University of Waterloo

ME201

ADVANCED CALCULUS
FINAL EXAMINATION

April 10, 2017 12:30 pm - 3:00 pm

Rooms: DWE 2402 & CPH 3679

Instructor: R. Culham

Name: ______________________________

Student ID Number: ______________________________

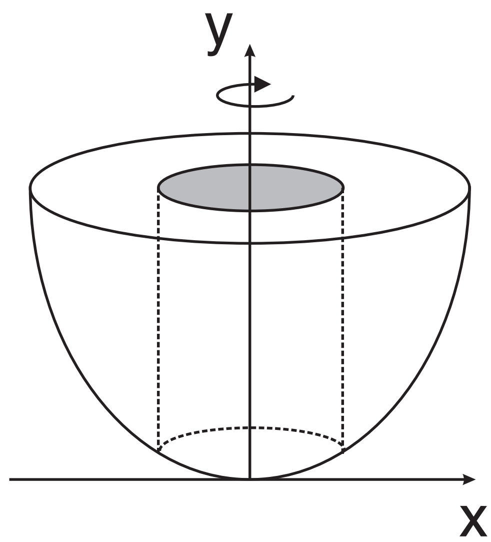
Instructions

1. This is a 2.5 hour, closed-book examination.
2. Permitted aids include:
   - one 8.5 in.$\times$ 11 in. crib sheet, (both sides)
   - Mathematical Handbook of Formulas and Tables, 4th ed., M.R. Spiegel, S. Lipschutz and J. Liu, Schaum's Outline Series, 2013.
   - calculator
3. Answer all questions in the space provided. If additional space is required, use the back of the pages or the blank pages included.
4. It is your responsibility to write clearly and legibly. Clearly state all assumptions. Part marks will be given for part answers, provided that your methodology is clear.

Question	**Marks**	**Grade**
1	9	
2	9	
3	10	
4	12	
5	15	
TOTAL	55	

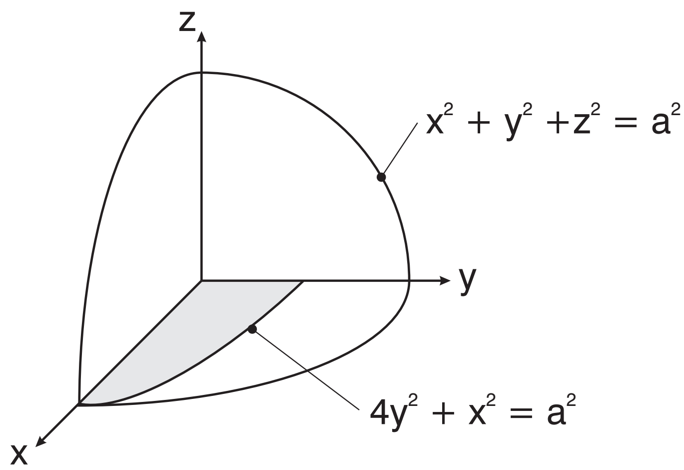
Question 1 (9 marks)

A solid is generated by revolving the region bounded by $y = x^2/4$ and $y = 4$ about the $y-$axis. A circular cylindrical hole, centered along the axis of revolution, is drilled through this solid so that 25% of the volume is removed. Find the diameter of the hole.



<u>**Question 2**</u> (9 marks)

Determine the surface area in the positive $\boldsymbol{x, y, z-}$ octant for the portion of the sphere, $\boldsymbol{x^2 + y^2 + z^2 = a^2}$, that lies inside an elliptic cylinder centered about the $\boldsymbol{z-}$axis, given as $\boldsymbol{4y^2 + x^2 = a^2}$.

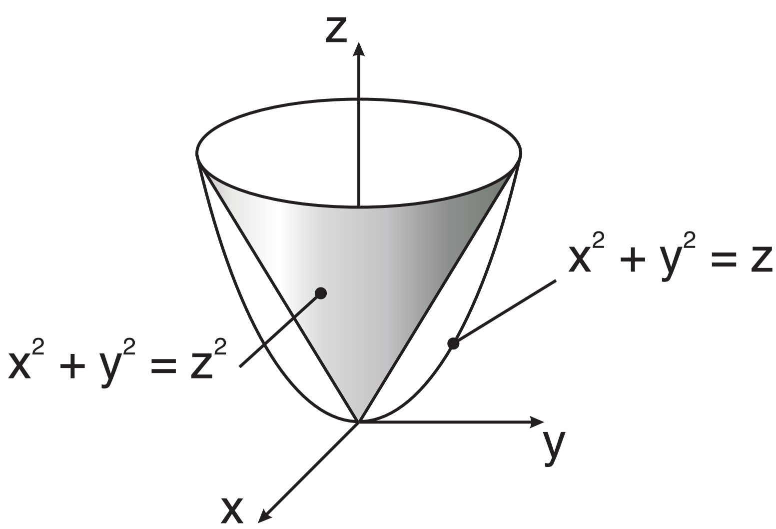


Question 3 (10 marks)

Consider the region bounded between the inside of the paraboloid, $x^2 + y^2 = z$, and the outside of the cone $x^2 + y^2 = z^2$, as shown in the figure below.

a) find the center of mass of the solid region between the paraboloid and the cone for a homogeneous solid with a density, ρ.

b) calculate the moment of inertia about the $z-$axis.



Question 4 (12 marks)

Given the vector function

$$\vec{F} = (3x^2yz)\,\hat{i} \,+\, (x^3z)\,\hat{j} \,+\, (x^3y - 4z)\,\hat{k}$$

i) determine if the field is independent of path

ii) if yes, find the scalar potential function, ϕ, such that $\vec{F} = \nabla\phi$

iii) find the work done ($Joules$) by the field if travelling along a path formed by the intersection of $y = 2x$ and $x^2 + y^2 + z^2 = 54$ from $(1, 2, 7)$ to $(3, 6, 3)$

<u>Question 5</u> (15 marks)

For the flow field

$$\vec{F} = \hat{i}(y^2) \; + \; \hat{j}(xy) \; - \; \hat{k}(2xz)$$

clearly show that Stokes' Theorem is valid for the triangular plane bounded by the sides C_1, C_2 and C_3 directed as shown in the figure below. (Show that LHS = RHS in Stokes' Theorem)