ME 201 ADVANCED CALCULUS

8 February 2006

Midterm Examination — R. Culham

- This is a two-hour, closed-book examination, where you are permitted to use:
  - Schaum's - Mathematical Handbook of Formulas and Tables
  - one 8.5 in.$\times$ 11 in. crib sheet. (one side)
- There are 6 questions to be answered. Read the questions very carefully.
- Clearly state all assumptions.
- It is your responsibility to write clearly and legibly.
- Reduce all equations to their simplest possible form.

Question 1 (11 marks)

For the curve given by

$$\vec{\boldsymbol{R}}(\boldsymbol{t}) = \hat{\boldsymbol{\imath}}(\mathbf{2}\ln \boldsymbol{t}) \; - \; \hat{\boldsymbol{\jmath}}\left(\frac{\mathbf{1}}{\boldsymbol{t}} + \boldsymbol{t}\right) \; - \; \hat{\boldsymbol{k}}$$

find:

i) velocity and acceleration vectors

ii) the angle (**radians**) between the velocity and acceleration vectors at $\boldsymbol{t} = \mathbf{1}$

iii) the unit tangent vector

iv) the principal unit normal vector

v) the length of the curve over the interval $\mathbf{1} \leq \boldsymbol{t} \leq \mathbf{2}$

Question 2 (7 marks)

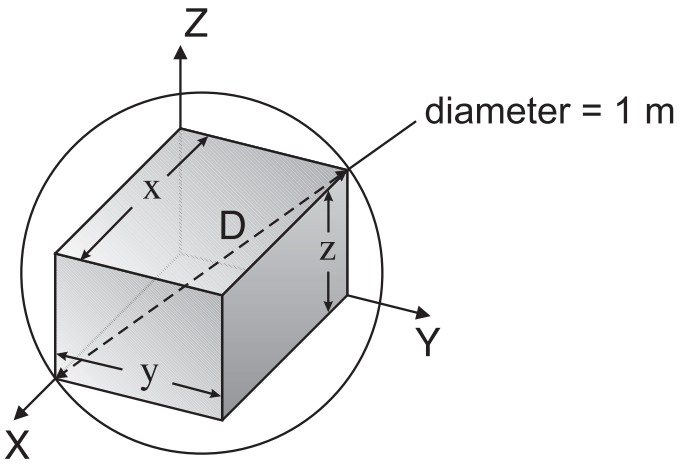
Find the curvature of the path

$$\vec{\boldsymbol{R}}(\boldsymbol{t}) = \hat{\boldsymbol{\imath}}\,(\boldsymbol{e}^{\boldsymbol{t}}\cos \boldsymbol{t}) \; + \; \hat{\boldsymbol{\jmath}}\,(\boldsymbol{e}^{\boldsymbol{t}}\sin \boldsymbol{t}) \; + \; \hat{\boldsymbol{k}}\,(\boldsymbol{e}^{\boldsymbol{t}})$$

when $\boldsymbol{t} = \mathbf{0}$.

Question 3 (11 marks)

Find the dimensions $(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z})$ of a rectangular box of maximum volume that can be contained within a sphere with a diameter of $\boldsymbol{D = 1\ m}$, as shown below. Clearly demonstrate that this is the maximum volume. Note: the diagonal of the box will equal the diameter of the sphere.



Question 4 (9 marks)

Given the following two equations:

$$\boldsymbol{x^2 + y^2 - z^2 + 2xy} \;=\; \boldsymbol{1}$$

$$\boldsymbol{x^3 + y^3 - 5y} \;=\; \boldsymbol{4}$$

use the chain rule to find $\dfrac{\boldsymbol{dz}}{\boldsymbol{dx}}$.

Question 5 (6 marks)

Find the distance between the planes:

$$\boldsymbol{x - 2y + 2z} \;=\; \boldsymbol{5}$$

$$\boldsymbol{3x - 6y + 6z} \;=\; \boldsymbol{30}$$

Question 6 (11 marks)

Given the density function

$$\boldsymbol{f(x,y) = 6 - x^2y - 3xy^2}$$

i) Find the maximum rate of density change at $\boldsymbol{(2,-1)}$.

ii) Find the direction(s) of no density change at $\boldsymbol{(2,-1)}$.

iii) Find the rate of density change at $\boldsymbol{(2,-1)}$ away from the origin.

iv) Find the equation of the tangent to the graph $\boldsymbol{z = f(x,y)}$ at the point $\boldsymbol{x = 2,\ y = -1}$.