University of Waterloo

ME201

ADVANCED CALCULUS
MIDTERM EXAMINATION

February 14, 2017 8:30 am - 10:30 am

Instructor: R. Culham

Name: ______________________________

Student ID Number: ______________________________

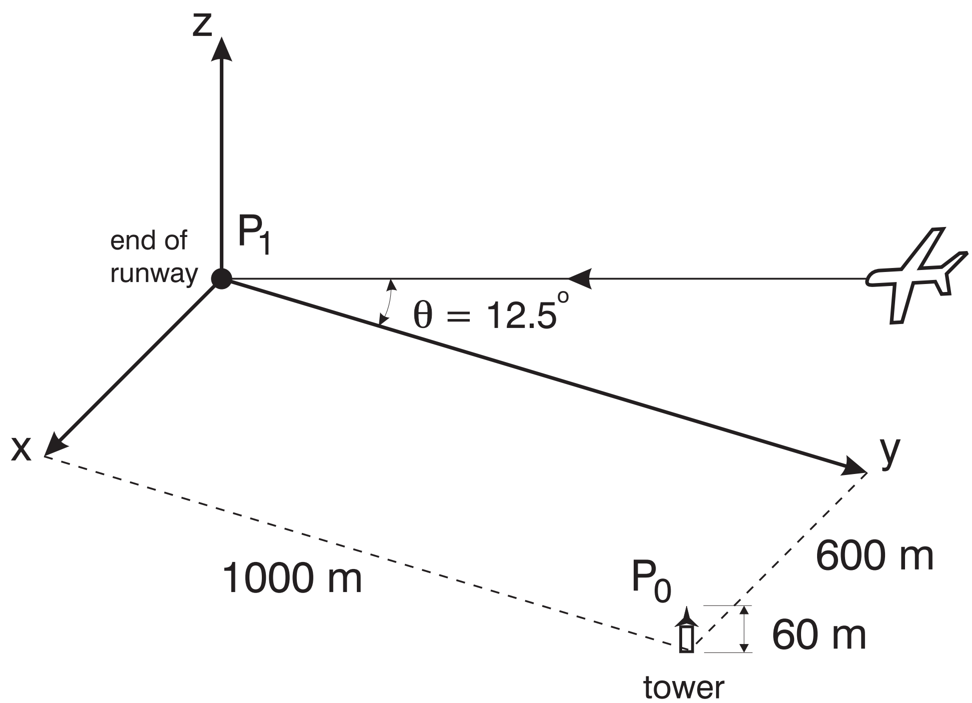
Instructions

1. This is a 2 hour, closed-book examination.
2. Permitted aids include:
   - one 8.5 in. × 11 in. crib sheet, (both sides)
   - Mathematical Handbook of Formulas and Tables, 4th ed., M.R. Spiegel, S. Lipschutz and J. Liu, Schaum's Outline Series, 2013.
   - calculator
3. Answer all questions in the space provided. If additional space is required, use the back of the pages or the blank pages included.
4. It is your responsibility to write clearly and legibly. Clearly state all assumptions. Part marks will be given for part answers, provided that your methodology is clear.

Question	**Marks**	**Grade**
1	8	
2	15	
3	8	
4	14	
5	10	
TOTAL	55	

<u>**Question 1**</u> (8 marks)

An airplane approaches the end of a runway from the east with an angle of descent of $\mathbf{12.5^\circ}$. A tower that is **60** $\boldsymbol{m}$ high is located **600** $\boldsymbol{m}$ south and **1000** $\boldsymbol{m}$ east of the end of the runway. How close does the plane come to a warning light that is at the top of the control tower?



<u>**Question 2**</u> (15 marks)

For the curve **C** formed by the intersection of $\boldsymbol{z = x^2 + 3y^2}$ and $\boldsymbol{y = x - 1}$:

(a) Express **C** in vector notation if the curve runs from $\mathbf{(0, -1, 3)}$ to $\mathbf{(1, 0, 1)}$

(b) Find the unit tangent vector to **C** at $\left(\frac{1}{2}, -\frac{1}{2}, 1\right)$

(c) Find the radius of curvature of **C** at $\left(\frac{1}{2}, -\frac{1}{2}, 1\right)$

(d) What is the unit tangent vector of **C** and the radius of curvature at $\left(\frac{1}{2}, -\frac{1}{2}, 1\right)$ if the curve, instead runs from $\mathbf{(1, 0, 1)}$ to $\mathbf{(0, -1, 3)}$

<u>Question 3</u> (8 marks)

Use the chain rule to show that the partial differential equation

$$\frac{\partial^2 y}{\partial u \, \partial v} = 0$$

can be transformed into the one dimensional wave equation i.e. a vibrating string, which takes the following form

$$\frac{\partial^2 y}{\partial t^2} = 4 \, \frac{\partial^2 y}{\partial x^2}$$

where the y is the amplitude of vibration, x is the position variable along the string given as $x = \frac{1}{2}(v + u)$ and t is the time given as $t = \frac{1}{4}(v - u)$.

Question 4 (14 marks)

The local air temperature field $T(x,y,z)$ of the air in a room is given by

$$T(x,y,z) = 10 + x^2 + y^2 + z \qquad (^\circ C)$$

where the origin of the co-ordinate system is one of the lower corners of the room. An insect is situated at $(1,1,1)$.

a) In which direction should the insect fly in order to warm up most rapidly? Specify the direction by a unit vector.
b) If the insect flies in this direction, at what rate will the air temperature seen by the insect increase? This increase should be specified in $^\circ C/m$.
c) In which direction should the insect fly in order to remain at a constant temperature?
d) Suppose instead the insect flew in the direction of vector $\vec{V} = \hat{\imath} - 3\hat{\jmath} - 5\hat{k}$. At what rate, in $^\circ C/m$, will the temperature seen by the insect change then?

Question 5 (10 marks)

Find the point(s) on the surface of the elliptic cone $\boldsymbol{3x^2 + 2y^2 = z^2}$ that are closest to the point $\boldsymbol{(1, 1, 0)}$. Hint: Consider locating a sphere of radius, $\boldsymbol{r}$, at point $\boldsymbol{(1, 1, 0)}$.