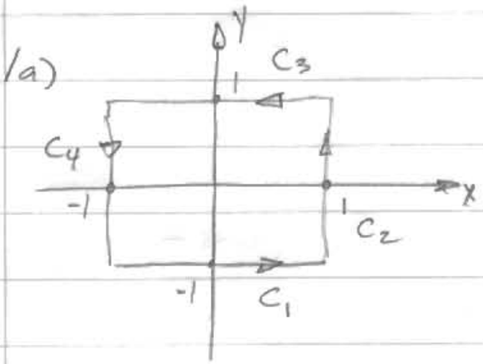
ASSIGNMENT #10 SOLUTIONS.

1a)



$$\oint_{ccw} (x^2+y^2)\,dS$$

parametric equations

C_1 : $y=-1 \quad x=t \quad -1 \le t \le 1$

C_2 : $x=1 \quad y=t \quad -1 \le t \le 1$

C_3 : $y=1 \quad x=-t \quad -1 \le t \le 1$

C_4 : $x=-1 \quad y=-t \quad -1 \le t \le 1$

$$\oint_{ccw} f\,dS = \int_{C_1} f\,dS + \int_{C_2} f\,dS + \int_{C_3} f\,dS + \int_{C_4} f\,dS$$

C_1: $\frac{dx}{dt} = 1 \quad \frac{dy}{dt} = 0 \qquad \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} = \sqrt{(1)^2} = 1$

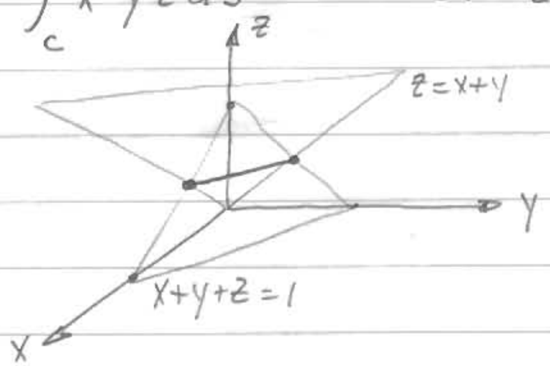
$$\int_{C_1} f\,dS = \int_{-1}^{1} t^2+1\,dS = \frac{t^3}{3} + t \Big|_{-1}^{1} = \frac{1}{3}+1-\left(-\frac{1}{3}-1\right) = \frac{8}{3}$$

C_2: $\frac{dy}{dt} = 1 \quad \frac{dx}{dt} = 0 \qquad \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} = \sqrt{(1)^2} = 1$

$$\int_{C_2} f\,dS = \int_{-1}^{1} 1+t^2\,dS = \frac{t^3}{3} + t \Big|_{-1}^{1} = \frac{8}{3}$$

C_3, C_4 same as C_1, C_2 $\quad \therefore \oint_{ccw} (x^2+y^2)\,dS = 4\left(\frac{8}{3}\right) = \frac{32}{3}$

b) $\int_C x^2yz\,ds \qquad C: \; z = x+y \quad x+y+z=1$



① form parametric equations

let $\boxed{x=-t}$ ← since x goes from 1 to -3

↳ substitute into plane equations

$$z - y = -t$$
$$z + y = 1 + t$$
$$2z = 1$$
$$\boxed{z = 1/2}$$

$$\boxed{y = z - x = \frac{1}{2} + t}$$
$$\boxed{-1 \le t \le 3}$$

② Substitute into integrand.

$$x^2yz = (t^2)\left(\frac{1}{2}+t\right)\left(\frac{1}{2}\right) = \frac{t^2}{4} + \frac{t^3}{2}$$

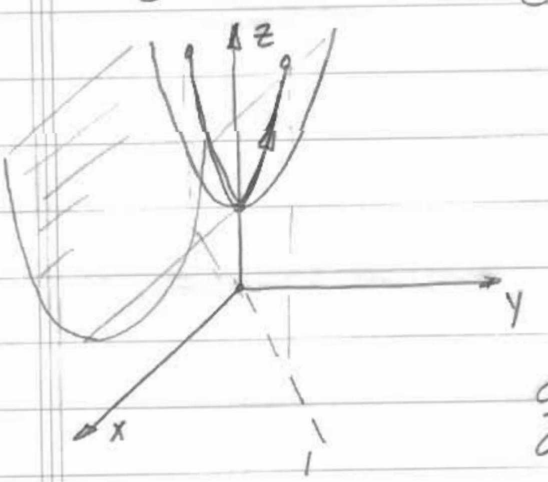
$$\frac{dx}{dt} = -1 \qquad \frac{dy}{dt} = 1 \qquad \frac{dz}{dt} = 0$$

③ form integral
$$\int_C f\,ds = \int_{-1}^{3} \frac{t^2}{4} + \frac{t^3}{2} \sqrt{(-1)^2+(1)^2}\, ds$$

$$= \sqrt{2}\left[\frac{t^3}{12} + \frac{t^4}{8}\right]\Big|_{-1}^{3} = \sqrt{2}\left[\frac{27}{12} + \frac{81}{8} + \frac{1}{12} - \frac{1}{8}\right]$$
$$= \frac{37\sqrt{2}}{3}$$

c) $\int_C (x+y)z\,dS$

C: $y = x$
$z = 1 + y^4$ let $x = t$ $-1 \le t \le 1$
$y = t$
$z = 1 + t^4$



$$(x+y)z = 2t(1+t^4) = 2t + 2t^5$$

$$\frac{dx}{dt} = 1 \qquad \frac{dy}{dt} = 1 \qquad \frac{dz}{dt} = 4t^3$$

$$\int_C f\,dS = \int_{-1}^{1} 2t + 2t^5 \sqrt{2 + 16t^6}\, dt$$

$$= 2\sqrt{2}\int_{-1}^{1} t\sqrt{1+8t^6}\,dt + 2\sqrt{2}\int_{-1}^{1} t^5\sqrt{1+8t^6}\,dt$$

2) $\bar{f} = \frac{1}{L} \int_C f\,dS$ C: $z = x^2$

$y = x^2$

let $x = t$ $y = t^2$ $0 \le t \le 1$

$z = t^2$

$f = xyz = (t)(t^2)(t^2) = t^5$

$\frac{dx}{dt} = 1$ $\frac{dy}{dt} = 2t$ $\frac{dz}{dt} = 2t$

$$\int_C f\,ds = \int_0^1 t^5 \sqrt{1+4t^2+4t^2}\,dt = \int_0^1 t^5\sqrt{1+8t^2}\,dt$$

substitution $u = 1+8t^2$ $t^2 = \left(\frac{u-1}{8}\right)$ $t^4 = \left(\frac{u-1}{8}\right)^2$

$du = 16t\,dt$

$t\,dt = \frac{du}{16}$

$$\int_0^1 t^5\sqrt{1+8t^2}\,dt = \int_1^9 \left(\frac{u-1}{8}\right)^2 \sqrt{u}\,\frac{du}{16}$$

$$= \frac{1}{1024}\int_1^9 u^{5/2} - 2u^{3/2} + u^{1/2}\,du$$

$$= \frac{1}{1024}\left[\frac{2}{7}u^{7/2} - \frac{4}{5}u^{5/2} + \frac{2}{3}u^{3/2}\right]\Bigg|_1^9 = \frac{1471}{3360}$$

$$L = \int_0^1 \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2}\,dt = \int_0^1 \sqrt{1+8t^2}\,dt$$

$$= 2\sqrt{2}\int_0^1 \sqrt{\frac{1}{8}+t^2}\,dt$$

<u>Schaums</u> $\int \sqrt{a^2+x^2}\,dx = \frac{x\sqrt{x^2+a^2}}{2} + \frac{a^2}{2}\ln\left(x+\sqrt{x^2+a^2}\right)$

$$L = 2\sqrt{2}\left[\frac{t\sqrt{t^2+\frac{1}{8}}}{2} + \frac{1}{16}\ln\left(t+\sqrt{t^2+\frac{1}{8}}\right)\right]\Big|_0^1$$

$$= 2\sqrt{2}\left[\frac{1}{2}\sqrt{\frac{9}{8}} + \frac{1}{16}\ln\left(1+\frac{\sqrt{9}}{\sqrt{8}}\right) - \frac{1}{16}\ln\sqrt{\frac{1}{8}}\right]$$

$$= \frac{3}{2} + \frac{\sqrt{2}}{8}\ln\left[\frac{\frac{\sqrt{8}+\sqrt{9}}{\sqrt{8}}}{\sqrt{\frac{1}{8}}}\right] = \frac{3}{2} + \frac{\sqrt{2}}{8}\ln\left(2\sqrt{2}+3\right)$$

$$\bar{f} = \frac{1}{L}\int_c f\,ds \cong 0.2417$$

3) given $T(x,y,z) = -10 + 2\sqrt{x^2+y^2+z}$

$$\bar{T} = \frac{1}{L}\int_c T\,ds \qquad C: \begin{array}{l} z = x^2 \\ y = 2x \end{array} \quad \text{from } (0,0,0) \text{ to } (3,6,9)$$

① parametric equations
$$\left.\begin{array}{l} x = t \\ y = 2t \\ z = t^2 \end{array}\right\} \quad 0 \le t \le 3$$

② substitute into integrand $\quad T = -10 + 2\sqrt{t^2+4t^2+t^2} = -10 + 2\sqrt{6}\,t$

$$\frac{dx}{dt} = 1 \qquad \frac{dy}{dt} = 2 \qquad \frac{dz}{dt} = 2t$$

$$\int_c T\,dS = \int_0^3 \left(-10 + 2\sqrt{6}\,t\right)\sqrt{5+4t^2}\,dt$$

$$= \underbrace{-20\int_0^3 \sqrt{\frac{5}{4}+t^2}\,dt}_{①} + \underbrace{4\sqrt{6}\int_0^3 t\sqrt{\frac{5}{4}+t^2}\,dt}_{②}$$

① from Schaums $\int \sqrt{x^2+a^2}\,dx = \frac{x\sqrt{x^2+a^2}}{2} + \frac{a^2}{2}\ln\left(x+\sqrt{x^2+a^2}\right)$

$$= -20\left[\frac{t\sqrt{5/4+t^2}}{2} + \frac{5}{8}\ln\left(t+\sqrt{5/4+t^2}\right)\right]\Big|_0^3$$

$$= -30\sqrt{\frac{5}{4}+9} + \frac{5}{8}\ln\left(3+\sqrt{\frac{5}{4}+9}\right) - \frac{5}{8}\ln\sqrt{\frac{5}{4}} \cong -117.46$$

② Substitute $u = \frac{5}{4}+t^2$

$du = 2t\,dt \quad t\,dt = \frac{du}{2}$

$$\frac{1}{2}\int u^{1/2}\,du = \frac{1}{3}u^{3/2}$$

$$4\sqrt{6}\int_0^3 t\sqrt{\frac{5}{4}+t^2}\,dt = \frac{4\sqrt{6}}{3}\left(\frac{5}{4}+t^2\right)^{3/2}\Big|_0^3 = \frac{4\sqrt{6}}{3}\left[\left(\frac{5}{4}+9\right)^{3/2} - \left(\frac{5}{4}\right)^{3/2}\right]$$

$$\cong 102.61$$

$$\int_c T\,dS \cong -14.85$$

$$L = \int_0^3 \sqrt{\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2+\left(\frac{dz}{dt}\right)^2}\,dt = \int_0^3 \sqrt{5+4t^2}\,dt$$

$$= 2\int_0^3 \sqrt{\frac{5}{4}+t^2}\,dt$$

from above $= 2\left[\frac{t\sqrt{5/4+t^2}}{2} + \frac{5}{8}\ln\left(t+\sqrt{\frac{5}{4}+t^2}\right)\right]\Big|_0^3$

$L \cong 11.74$ $\quad \bar{T} = \frac{1}{L}\int_c T\,ds = \frac{-14.85}{11.74} \cong -1.265$

$\therefore$ since $\bar{T} < 0$, ice will form on wings

4a) $\int_C x\,dx + yz\,dy + x^2\,dz$ $\quad$ $C: \ y = x, \ z = x^2$ $\quad$ $(-1,-1,1)$ to $(2,2,4)$

① parametric equations $\quad x = t, \quad y = t, \quad z = t^2 \qquad -1 \le t \le 2$

② $\dfrac{dx}{dt} = 1 \quad \dfrac{dy}{dt} = 1 \quad \dfrac{dz}{dt} = 2t$

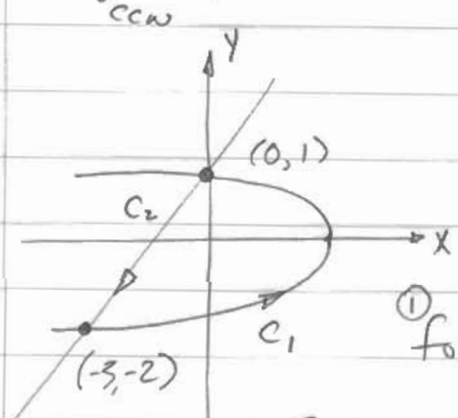
$$\int_C \vec{F}\cdot d\vec{R} = \int_{-1}^{2} (t)(1) + (t)(t^2)(1) + (t)^2(2t)\ dt$$

$$= \int_{-1}^{2} t + 3t^3\,dt = \frac{t^2}{2} + \frac{3t^4}{4}\Big|_{-1}^{2}$$

$$= 2 + 12 - \frac{1}{2} - \frac{3}{4} = \frac{51}{4}$$

b) $\oint_{ccw} x^2y\,dx + (x-y)\,dy$

$C: \ x = 1-y^2, \ y = x+1$



intersection points $\quad x = 1-y^2 = y-1$

$y^2 + y - 2 = 0$

$(y+2)(y-1) = 0$

$y = 1, -2$

$x = 0, -3$

① form parametric eqs.

$C_1: \ y = t, \quad x = 1-t^2 \qquad -2 \le t \le 1$

$C_2: \ x = -t, \quad y = 1-t \qquad 0 \le t \le 3$

② Substitute into integrand

C_1: $(x^2y)\left(\frac{dx}{dt}\right) + (x-y)\left(\frac{dy}{dt}\right) = (1-t^2)^2(t)(-2t) + (1-t^2-t)(1)$

$= -2t^2 + 4t^4 - 2t^6 + 1 - t^2 - t = -2t^6 + 4t^4 - 3t^2 - t + 1$

③ integrate

$$\int_{C_1} \vec{F} \cdot d\vec{R} = \int_{-2}^{1} -2t^6 + 4t^4 - 3t^2 - t + 1 \, dt$$

$$= -\frac{2}{7}t^7 + \frac{4}{5}t^5 - t^3 - \frac{t^2}{2} + t \Big|_{-2}^{1} = -\frac{1047}{70}$$

repeat for C_2

C_2: $= (t^2)(1-t)(-1) + (-t-1+t)(-1) = t^3 - t^2 + 1$

$$\int_{C_2} \vec{F} \cdot d\vec{R} = \int_0^3 t^3 - t^2 + 1 \, dt = \frac{t^4}{4} - \frac{t^3}{3} + t \Big|_0^3 = \frac{81}{4} - \frac{27}{3} + 3$$

$$= \frac{57}{4}$$

$$\therefore \oint_{ccw} \vec{F} \cdot d\vec{R} = -\frac{1047}{70} + \frac{57}{4} = -\frac{99}{140}.$$

5a) $W = \int_C \vec{F} \cdot d\vec{R}$ $\qquad$ C: straight line $(1,0)$ to $(6,5)$

$x = 1 + t$

$y = t$ $\qquad 0 \le t \le 5$

$$\int_C \vec{F} \cdot d\vec{R} = \int_C (x^2y)\frac{dx}{dt} + x\frac{dy}{dt} \, dt$$

$$= \int_0^5 (1+t)^2 t + 1 + t \, dt = \int_0^5 t^3 + 2t^2 + 2t + 1 \, dt$$

$$= \frac{t^4}{4} + \frac{2t^3}{3} + t^2 + t \Big|_0^5 = \frac{625}{4} + \frac{250}{3} + 25 + 5$$

$$= \frac{3235}{12}$$

5b) $W = \oint_{CCW} \vec{F} \cdot d\vec{R}$ $\quad C: b^2x^2 + a^2y^2 = a^2b^2$

let $x = a\cos t$, $y = b\sin t$ $\Big\}$ $b^2a^2\cos^2 t + a^2b^2\sin^2 t = a^2b^2$

$0 \le t \le 2\pi$

$$W = \int_0^{2\pi} (a\cos t)(-a\sin t) + (b\sin t)(b\cos t)\,dt$$

$$= (b^2 - a^2)\int_0^{2\pi} \sin t \cos t\,dt \qquad u = \sin t,\ du = \cos t\,dt$$

$$= (b^2 - a^2)\left.\frac{\sin^2 t}{2}\right|_0^{2\pi} = 0$$

6a) $\vec{F} = (3x^2yz,\ x^3z,\ x^3y - 4z)$ $\quad$ check $\nabla \times \vec{F}$

$$\nabla \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ 3x^2yz & x^3z & x^3y - 4z \end{vmatrix} = (x^3 - x^3)\hat{i} - (3x^2y - 3x^2y)\hat{j} + (3x^2z - 3x^2z)\hat{k} = 0$$

Since $\nabla \times \vec{F} = 0$, $\vec{F}$ is conservative, independent of path

$\vec{F} = \nabla\phi$

$$\frac{\partial\phi}{\partial x} = 3x^2yz \qquad \phi = x^3yz + f(y,z) + C$$

$$\frac{\partial\phi}{\partial y} = x^3z \qquad \phi = x^3yz + f(x,z) + C$$

$$\frac{\partial\phi}{\partial z} = x^3y - 4z \qquad \phi = x^3yz - 2z^2 + f(x,y) + C$$

$$\phi = x^3yz - 2z^2 + C$$

$\phi(B) - \phi(A) = \phi(1,1,-1) - \phi(-1,-1,1)$

$= (1)^3(1)(-1) - 2(-1)^2 - (-1)^3(-1)(1) + 2(1)^2 = -1-1 = -2$

6b) $\vec{F} = (y\cos x, \sin x)$

$$\nabla \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ y\cos x & \sin x & 0 \end{vmatrix} = (\cos x - \cos x)\hat{k} = 0$$

since $\nabla \times \vec{F} = 0$
$\vec{F}$ is conservative, line integral is independent of path

by definition $\oint \vec{F} \cdot d\vec{R} = 0$ when $\vec{F}$ conservative

6c) $\vec{F} = (3x^2y^3, 3x^3y^2)$

$$\nabla \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ 3x^2y^3 & 3x^3y^2 & 0 \end{vmatrix} = (9x^2y^2 - 9x^2y^2)\hat{k}$$

$= 0$

since $\nabla \times \vec{F} = 0$
$\vec{F}$ is conservative, line integral is independent of path

$\vec{F} = \nabla \phi$

$\frac{\partial \phi}{\partial x} = 3x^2y^3 \quad \phi = x^3y^3 + f(y) + C$

$\frac{\partial \phi}{\partial y} = 3x^3y^2 \quad \phi = x^3y^3 + f(x) + C$

$\therefore \phi = x^3y^3 + C$

$\phi(B) - \phi(A) = (1)^3(e)^3 - (0) = e^3$

d) $\vec{F} = (y\tan x + xy\sec^2 x,\ x\tan x,\ 1)$

$$\nabla \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ y\tan x + xy\sec^2 x & x\tan x & 1 \end{vmatrix}$$

$$= (0-0)\hat{i} - (0-0)\hat{j} + (\tan x + x\sec^2 x - \tan x - x\sec^2 x)\hat{k}$$

$\nabla \times \vec{F} = 0 \quad \therefore \vec{F}$ is conservative and $\oint_{cw} \vec{F}\cdot d\vec{R} = 0$

7) $\phi = 3x^2y - y^4 + x^3$ for $\vec{F} = \nabla\phi$

from $A(0,0)$ to $B(2,4)$ $\rightarrow$ Path #1 $\quad x = t \quad 0 \le t \le 2$
$y = 2t$

$$\vec{F} = \nabla\phi = (6xy + 3x^2)\hat{i} + (3x^2 - 4y^3)\hat{j}$$

$$\int_C \vec{F}\cdot d\vec{R} = \int_0^2 (6t(2t) + 3t^2)(1) + (3t^2 - 32t^3)(2)\ dt$$

$$= \int_0^2 21t^2 - 64t^3\,dt = 7t^3 - 16t^4 \Big|_0^2 = -200$$

$\rightarrow$ Path 2 $\quad x = t \quad 0 \le t \le 2$
$y = t^2$

$$\int_C \vec{F}\cdot d\vec{R} = \int_0^2 (6t^3 + 3t^2) + (3t^2 - 4t^6)(2t)\ dt$$

$$= \int_0^2 3t^2 + 12t^3 - 8t^7\,dt$$

$$= t^3 + 3t^4 - t^8 \Big|_0^2 = 8 + 48 - 256 = -200$$

Check.

$$\int_C \vec{F}\cdot d\vec{R} = \phi(B) - \phi(A) = 3(4)(4) - (4)^4 + (2)^3 = -200$$

proves path independence

8a) $\vec{F} = mx\,\hat{i} + xy\,\hat{j}$ $\quad \nabla \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ mx & xy & 0 \end{vmatrix}$

$\nabla \times \vec{F} = 0\hat{i} - 0\hat{j} + (y-0)\hat{k} = y\hat{k}$

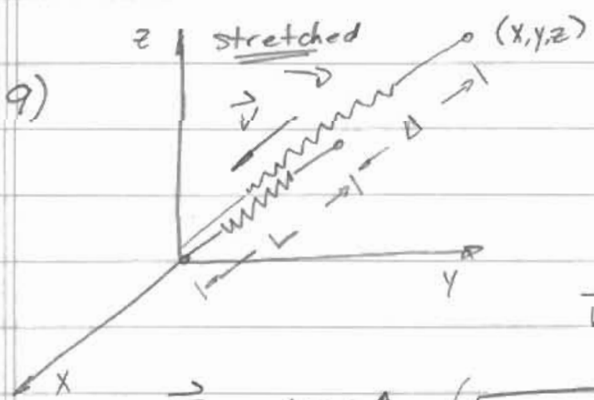
Since $\nabla \times \vec{F} \neq 0$ force field <u>not</u> conservative,
no potential function exists.

b) $\vec{F} = -mg\hat{k}$ $\quad \nabla \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ 0 & 0 & -mg \end{vmatrix} = 0$

$\vec{F}$ is conservative force field
potential function ϕ from $\vec{F} = \nabla\phi$

$$\left.\begin{aligned} \frac{\partial \phi}{\partial x} &= 0 & \phi &= f(y,z) \\ \frac{\partial \phi}{\partial y} &= 0 & \phi &= f(x,z) \\ \frac{\partial \phi}{\partial z} &= -mg & \phi &= -mgz + f(x,y) \end{aligned}\right\} \phi = -mgz$$

9)



Spring $|F| = k\Delta$

$\Delta = \sqrt{x^2+y^2+z^2} - L$

$\vec{F}$ points towards origin

$\vec{v} = -\vec{R} = -(x,y,z)$

$$\vec{F} = |F|\,\hat{v} = \left[\sqrt{x^2+y^2+z^2} - L\right]\left(\frac{-(x,y,z)}{\sqrt{x^2+y^2+z^2}}\right)$$

$$P = \frac{x\left(L - \sqrt{x^2+y^2+z^2}\right)}{\sqrt{x^2+y^2+z^2}} \quad Q = \frac{y\left(L - \sqrt{x^2+y^2+z^2}\right)}{\sqrt{x^2+y^2+z^2}} \quad R = \frac{z\left(L - \sqrt{x^2+y^2+z^2}\right)}{\sqrt{x^2+y^2+z^2}}$$

$$\nabla \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ P & Q & R \end{vmatrix} = \left(\frac{\partial R}{\partial y} - \frac{\partial Q}{\partial z}\right)\hat{i} - \left(\frac{\partial R}{\partial x} - \frac{\partial P}{\partial z}\right)\hat{j} + \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right)\hat{k}$$

$$\frac{\partial R}{\partial y} = \frac{\partial}{\partial y}\left(\frac{zL}{\sqrt{x^2+y^2+z^2}}\right) - \frac{\partial z}{\partial y} = (zL)\left(-\frac{1}{2}\frac{2y}{(x^2+y^2+z^2)^{3/2}}\right)$$

$$= \frac{-yzL}{(x^2+y^2+z^2)^{3/2}}$$

$$\frac{\partial Q}{\partial z} = \frac{\partial}{\partial z}\left(\frac{yL}{\sqrt{x^2+y^2+z^2}}\right) - \frac{\partial y}{\partial z} = (yL)\left(\frac{-1}{2}\cdot\frac{2z}{(x^2+y^2+z^2)^{3/2}}\right)$$

$$= \frac{-yzL}{(x^2+y^2+z^2)^{3/2}}$$

$$\left(\frac{\partial R}{\partial y} - \frac{\partial Q}{\partial z}\right)\hat{i} = 0$$

from symmetry, remaining components of $\nabla \times \vec{F}$ will also be 0 and $\vec{F}$ is conservative

10a) $\vec{F} = \left(\frac{-y}{z}\sin x, \frac{\cos x}{z}, \frac{-y}{z^2}\cos x\right)$

check for path independence, $\nabla \times \vec{F} = 0$

$$\nabla \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ \frac{-y}{z}\sin x & \frac{\cos x}{z} & \frac{-y}{z^2}\cos x \end{vmatrix} = \left(\frac{-\cos x}{z^2} - \left(\frac{-\cos x}{z^2}\right)\right)\hat{i} - \left(\frac{y}{z^2}\sin x - \frac{y}{z^2}\sin x\right)\hat{j}$$

$$+ \left(\frac{-\sin x}{z} - \left(\frac{-\sin x}{z}\right)\right)\hat{k} = 0$$

$\therefore$ $\vec{F}$ conservative, line integral independent of path.

if $\vec{F} = \nabla\phi$

$$\frac{\partial\phi}{\partial x} = -\frac{y}{z}\sin x \qquad \phi = \frac{y}{z}\cos x + f(y,z) + C$$

$$\frac{\partial\phi}{\partial y} = \frac{\cos x}{z} \qquad \phi = \frac{y}{z}\cos x + f(x,z) + C$$

$$\frac{\partial\phi}{\partial z} = -\frac{y}{z^2}\cos x \qquad \phi = \frac{y}{z}\cos x + f(x,y) + C$$

$$\phi = \frac{y}{z}\cos x$$

$$\int_C \vec{F}\cdot d\vec{R} = \phi(B) - \phi(A) = \frac{\sqrt{2}}{4\pi}\cos\sqrt{2} - \frac{\sqrt{2}}{2\pi}\cos\sqrt{2}$$

$$= -\frac{\sqrt{2}}{4\pi}\cos\sqrt{2} \cong -0.0175$$

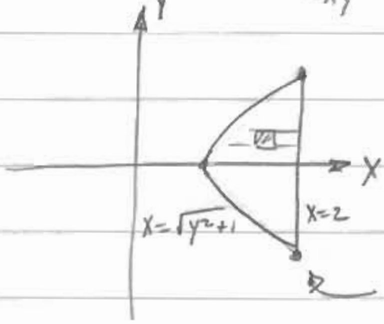
10b) since $\vec{F}$ conservative, line integral is independent of path, $\therefore \int_C \vec{F}\cdot d\vec{R} \cong -0.0175$

c) Since $\vec{F}$ conservative $\oint \vec{F}\cdot d\vec{R} = 0$

11a) $\oint_{ccw} xy^3\,dx + x^2\,dy \qquad P = xy^3 \quad \partial P/\partial y = 3xy^2$

$\qquad Q = x^2 \quad \partial Q/\partial x = 2x$

$$\oint_{ccw} \vec{F}\cdot d\vec{R} = \iint_{R_{xy}} (2x - 3xy^2)\,dA$$



C: $x = \sqrt{y^2+1}$

$x = 2$

intersection points $\quad x = 2 = \sqrt{y^2+1}$

$y^2 = 3 \quad y = \pm\sqrt{3}$

$$= \int_{-\sqrt{3}}^{\sqrt{3}} \int_{\sqrt{y^2+1}}^{2} 2x - 3xy^2 \, dx\, dy = \int_{-\sqrt{3}}^{\sqrt{3}} x^2 - \frac{3x^2y^2}{2} \Big|_{\sqrt{y^2+1}}^{2} dy$$

$$= \int_{-\sqrt{3}}^{\sqrt{3}} 4 - 6y^2 - y^2 - 1 + \frac{3}{2}(y^2+1)(y^2)\,dy$$

$$= \int_{-\sqrt{3}}^{\sqrt{3}} 3 - \frac{11}{2}y^2 + \frac{3}{2}y^4\,dy = 3y - \frac{11}{6}y^3 + \frac{3}{10}y^5 \Big|_{-\sqrt{3}}^{\sqrt{3}}$$

$$= 3\sqrt{3} - \frac{11(3\sqrt{3})}{6} + \frac{3(9\sqrt{3})}{10} - \left(-3\sqrt{3} + \frac{11(3\sqrt{3})}{6} - \frac{3(9\sqrt{3})}{10}\right)$$

$$= \frac{2\sqrt{3}}{5}$$

b) $\oint_{ccw} 2\tan^{-1}\left(\frac{y}{x}\right)dx + \ln(x^2+y^2)dy$ $\quad P = 2\tan^{-1}\left(\frac{y}{x}\right)$

$Q = \ln(x^2+y^2)$

$$\frac{\partial P}{\partial y} = 2\frac{\partial}{\partial y}\tan^{-1}\left(\frac{y}{x}\right) = \frac{2}{x}\cdot\frac{1}{1+\left(\frac{y}{x}\right)^2} = \frac{2}{x}\cdot\frac{x^2}{x^2+y^2} = \frac{2x}{x^2+y^2}$$

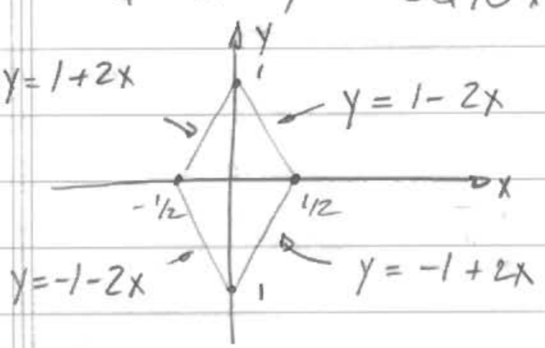
$$\frac{\partial Q}{\partial z} = \frac{2x}{x^2+y^2} \qquad \oint_{ccw} \vec{F}\cdot d\vec{R} = \iint_{Rxy}\left(\frac{2x}{x^2+y^2} - \frac{2x}{x^2+y^2}\right)dA = 0$$

$\therefore \vec{F}$ is conservative.

c) $\oint_{ccw} (x^3+y^3)\,dx + (x^3-y^3)dy \qquad \oint_{ccw} \vec{F}\cdot d\vec{R}$

$P = x^3+y^3 \quad \partial P/\partial y = 3y^2 \quad = \iint_{Rxy} 3x^2 - 3y^2\,dA$

$Q = x^3 - y^3 \quad \partial Q/\partial x = 3x^2$



$$= \int_{0}^{1/2}\int_{-1+2x}^{1-2x} 3x^2 - 3y^2\,dy\,dx$$

$$+ \int_{-1/2}^{0}\int_{-1-2x}^{1+2x} 3x^2 - 3y^2\,dy\,dx$$

$$= \int_0^{1/2} 3x^2y - y^3 \Big|_{-1+2x}^{1-2x} dx + \int_{-1/2}^{0} 3x^2y - y^3 \Big|_{-1-2x}^{1+2x} dx$$

$$= \int_0^{1/2} 3x^2(1-2x) - (1-2x)^3 - 3x^2(-1+2x) + (-1+2x)^3 \, dx$$

$$+ \int_{-1/2}^{0} 3x^2(1+2x) - (1+2x)^3 - 3x^2(-1-2x) + (-1-2x)^3 \, dx$$

$$= \int_0^{1/2} 4x^3 - 18x^2 + 12x - 2 \, dx - \int_{-1/2}^{0} 4x^3 + 18x^2 + 12x + 2$$

$$= x^4 - 6x^3 + 6x^2 - 2x \Big|_0^{1/2} - x^4 - 6x^3 - 6x^2 - 2x \Big|_{-1/2}^{0}$$

$$= -\frac{3}{16} - \frac{3}{16} = -\frac{3}{8}$$