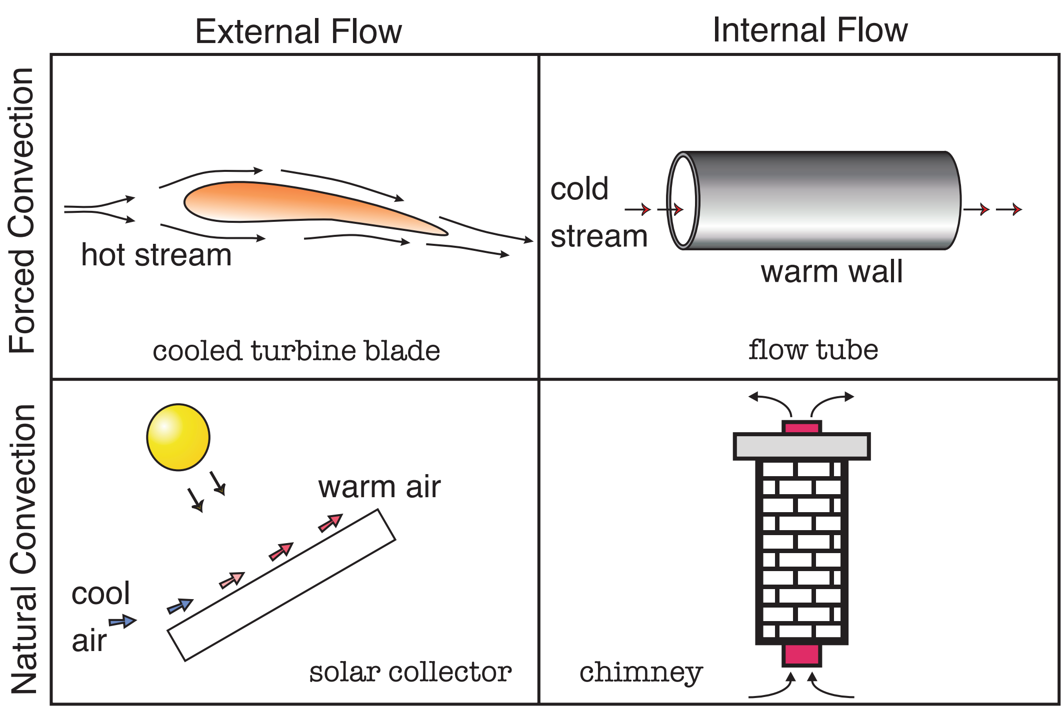
Convection Heat Transfer



Reading	Problems
10-1 ⟶ 10-7	10-22, 10-31, 10-33, 10-43, 10-49, 10-51, 10-55, 10-58, 10-83, 10-108,
11-1 ⟶ 11-2, 11-4	11-18, 11-33, 11-55, 11-56

Introduction

- *convection heat transfer* is the transport mechanism made possible through the *motion of fluid*
- fluid acts as a carrier or conveyor belt for the energy that it draws from (or delivers to) a solid wall
- the types of flow arrangements and heat transfer configurations associated with convection are extremely diverse including forced, natural and mixed convection for both internal and external flow geometries
- in addition to these convection mechanisms, flow can be laminar or turbulent and fluids can be single or multi-phase as in the case of boiling



- in convective heat transfer, the bulk fluid motion of the fluid plays a major role in the overall energy transfer process. Therefore, knowledge of the velocity distribution near a solid surface is essential.
- the controlling equation for convection is *Newton's Law of Cooling*

$$\dot{Q}_{conv} = \frac{\Delta T}{R_{conv}} = hA(T_w - T_\infty) \qquad \Rightarrow \quad R_{conv} = \frac{1}{hA}$$

where

$$A = \text{total convective area},\ m^2$$

$$h = \text{heat transfer coefficient},\ W/(m^2 \cdot K)$$

$$T_w = \text{surface temperature},\ ^\circ C$$

$$T_\infty = \text{fluid temperature},\ ^\circ C$$

- the heat transfer coefficient, h is a complex parameter that depends on geometry, thermal and fluid properties, fluid flow and boundary conditions

Factors Affecting Convective Heat Transfer

Geometry: flat plate, circular cylinder, sphere, spheroids plus many other shapes. In addition to the general shape, size, aspect ratio (thin or thick) and orientation (vertical or horizontal) play a significant role in convective heat transfer.

Type of flow: forced, natural, mixed convection as well as laminar, turbulent and transitional flows. These flows can also be considered as developing, fully developed, steady or transient.

Boundary condition: (i) isothermal wall (T_w = constant) or
(ii) isoflux wall ($\dot{q}_w$ = constant)

Type of fluid: viscous oil, water, gases (air) or liquid metals.

Fluid properties: symbols and units

mass density	:	$\rho,\ (kg/m^3)$
specific heat capacity	:	$C_p,\ (J/kg \cdot K)$
dynamic viscosity	:	$\mu,\ (N \cdot s/m^2)$
kinematic viscosity	:	$\nu,\ \equiv \mu/\rho\ (m^2/s)$
thermal conductivity	:	$k,\ (W/m \cdot K)$
thermal diffusivity	:	$\alpha,\ \equiv k/(\rho \cdot C_p)\ (m^2/s)$
Prandtl number	:	$Pr,\ \equiv \nu/\alpha\ (--)$
volumetric compressibility	:	$\beta,\ (1/K)$

All properties are temperature dependent and are usually determined at the film temperature, $T_f = (T_w + T_\infty)/2$

External Flow: the flow engulfs the body with which it interacts thermally

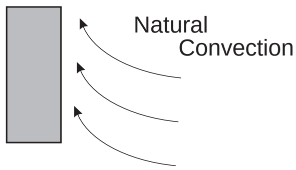
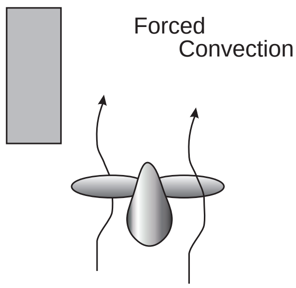
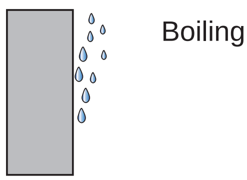
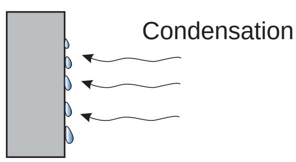
Internal Flow: the heat transfer surface surrounds and guides the convective stream

Forced Convection: flow is induced by an external source such as a pump, compressor, fan, etc.

Natural Convection: flow is induced by natural means without the assistance of an external mechanism. The flow is initiated by a change in the density of fluids incurred as a result of heating.

Mixed Convection: combined forced and natural convection

The following table gives the range of heat transfer coefficient expected for different convection mechanisms and fluid types.

Process	$h\ [W/(m^2 \cdot K)]$
 Natural Convection	• gases 3 - 20 • water 60 - 900
 Forced Convection	• gases 30 - 300 • oils 60 - 1 800 • water 100 - 1 500
 Boiling	• water 3 000 - 100 000
 Condensation	• steam 3 000 - 100 000

Dimensionless Groups

In the study and analysis of convection processes it is common practice reduce the total number of functional variables by forming dimensionless groups consisting of relevant thermophysical properties, geometry, boundary and flow conditions.

Prandtl number: $Pr = \nu/\alpha$ where $0 < Pr < \infty$ ($Pr \rightarrow 0$ for liquid metals and $Pr \rightarrow \infty$ for viscous oils). A measure of ratio between the diffusion of momentum to the diffusion of heat.

Oils	$Pr \approx 10^3$
Water	$Pr \approx 5$
Air	$Pr \approx 0.7$
Liquid Metals	$Pr \approx 10^{-2}$

Reynolds number: $Re = \rho U \mathcal{L}/\mu \equiv U\mathcal{L}/\nu$ (forced convection). A measure of the balance between the inertial forces and the viscous forces.

Peclet number: $Pe = U\mathcal{L}/\alpha \equiv RePr$

Grashof number: $Gr = g\beta(T_w - T_f)\mathcal{L}^3/\nu^2$ (natural convection)

Rayleigh number: $Ra = g\beta(T_w - T_f)\mathcal{L}^3/(\alpha \cdot \nu) \equiv GrPr$

Nusselt number: $Nu = h\mathcal{L}/k_f$ This can be considered as the dimensionless heat transfer coefficient.

Stanton number: $St = h/(U\rho C_p) \equiv Nu/(RePr)$

While for certain geometries, analytical solutions are available, typically because of the complexity of natural convection, *empirical correlations* based on experimental data are used.

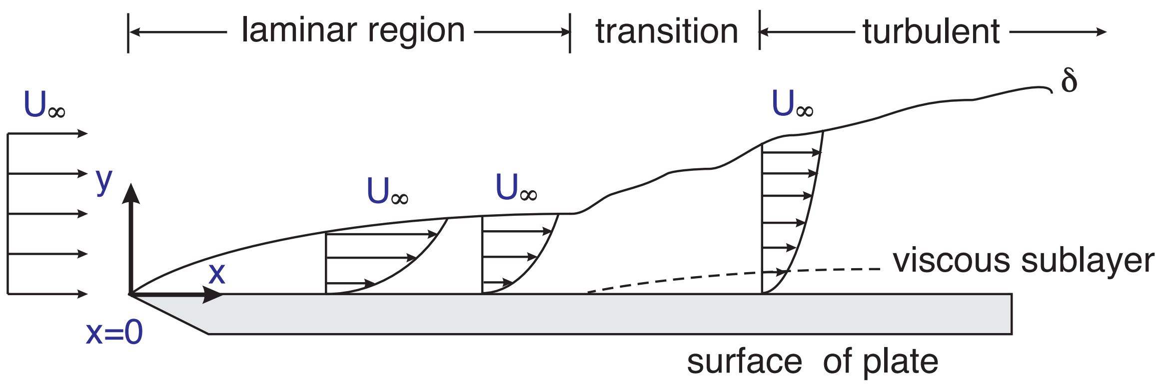
- specific to geometry, boundary conditions, etc.
- we need a separate correlation for each configuration

Correlations consist of the dimensionless groups described above and are available for

fluid	air, general
flow type	laminar, turbulent
geometry	plate, cylinder, sphere
orientation	vertical, horizontal, inclined
boundary condition	isothermal (UWT), isoflux (UWF)

Forced Convection

The simplest forced convection configuration to consider is the flow of mass and heat near a flat plate as shown below.



- fluid approaches the plate with a uniform velocity profile, U_∞
- we will assume that at $x = 0$ the plate is infinitely thin and the flow field will develop in a *steady, two-dimensional, laminar* manner
- flow forms thin layers that can slip past one another at different velocities
- as Reynolds number increases the flow has a tendency to become more chaotic resulting in disordered motion known as turbulent flow
  - transition from laminar to turbulent is called the critical Reynolds number, Re_{cr}

$$Re_{cr} = \frac{U_\infty x_{cr}}{\nu}$$

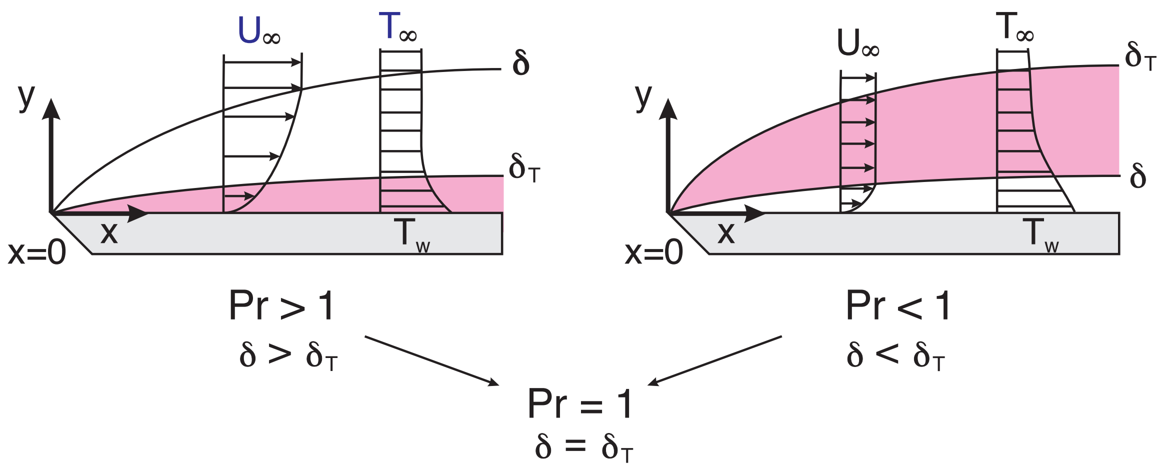
  - for flow over a flat plate $Re_{cr} \approx 500,000$
  - for engineering calculations, the transition region is usually neglected, so that the transition from laminar to turbulent flow occurs at a critical location from the leading edge, x_{cr}
  - $x < x_{cr}$ the boundary layer is laminar; $x > x_{cr}$ the boundary layer is turbulent

$$\text{water at } 3\ m/s: \quad \frac{U_\infty x_{cr}}{\nu} = 500,000 \quad \Rightarrow \quad x_c = 0.17\ m$$

$$\text{air at } 3\ m/s: \quad \frac{U_\infty x_{cr}}{\nu} = 500,000 \quad \Rightarrow \quad x_c = 2.5\ m$$

- the thin layer immediately adjacent to the wall where viscous effects dominate is known as the *laminar sublayer*

Boundary Layers



Velocity Boundary Layer

- the region of fluid flow over the plate where viscous effects dominate is called the *velocity* or *hydrodynamic* boundary layer
- the velocity at the surface of the plate, $y = 0$, is set to zero, $U_{@y=0} = 0\ m/s$ because of the *no slip condition* at the wall
- the velocity of the fluid progressively increases away from the wall until we reach approximately $0.99\ U_\infty$ which is denoted as the δ, the *velocity boundary layer thickness*. Note: 99% is an arbitrarily selected value.
- the region beyond the velocity boundary layer is denoted as the *inviscid flow* region, where frictional effects are negligible and the velocity remains relatively constant at U_∞

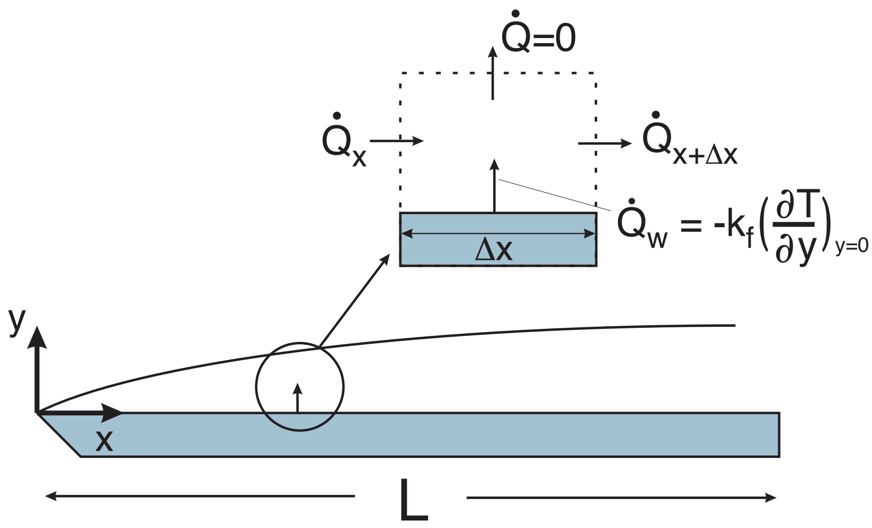
Thermal Boundary Layer

- the thermal boundary layer is arbitrarily selected as the locus of points where

$$\frac{T - T_w}{T_\infty - T_w} = 0.99$$

- for *low Prandtl number* fluids, i.e. liquid metals, momentum diffuses much slower than heat flow (remember $Pr = \nu/\alpha$) and the velocity boundary layer is fully contained within the thermal boundary layer
- conversely, for *high Prandtl number* fluids, i.e. oils, heat diffuses slower than the momentum and the thermal boundary layer is contained with in the velocity boundary layer

Heat Transfer Coefficient



Performing an energy balance over a control volume of width Δx and height δ_T, as shown above we obtain

$$\dot{Q}_w + \dot{Q}_x = \dot{Q}_{x+\Delta x} \qquad \Rightarrow \quad \dot{Q}_w = \dot{Q}_{\Delta x} = \dot{Q}_{conv}$$

We should note that the heat transfer from the edge of the thermal boundary is zero because the temperature at this point is invariant in y and there is no heat transfer in the y direction at $y = \delta_T$.

From Newton's law of cooling we know that

$$\dot{Q}_{conv} = hA(T_w - T_\infty)$$

Since the velocity at the wall is zero (no slip condition) heat transfer at $y = 0$ is by conduction only and from Fourier's law of conduction the heat flux is

$$\dot{Q}_w = -k_f A \left(\frac{\partial T}{\partial y}\right)_{y=0}$$

Substituting into our energy balance we get

$$-k_f \left(\frac{\partial T}{\partial y}\right)_{y=0} = h(T_w - T_\infty)$$

and the local heat transfer coefficient can be written as

$$h = \frac{-k_f \left(\frac{\partial T}{\partial y}\right)_{y=0}}{(T_w - T_\infty)} \equiv h(x) = h_x$$

The average heat transfer coefficient is determined using the mean value theorem such that

$$h_{av} = \frac{1}{L}\int_0^L h(x)\; dx$$

The energy leaving a differential element on the surface of the plate can be written as

$$\delta\dot{Q}_{conv} = \dot{q}_{conv}\; dA \qquad \text{where} \;\; dA = W\; dx$$

The local skin friction coefficient, $C_{f,x}$ is a dimensionless form of the wall shear stress, $\tau_{w,x}$, associated with resistance to flow at the wall.

$$C_{f,x} = \frac{\tau_{w,x}}{\frac{1}{2}\rho U_\infty^2}$$

The Nusselt number is a measure of the dimensionless heat transfer coefficient given as

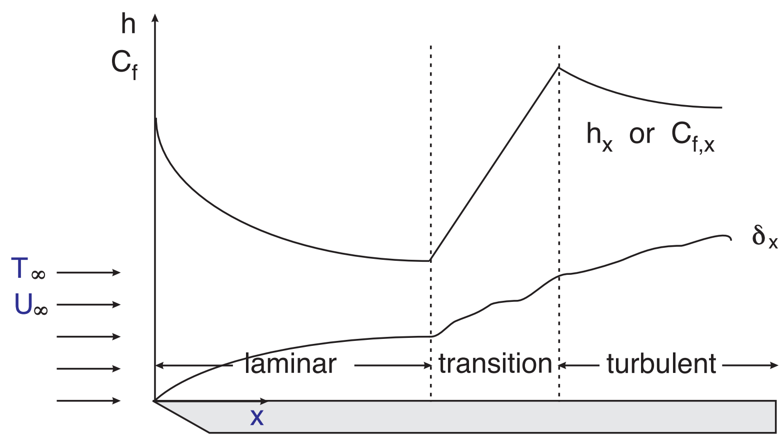
$$Nu = f(Re, Pr)$$

While both the skin friction coefficient and the Nusselt number can be determine analytically through the conservations equations for mass, momentum and energy, it is beyond the scope of this course. Instead we will use *empirical correlations* based on experimental data where

$$C_{f,x} = C_1 \cdot Re^{-m}$$

$$Nu_x = C_2 \cdot Re^m \cdot Pr^n$$

Flow Over Plates



1. Laminar Boundary Layer Flow, Isothermal (UWT)

The local values of the skin friction and the Nusselt number are given as

$$\boxed{C_{f,x} = \frac{0.664}{Re_x^{1/2}}}$$

$$\boxed{Nu_x = 0.332\, Re_x^{1/2}\, Pr^{1/3}} \quad \Rightarrow \text{local, laminar, UWT, } Pr \geq 0.6$$

where x is the distance from the leading edge of the plate.

The Stanton number is given as

$$\frac{\mathrm{Nu}_x}{\mathrm{Re}_x\, \mathrm{Pr}} = \mathrm{St}_x = \frac{hx}{k\dfrac{\rho U_\infty x}{\mu} \cdot \dfrac{\mu c_p}{k}} = \frac{h}{\rho U_\infty c_p}$$

If we multiply by $Pr^{2/3}$

$$Pr^{2/3}\mathrm{St} = \frac{0.332}{\sqrt{Re_x}} = \frac{1}{2}\left[\frac{0.664}{\sqrt{Re_x}}\right] = \frac{1}{2}\, C_{f,x}$$

$$Pr^{2/3}\mathrm{St} = C_{f,x}/2$$

- Note: the heat transfer coefficient is directly connected to the surface friction

- this relation has a wide validity – well beyond the flat plate. It also is very nearly true for turbulent flow

Average Skin Friction Coefficient

Using the mean value theorem

$$
\begin{aligned}
C_f &= \frac{1}{L}\int_0^L C_{f,x}\, dx \\
&= \frac{1}{L}\int_0^L \frac{0.664}{Re_x^{1/2}}\, dx \\
&= \frac{0.664}{L}\int_0^L \left(\frac{U_\infty x}{\nu}\right)^{-1/2} dx \\
&= \frac{0.664}{L}\left(\frac{U_\infty}{\nu}\right)^{-1/2} \cdot 2x^{1/2}\Big|_0^L \\
&= \frac{1.328}{L}\left(\frac{U_\infty L}{\nu}\right)^{-1/2} = \frac{1.328}{Re_L^{1/2}} \Leftarrow
\end{aligned}
$$

Average Heat Transfer Coefficient

Using the mean value theorem again

$$
\begin{aligned}
\frac{h_x x}{k} &= 0.332\left(\frac{U_\infty x}{\nu}\right)^{1/2} \mathrm{Pr}^{1/3} \\
h_x &= 0.332\left(\frac{U_\infty x}{\nu}\right)^{1/2} \mathrm{Pr}^{1/3}\,\frac{k}{x} \\
h_x &= 0.332\left(\underbrace{\frac{U_\infty L}{\nu}}_{Re_L}\right)^{1/2} \mathrm{Pr}^{1/3}\,\frac{k}{x}\left(\frac{x}{L}\right)^{1/2} \\
h_x &= [0.332\,\mathrm{Re}_L^{1/2}\,\mathrm{Pr}^{1/3} k/L]\frac{1}{(x/L)^{1/2}}
\end{aligned}
$$

$$\underbrace{\frac{1}{L}\int_0^L h_x dx}_{h_L} = 0.333 Re_L^{1/2} Pr^{1/3} \frac{k}{L^{1/2}} \left[\frac{1}{L}\int_0^L x^{-1/2} dx\right]$$

$$\overline{h}_L = 0.332 Re_L^{1/2} Pr^{1/3} \left(\frac{2k}{L}\right)$$

$$\overline{Nu}_L = 0.664 Re_L^{1/2} Pr^{1/3}$$

$$\boxed{Nu_L = \frac{h_L L}{k_f} = 0.664\ \mathrm{Re}_L^{1/2}\ \mathrm{Pr}^{1/3}} \Rightarrow \text{average, laminar, UWT, } Pr \geq 0.6$$

For low Prandtl numbers, i.e. liquid metals

$$\boxed{Nu_x = 0.565\ Re_x^{1/2}\ Pr^{1/2}} \Rightarrow \text{local, laminar, UWT, } Pr \leq 0.6$$

2. Turbulent Boundary Layer Flow, Isothermal (UWT)

The local skin friction is given as

$$\boxed{C_{f,x} = \frac{\tau_w}{(1/2)\rho U_\infty^2} = \frac{0.0592}{Re_x^{0.2}}} \Rightarrow \text{local, turbulent, UWT, } Pr \geq 0.6$$

As mentioned previously, we know that

$$Pr^{2/3}\mathrm{St} = \frac{C_{f,x}}{2}$$

Therefore

$$Pr^{2/3}\frac{Nu_x}{Re_x\ Pr} = \frac{1}{2}\left(\frac{0.0592}{Re_x^{0.2}}\right)$$

and

$$\boxed{Nu_x = 0.0296\; Re_x^{0.8}\; Pr^{1/3}} \;\Rightarrow\; \begin{array}{l}\text{local, turbulent, UWT,}\\ 0.6 < Pr < 100, \;\; Re_x > 500,000\end{array}$$

$$\boxed{Nu_L = 0.037\; Re_L^{0.8}\; Pr^{1/3}} \;\Rightarrow\; \begin{array}{l}\text{average, turbulent, UWT,}\\ 0.6 < Pr < 100, \;\; Re_x > 500,000\end{array}$$

3. Combined Laminar and Turbulent Boundary Layer Flow, Isothermal (UWT)

When $(T_w - T_\infty)$ constant

$$h_L = \frac{1}{L}\int_0^L h dx = \frac{1}{L}\left\{\int_0^{x_{cr}} h_x^{lam} dx + \int_{x_{cr}}^L h_x^{tur} dx\right\}$$

$$\boxed{Nu_L = \frac{h_L L}{k} = (0.037\; Re_L^{0.8} - 871)\; Pr^{1/3}} \;\Rightarrow\; \begin{array}{l}\text{average, turbulent, UWT,}\\ 0.6 < Pr < 100, \;\; Re_L > 500,000\end{array}$$

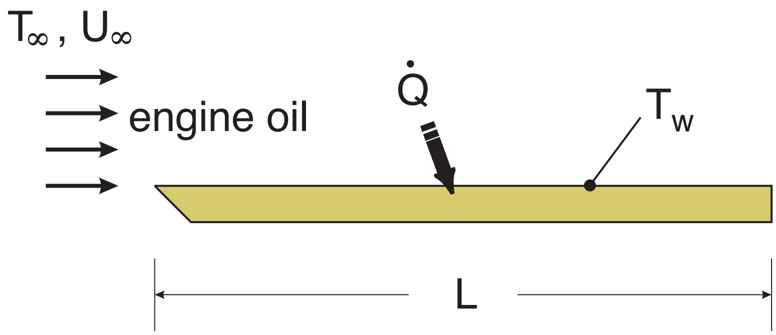
4. Laminar Boundary Layer Flow, Isoflux (UWF)

$$\boxed{Nu_x = 0.453\; Re_x^{1/2}\; Pr^{1/3}} \;\Rightarrow\; \text{local, laminar, UWF, } Pr \geq 0.6$$

5. Turbulent Boundary Layer Flow, Isoflux (UWF)

$$\boxed{Nu_x = 0.0308\; Re_x^{4/5}\; Pr^{1/3}} \;\Rightarrow\; \text{local, turbulent, UWF, } Pr \geq 0.6$$

Example: The flow of hot engine oil over a flat plate



Given: $T_\infty = 60\,°C, \quad U_\infty = 2\,m/s, \quad T_w = 20\,°C, \quad L = 5\,m$

Find: $\dot{Q}$ per unit width of the plate

Solution: Assume steady state; width $W = 1\,m$

The film temperature is

$$T_f = \frac{1}{2}(T_w + T_\infty) = \frac{1}{2}(20 + 60) = 40\,°C$$

From Table A-18, for unused engine oil

$$\rho = 876\,kg/m^3$$

$$k = 0.144\,W/(m \cdot K)$$

$$\nu = 242 \times 10^{-6}\,m^2/s$$

$$Pr = 2870$$

The Reynolds number is

$$Re_L = \frac{U_\infty L}{\nu} = \frac{2\,m/s \times 5\,m}{242 \times 10^{-6}\,m^2/s} = 4.13 \times 10^4 < Re_{cr} = 500,000$$

Therefore we are in the laminar regime over the entire length of the plate.

The Nusselt number is given as

$$Nu_L = \frac{h_{av} L}{k_f} = 0.664\,Re_L^{1/2} Pr^{1/3}$$

and the average heat transfer coefficient is

$$
\begin{aligned}
h_{av} &= \left(\frac{k}{L}\right) 0.664\, Re_L^{1/2}\, Pr^{1/3} \\
&= \frac{0.144\, W/(m \cdot K)}{5\, m}\, 0.664 \times (4.13 \times 10^4)^{1/2} \times 2870^{1/3} \\
&= 55.2\, W/(m^2 \cdot K)
\end{aligned}
$$

The heat flow rate is

$$
\begin{aligned}
\dot{Q} &= h_{av}\, W \cdot L (T_\infty - T_w) \\
&= 55.2\, W/(m^2 \cdot K) \;\; \times \;\; (1 m \times 5\, m) \;\; \times \;\; (60 - 20)\, K \\
&= 11,040\, W
\end{aligned}
$$

Flow Over Cylinders and Spheres

1. Boundary Layer Flow Over Circular Cylinders, Isothermal (UWT)

The Churchill-Berstein (1977) correlation for the average Nusselt number for long ($L/D > 100$) cylinders is

$$\boxed{Nu_D = S_D^* + f(Pr)\ Re_D^{1/2}\left[1 + \left(\frac{Re_D}{28200}\right)^{5/8}\right]^{4/5}} \Rightarrow \begin{array}{l}\text{average, UWT, } Re < 10^7 \\ 0 \le Pr \le \infty, \ \ Re \cdot Pr > 0.2\end{array}$$

where S_D^* is the diffusive term associated with $Re_D \rightarrow 0$ and is given as

$$S_D^* \ = \ 0.3$$

and the Prandtl number function is

$$f(Pr) = \frac{0.62\ Pr^{1/3}}{[1 + (0.4/Pr)^{2/3}]^{1/4}}$$

All fluid properties are evaluated at $T_f = (T_w + T_\infty)/2$.

2. Boundary Layer Flow Over Non-Circular Cylinders, Isothermal (UWT)

The empirical formulations of Zhukauskas and Jakob given in Table 10-3 are commonly used, where

$$\boxed{Nu_D \approx \frac{\overline{h}D}{k} = C\ Re_D^m\ Pr^{1/3}} \ \Rightarrow \text{see Table 10-3 for conditions}$$

3. Boundary Layer Flow Over a Sphere, Isothermal (UWT)

For flow over an isothermal sphere of diameter D

$$\boxed{Nu_D = S_D^* + 0.4\, Re_D^{1/2} + 0.06\, Re_D^{2/3}\, Pr^{0.4} \left(\frac{\mu_\infty}{\mu_w}\right)^{1/4}} \Rightarrow$$

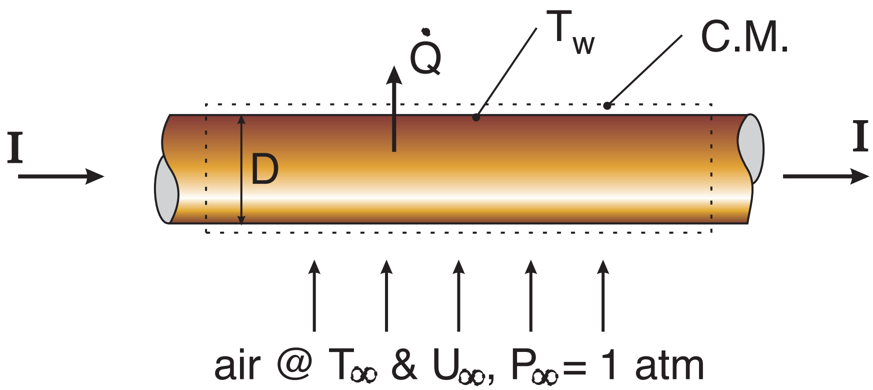
average, UWT,
$0.7 \leq Pr \leq 380$
$3.5 < Re_D < 80,000$

where the diffusive term at $Re_D \rightarrow 0$ is

$$S_D^* = 2$$

and the dynamic viscosity of the fluid in the bulk flow, μ_∞ is based on T_∞ and the dynamic viscosity of the fluid at the surface, μ_w, is based on T_w. All other properties are based on T_∞.

Example: Air flow across an electric wire



Given: $T_\infty = 275\ K,\ D = 1\ mm,\ T_w = 325\ K,\ \dfrac{\dot{Q}}{L} = 70\ W/m,$ steady state

Find: U_∞

Solution: Assume that air is an ideal gas.

The film temperature is

$$T_f = \frac{1}{2}(T_w + T_\infty) = \frac{1}{2}(325 + 275) = 300\ K$$

From Table A-19:

$$\begin{aligned} k &= 0.0261\ W/(m \cdot K) \\ \nu &= 1.57 \times 10^{-5}\ m^2/s \\ Pr &= 0.712 \end{aligned}$$

From Newton's law of cooling

$$\dot{Q} = hA(T_w - T_\infty)$$

where $A = \pi DL$. Therefore

$$\begin{aligned} h &= \frac{\dot{Q}/L}{\pi D(T_w - T_\infty)} = \frac{70\ W/m}{\pi(1 \times 10^{-3}\ m)(325 - 275)\ K} \\ &= 445.6\ W/(m^2 \cdot K) \end{aligned}$$

The Nusselt number is

$$Nu_D = \frac{hD}{k} = \frac{445.6\ W/(m^2 \cdot K) \times 10^{-3}}{0.261\ W/(m \cdot K)} = 17.07$$

From the Churchill=Bernstein correlation

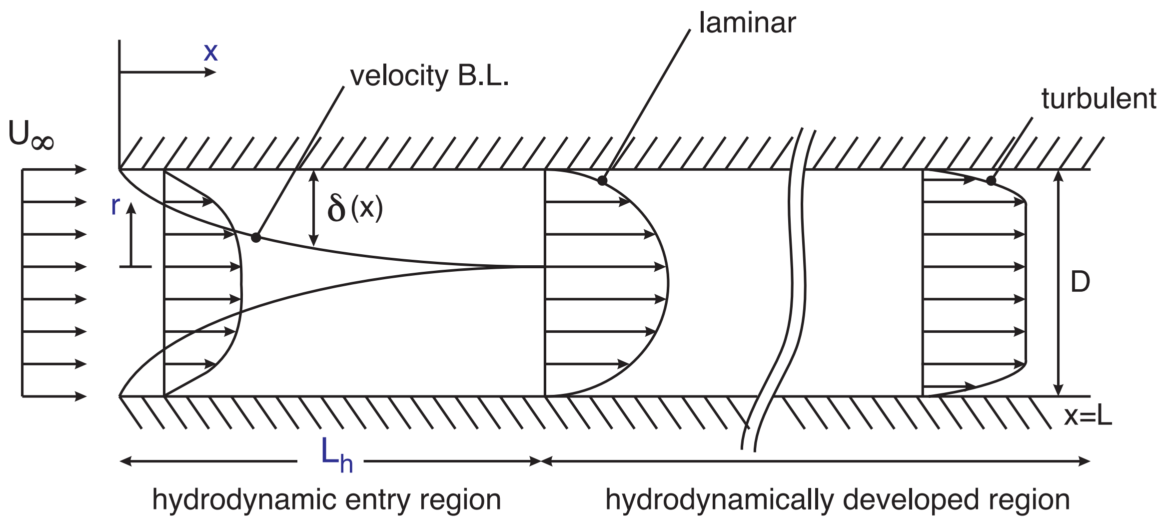
$$\begin{aligned}
Nu_D &= 0.3 + \frac{0.62 Re_D^{1/2} Pr^{1/3}}{\left[1 + \left(\frac{0.4}{Pr}\right)^{2/3}\right]^{1/4}} \left[1 + \left(\frac{Re_D}{28,200}\right)^{5/8}\right]^{4/5} \\
17.07 &= 0.3 + 0.4862 Re_D^{1/2} \left[1 + \left(\frac{Re_D}{28,200}\right)^{5/8}\right]^{4/5} \\
34.49 &= Re_D^{1/2} \left[1 + \left(\frac{Re_D}{28,200}\right)^{5/8}\right]^{4/5} \\
Re_D &\approx 990.0 = \frac{U_\infty D}{\nu} \\
U_\infty &= 990 \left(\frac{\nu}{D}\right) = 990 \left(\frac{1.57 \times 10^{-5}\ m^2/s}{10^{-3}\ m}\right) = 15.54\ m/s \Leftarrow
\end{aligned}$$

Note:

- for this flow, the frequency of vortex shedding behind the cylinder (wire) can be estimated as $3.3\ kHz$ which is in the audible range. Hence the name “the singing wire”.
- when T_w is not known, the properties cannot be evaluated. Then you have to assume a T_w value to start the calculation and in the end calculate T_w. If these T_w values do not match, guess a new value for T_w and iterate until convergence. Since the properties are usually weak functions of temperature, typically one iteration is sufficient.

Internal Flow

Lets consider fluid flow in a duct bounded by a wall that is at a different temperature than the fluid. For simplicity we will examine a round tube of diameter D as shown below



The velocity profile across the tube changes from $U = 0$ at the wall to a maximum value along the center line. The average velocity, obtained by integrating this velocity profile, is called the *mean velocity* and is given as

$$U_m = \frac{1}{A_c}\int_{A_c} u \, dA = \frac{\dot{m}}{\rho_m A_c}$$

where the area of the tube is given as $A_c = \pi D^2/4$ and the fluid density, ρ_m is evaluated at T_f.

The Reynolds number is given as

$$Re_D = \frac{U_m D}{\nu}$$

For flow in a tube:

$Re_D < 2300$	laminar flow
$2300 < Re_D < 4000$	transition to turbulent flow
$Re_D > 4000$	turbulent flow

For engineering calculations, we typically assume that $Re_{cr} \approx 2300$, therefore

$$Re_D \begin{cases} < Re_{cr} & \text{laminar} \\ > Re_{cr} & \text{turbulent} \end{cases}$$

<u>Hydrodynamic (Velocity) Boundary Layer</u>

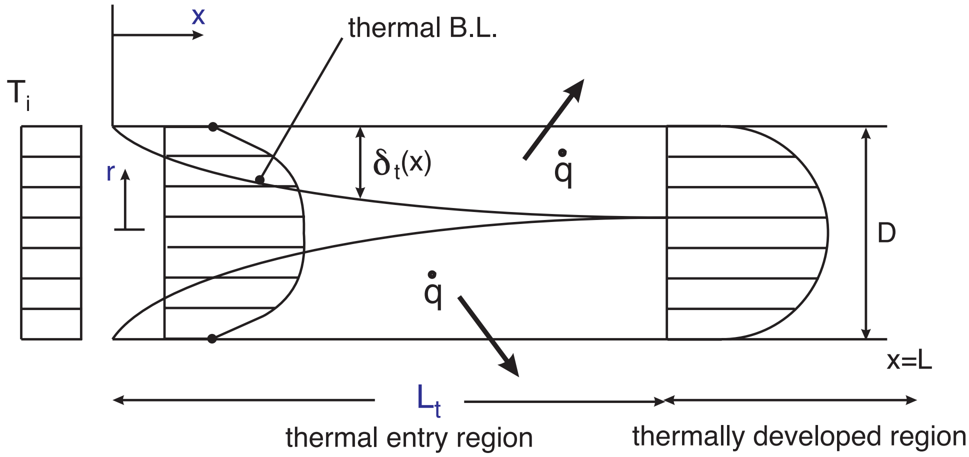
- the boundary layer, δ, is initiated at the entrance to the duct and grows as $x^{1/2}$
- when the boundary layer grows to the tube radius, r, the boundary layers merge
  - this flow length is called the flow entrance length, L_h
  - $0 \le x \le L_h$ is the hydrodynamic entrance region
  - $L_h < x \le L$ is the fully developed region or hydrodynamically developed region
- the hydrodynamic boundary layer thickness can be approximated as

$$\delta(x) \approx 5x \left(\frac{U_m x}{\nu}\right)^{-1/2} = \frac{5x}{\sqrt{Re_x}}$$

- the hydrodynamic entry length can be approximated as

$$L_h \approx 0.05 Re_D D \qquad \text{(laminar flow)}$$

Thermal Boundary Layer



- the entering fluid has an initial temperature of T_i at $x = 0$
- heat transfer occurs between the duct wall and the fluid
- a thermal entrance region develops from $0 \leq x \leq L_t$
- when the thermal boundary layers, δ_t merge at the thermal entry length L_t, the flow is considered thermally fully developed
- the thermal entry length can be approximated as

$$L_t \approx 0.05 Re_D Pr D \qquad \text{(laminar flow)}$$

- for turbulent flow $L_h \approx L_t \approx 10D$

Wall Boundary Conditions

1. Uniform Wall Heat Flux: The total heat transfer from the wall to the fluid stream can be determined by performing an energy balance over the tube

$$\frac{dE_{cv}}{dt} = \dot{Q} - \dot{W} + \left[\dot{m}(h + KE + PE)_{in} - \dot{m}(h + KE + PE)_{out}\right]$$

if we assume:

- steady state $\longrightarrow$ $dE_{cv}/dt = 0$

- $\Delta KE = \Delta PE \rightarrow 0$
- no work done on or by the tube, $\dot{W} = 0$
- steady flow conditions, $\dot{m}_{in} = \dot{m}_{out} = \dot{m}$

then the energy balance becomes

$$\dot{Q} = \dot{q}_w A = \dot{m}(h_{out} - h_{in}) = \dot{m} C_p (T_{out} - T_{in})$$

Since the wall flux $\dot{q}_w$ is uniform, the local mean temperature denoted as

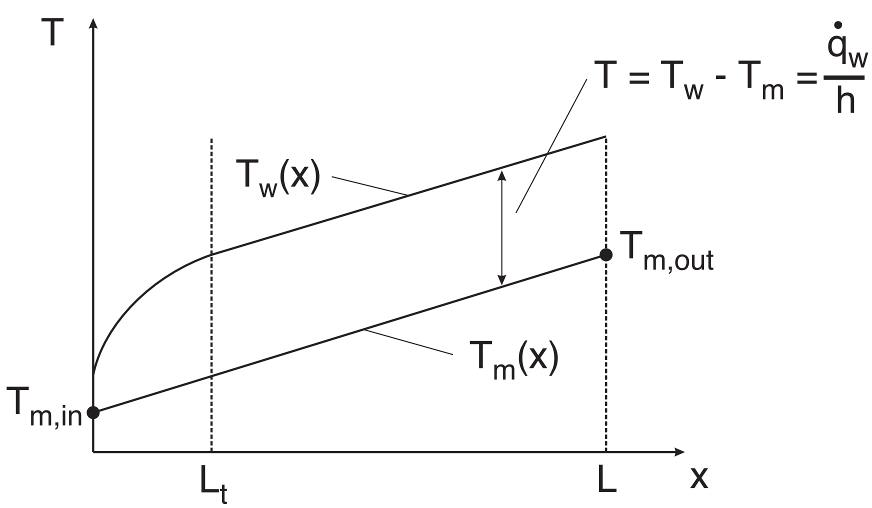
$$T_{m,x} = T_{m,i} + \frac{\dot{q}_w A}{\dot{m} C_p}$$

will increase in a linear manner with respect to x.

The surface temperature can be determined from

$$T_w = T_m + \frac{\dot{q}_w}{h}$$

- in the developing region where h is changing in a non linear manner, T_w will also change in a non linear manner
- in the fully developed region where h is constant, T_w will increase in a linear manner in proportion to T_m



2. Isothermal Wall: Using Newton's law of cooling we can determine the average rate of heat transfer to or from a fluid flowing in a tube

$$\dot{Q} = hA \underbrace{(T_w - T_m)}_{average\ \Delta T}$$

From an energy balance over a control volume in the fluid, we can determine

$$\dot{Q} = \dot{m} C_p dT_m$$

Equating the two equations above we find

$$\dot{m} C_p dT_m = hA \underbrace{(T_w - T_m)}_{average\ \Delta T}$$

By isolating the temperature terms and integrating we obtain

$$\ln\left(\frac{T_w - T_{out}}{T_w - T_{in}}\right) = -\frac{hA}{\dot{m} C_p}$$

where

$$A = \text{surface area of the tube}$$

$$h = \text{average heat transfer coefficient}$$

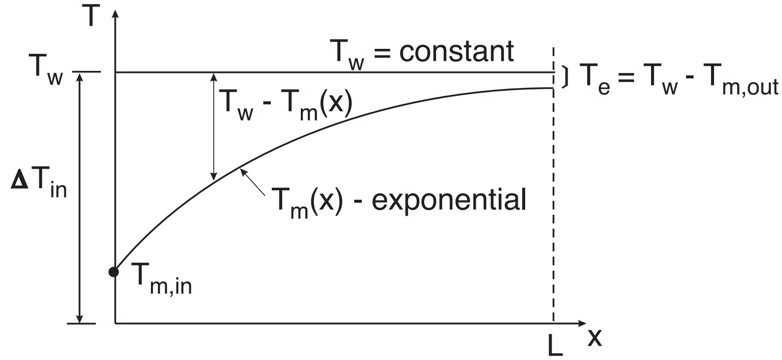
We can now determine the outlet temperature of the tube as

$$T_{out} = T_w - (T_w - T_{in}) \exp[-hA/(\dot{m} C_p)]$$

Because of the exponential temperature decay within the tube, it is common to present the mean temperature from inlet to outlet as a log mean temperature difference where

$$\dot{Q} = hA\Delta T_{ln}$$

$$\Delta T_{ln} = \frac{T_{out} - T_{in}}{\ln\left(\dfrac{T_w - T_{out}}{T_w - T_{in}}\right)} = \frac{T_{out} - T_{in}}{\ln(\Delta T_{out}/\Delta T_{in})}$$



1. Laminar Flow in Circular Tubes, Isothermal (UWT) and Isoflux (UWF)

For laminar flow where $Re_D \leq 2300$

$$\boxed{Nu_D = 3.66} \quad \Rightarrow \text{ fully developed, laminar, UWT, } L > L_t \text{ \& } L_h$$

$$\boxed{Nu_D = 4.36} \quad \Rightarrow \text{ fully developed, laminar, UWF, } L > L_t \text{ \& } L_h$$

$$\boxed{Nu_D = 1.86 \left(\frac{Re_D Pr D}{L}\right)^{1/3} \left(\frac{\mu_b}{\mu_w}\right)^{0.4}} \quad \Rightarrow$$

developing laminar flow, UWT,
$Pr > 0.5$
$L < L_h$ or $L < L_t$

For non-circular tubes the hydraulic diameter, $D_h = 4A_c/P$ can be used in conjunction with Table 10-4 to determine the Reynolds number and in turn the Nusselt number.

In all cases the fluid properties are evaluated at the mean fluid temperature given as

$$T_{mean} = \frac{1}{2}\left(T_{m,in} + T_{m,out}\right)$$

except for μ_w which is evaluated at the wall temperature, T_w.

2. Turbulent Flow in Circular Tubes, Isothermal (UWT) and Isoflux (UWF)

For turbulent flow where $Re_D \geq 2300$ the Dittus-Bouler equation (Eq. 10-76) can be used

$$\boxed{Nu_D = 0.023\, Re_D^{0.8}\, Pr^n} \Rightarrow \begin{array}{l} \text{turbulent flow, UWT or UWF,} \\ 0.7 \leq Pr \leq 160 \\ Re_D > 2,300 \\ n = 0.4 \text{ heating} \\ n = 0.3 \text{ cooling} \end{array}$$

For non-circular tubes, again we can use the hydraulic diameter, $D_h = 4A_c/P$ to determine both the Reynolds and the Nusselt numbers.

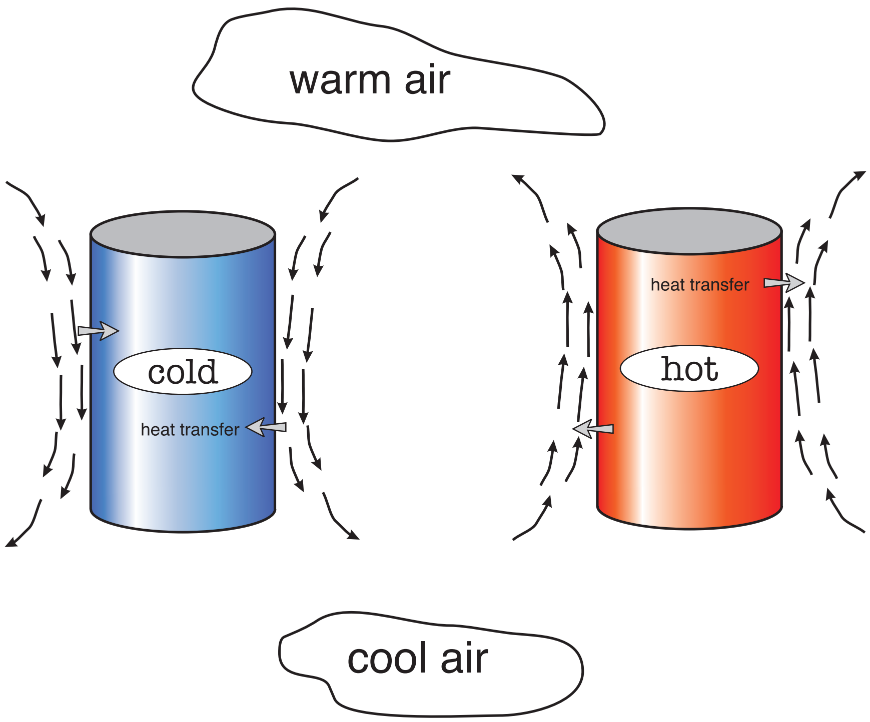
In all cases the fluid properties are evaluated at the mean fluid temperature given as

$$T_{mean} = \frac{1}{2}\left(T_{m,in} + T_{m,out}\right)$$

Natural Convection

What Drives Natural Convection?

- fluid flow is driven by the effects of buoyancy
- fluids tend to expand when heated and contract when cooled at constant pressure
- therefore a fluid layer adjacent to a surface will become lighter if heated and heavier if cooled by the surface



- a lighter fluid will flow upward and a cooler fluid will flow downward
- as the fluid sweeps the wall, heat transfer will occur in a similar manner to boundary layer flow however in this case the bulk fluid is stationary as opposed to moving at a constant velocity in the case of forced convection

Recall from forced convection that the flow behavior is determined by the Reynolds number. In natural convection, we do not have a Reynolds number but we have an analogous dimensionless group called the *Grashof number*

$$Gr = \frac{\text{buouancy force}}{\text{viscous force}} = \frac{g\beta(T_w - T_\infty)\mathcal{L}^3}{\nu^2}$$

where

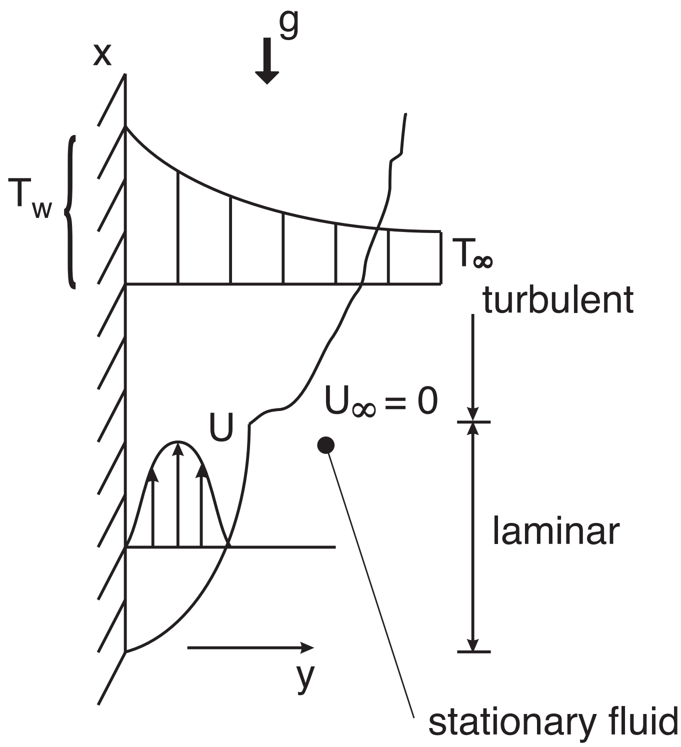
g	$=$	gravitational acceleration, m/s^2
β	$=$	volumetric expansion coefficient, $\beta \equiv 1/T$ (T is ambient temp. in K)
T_w	$=$	wall temperature, K
T_∞	$=$	ambient temperature, K
$\mathcal{L}$	$=$	characteristic length, m
ν	$=$	kinematic viscosity, m^2/s

The volumetric expansion coefficient, β, is used to express the variation of density of the fluid with respect to temperature and is given as

$$\beta = -\frac{1}{\rho}\left(\frac{\partial \rho}{\partial T}\right)_P$$

Natural Convection Over Surfaces

- few general purpose analytical models exist for natural convection
- there are many empirically based correlations specific to geometry and boundary conditions
- natural convection heat transfer depends on geometry and orientation
- the velocity and temperature profiles within a boundary layer formed on a vertical plate in a stationary fluid looks as follows:



- note that unlike forced convection, the velocity at the edge of the boundary layer goes to zero

Natural Convection Heat Transfer Correlations

The general form of the Nusselt number for natural convection is as follows:

$$Nu = f(Gr, Pr) \equiv C Ra^m Pr^n \qquad \text{where} \;\; Ra = Gr \cdot Pr$$

- C depends on geometry, orientation, type of flow, boundary conditions and choice of characteristic length.
- m depends on type of flow (laminar or turbulent)
- n depends on the type of fluid and type of flow

1. Laminar Flow Over a Vertical Plate, Isothermal (UWT)

The general form of the Nusselt number is given as

$$Nu_{\mathcal{L}} = \frac{h\mathcal{L}}{k_f} = C \left(\underbrace{\frac{g\beta(T_w - T_\infty)\mathcal{L}^3}{\nu^2}}_{\equiv Gr} \right)^{1/4} \left(\underbrace{\frac{\nu}{\alpha}}_{\equiv Pr} \right)^{1/4} = C \; \underbrace{Gr_{\mathcal{L}}^{1/4} \, Pr^{1/4}}_{Ra^{1/4}}$$

where

$$Ra_{\mathcal{L}} = Gr_{\mathcal{L}} Pr = \frac{g\beta(T_w - T_\infty)\mathcal{L}^3}{\alpha\nu}$$

for gases $\beta = 1/T_\infty, \;\; (1/K)$.

The correlation equation for UWT with a vertical plate and laminar flow is

$$\boxed{Nu_{\mathcal{L}} = \frac{0.67 \, Ra_{\mathcal{L}}^{1/4}}{[1 + (0.5/Pr)^{9/16}]^{4/9}}} \;\; \Rightarrow \;\; \begin{array}{l} \text{average, UWT,} \\ 0 \le Pr \le \infty \\ 10^4 < Gr_{\mathcal{L}} < 10^8 \end{array}$$

Other useful relations for UWT, laminar flow of air at a $T_f = 300 \; K$ and $T_f = 350 \; K$

$$h = 5.48 \left(\frac{T_w - T_\infty}{\mathcal{L} T_\infty} \right)^{1/4} \qquad \text{at} \;\; T_f = 300 \; K$$

and

$$h = 5.45 \left(\frac{T_w - T_\infty}{\mathcal{L} T_\infty} \right)^{1/4} \qquad \text{at} \quad T_f = 350 \ K$$

Note the negligible difference in the coefficients for the two film temperatures.

We can then use Newton's law of Cooling for a vertical UWT plate to find the overall heat transfer

$$\dot{Q}_{conv} = hA(T_w - T_\infty) = 5.45 \left(\frac{T_w - T_\infty}{\mathcal{L} T_\infty} \right)^{1/4} \mathcal{L} W (T_w - T_\infty)$$

2. Laminar Flow Over a Long Horizontal Circular Cylinder, Isothermal (UWT)

The general boundary layer correlation is

$$Nu_D = \frac{hD}{k_f} = C \left(\underbrace{\frac{g\beta(T_w - T_\infty)D^3}{\nu^2}}_{\equiv Gr} \right)^{1/4} \left(\underbrace{\frac{\nu}{\alpha}}_{\equiv Pr} \right)^{1/4} = C \ \underbrace{Gr_D^{1/4} \ Pr^{1/4}}_{Ra_D^{1/4}}$$

where

$$Ra_D = Gr_D Pr = \frac{g\beta(T_w - T_\infty)\mathcal{L}^3}{\alpha \nu}$$

and for ideal gases $\beta = 1/T_\infty, \ \ (1/K)$.

For a horizontal, isothermal (UWT) circular cylinder with air where $Pr = 0.71$

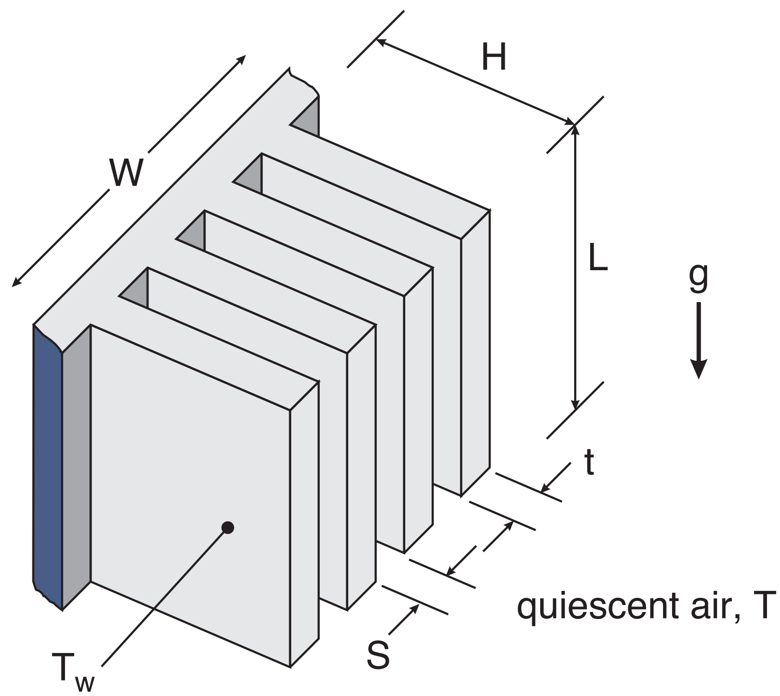
Range of Ra_D	C	n	**Flow Type**
$10^4 - 10^7$	0.54	1/4	laminar
$10^7 - 10^{11}$	0.15	1/3	turbulent

All fluid properties are evaluated at the film temperature, $T_f = (T_w + T_\infty)/2$.

Natural Convection From Plate Fin Heat Sinks

Plate fin heat sinks are often used in natural convection to increase the heat transfer surface are and in turn reduce the boundary layer resistance

$$R \downarrow = \frac{1}{hA \uparrow}$$



For a given baseplate area, $W \times L$, two factors must be considered in the selection of the number of fins

- more fins results in added surface area and reduced boundary layer resistance,

$$R \downarrow = \frac{1}{hA \uparrow}$$

- more fins results in a decrease fin spacing, S and in turn a decrease in the heat transfer coefficient

$$R \uparrow = \frac{1}{h \downarrow A}$$

A basic optimization of the fin spacing can be obtained as follows:

$$\dot{Q} = hA(T_w - T_\infty)$$

where the fins are assumed to be isothermal and the surface area is $2nHL$, with the area of the fin edges ignored.

For isothermal fins with $t < S$

$$S_{opt} = 2.714 \left(\frac{L}{Ra^{1/4}} \right)$$

with

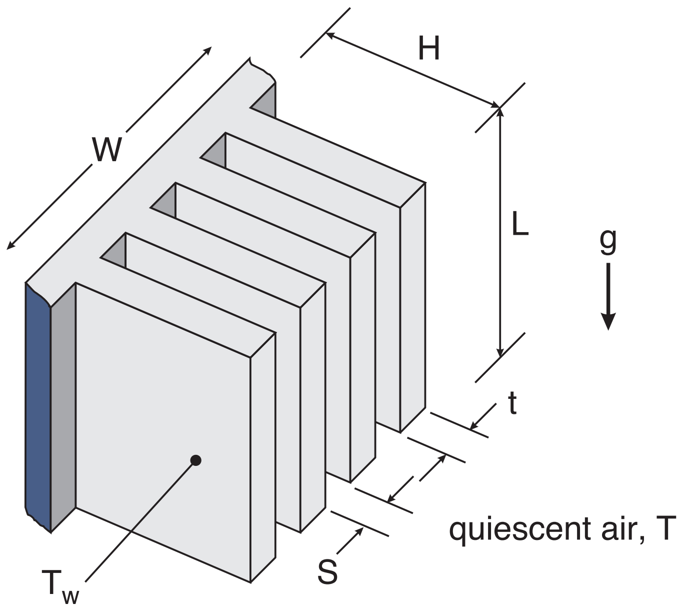
$$Ra = \frac{g\beta(T_w - T_\infty L^3)}{\nu^2} Pr$$

The corresponding value of the heat transfer coefficient is

$$h = 1.31k/S_{opt}$$

All fluid properties are evaluated at the film temperature.

Example: Plate Fin Heat Sink



Given:

$$\begin{aligned} W &= 120\, mm & H &= 24\, mm \\ L &= 18\, mm & t &= 1\, mm \\ T_w &= 80\, °C & T_\infty &= 25\, °C \\ P_\infty &= 1\, atm & \text{fluid} &= \text{air} \end{aligned}$$

Find: S_{opt} and the corresponding heat transfer, $\dot{Q}$

Solution: Assume steady state, and that air is an ideal gas $\Rightarrow \beta = 1/T_\infty$

The film temperature is given as

$$T_f = \frac{1}{2}(T_w + T_\infty) = \frac{1}{2}(80 + 25) = 52.5\, °C = 325.5\, K$$

From Table A-19 we find through interpolation

$$k = 0.0279\, W/(m \cdot K)$$

$$\nu = 1.82 \times 10^{-5}\, m^2/s$$

$$Pr = 0.709$$

$$\beta = \frac{1}{T_\infty} = \frac{1}{298\, K}$$

The Rayleigh number is

$$Ra = \frac{g\beta(T_w - T_\infty)L^3}{\nu^2} Pr = \frac{9.8 \times (80 - 25) \times 0.018^3}{298 \times (1.82 \times 10^{-5})^2} \times 0.709 = 2.258 \times 10^4$$

and the optimum fin spacing

$$S_{opt} = 2.714 \left(\frac{L}{Ra^{1/4}} \right) = 2.714 \left(\frac{0.018}{(2.258 \times 10^4)^{1/4}} \right) = 0.004\, m = 4.0\, mm$$

Since $S_{opt} = 4.0\, mm > t = \ 1\, mm$ the assumption of $t < S$ is satisfied.
The number of fins is given as

$$n = \frac{W}{S_{opt} + t} = \frac{0.12\, m}{(4.0 + 1) \times 10^{-3}\, m} \approx 24 fins$$

The heat transfer coefficient is

$$h = 1.31 \left(\frac{k}{S_{opt}} \right) = 1.31 \times \left(\frac{0.0279\, W/(m \cdot K)}{4.0 \times 10^{-3}\, m} \right) \approx 9.14\, W/(m^2 \cdot K)$$

and the total heat transfer is

$$\begin{aligned} \dot{Q} = hA(T_w - T_\infty) &= h(2nLH)(T_w - T_\infty) \\ &= 9.14 \times 2 \times 24 \times 0.018 \times 0.024 \times (80 - 25) \\ &= 10.4\, W \end{aligned}$$