ASSIGNMENT #1 SOLUTIONS.

1a) $\vec{w} = (4,3,-2) \qquad \vec{v} = (-2,0,4)$

$$2\vec{w} + 3\vec{v} = 2(4,3,-2) + 3(-2,0,4)$$
$$= (8,6,-4) + (-6,0,12)$$
$$= (8-6,\ 6+0,\ -4+12) = (2,6,8)$$

b) $|\vec{v}| = \sqrt{(-2)^2+(0)^2+(4)^2} = \sqrt{4+16} = \sqrt{20} = 2\sqrt{5}$

$|\vec{v}|\vec{v} - 2|\hat{v}|\vec{w}$

magnitude of unit vector $|\hat{v}| = 1$

$$2\sqrt{5}(-2,0,4) - 2(4,3,-2)$$
$$= (-4\sqrt{5}, 0, 8\sqrt{5}) - (8,6,-4)$$
$$= (-4\sqrt{5}-8,\ -6,\ 8\sqrt{5}+4)$$

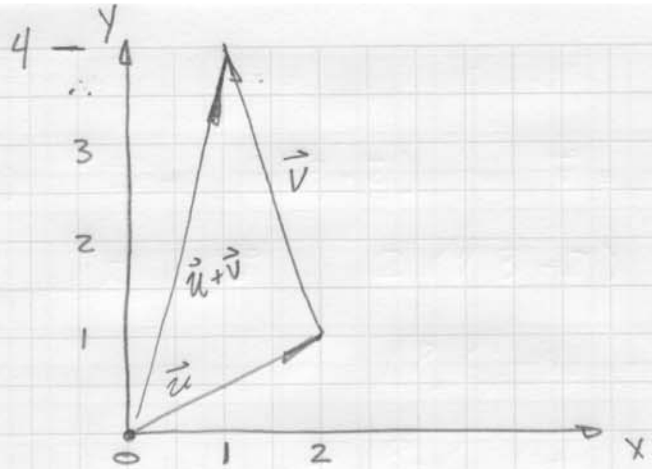
c) $\vec{v} - \vec{w} = (-2,0,4) - (4,3,-2)$
$$= (-2-4,\ 0-3,\ 4-(-2)) = (-6,-3,6)$$

$$|\vec{v}+\vec{w}| = |(-2,0,4) + (4,3,-2)|$$
$$= |(2,3,2)| = \sqrt{(2)^2+(3)^2+(2)^2}$$
$$= \sqrt{17}$$

$$\therefore \frac{\vec{v}-\vec{w}}{|\vec{v}+\vec{w}|} = \frac{(-6,-3,6)}{\sqrt{17}}$$

2a) $\vec{u} = 2\hat{i} + \hat{j} \qquad \vec{v} = -\hat{i} + 3\hat{j}$

$$\vec{u}+\vec{v} = (2-1)\hat{i} + (1+3)\hat{j}$$
$$= \hat{i} + 4\hat{j}$$



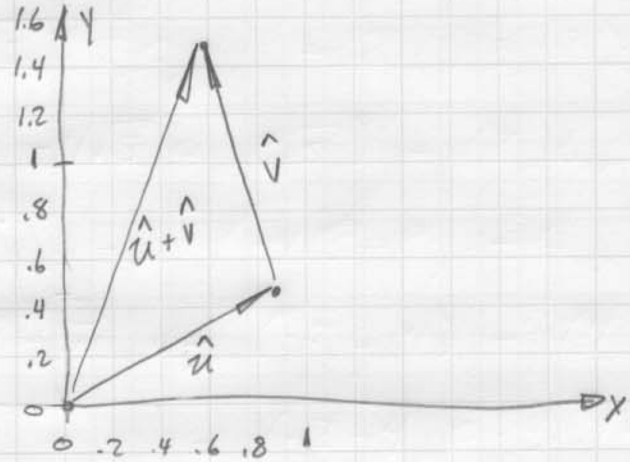
2b) $\hat{v} + \hat{u}$

$$\hat{v} = \frac{-\hat{i} + 3\hat{j}}{\sqrt{(-1)^2 + (3)^2}} = \frac{-\hat{i} + 3\hat{j}}{\sqrt{10}}$$

$$\hat{u} = \frac{2\hat{i} + \hat{j}}{\sqrt{(2)^2 + (1)^2}} = \frac{2\hat{i} + \hat{j}}{\sqrt{5}}$$

$$\therefore\ \hat{v} + \hat{u} = \left(\frac{-1}{\sqrt{10}} + \frac{2}{\sqrt{5}}\right)\hat{i} + \left(\frac{3}{\sqrt{10}} + \frac{1}{\sqrt{5}}\right)\hat{j}$$

$\hat{v} \approx -0.316\,\hat{i} + 0.949\,\hat{j}$ $\quad \hat{u} \approx 0.894\,\hat{i} + 0.447\,\hat{j}$



3) $\overrightarrow{PQ} = (0-3,\ 1-2,\ 4-(-1)) = (-3, -1, 5)$

find S such that $\overrightarrow{RS} = \overrightarrow{PQ}$

& components equal

$\therefore$ if $S = (x, y, z)$ then

$\overrightarrow{RS} = (x-6,\ y-5,\ z-(-2)) = (-3, -1, 5)$

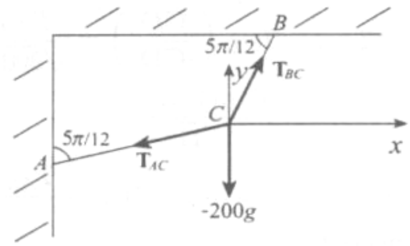
$$\begin{aligned} \therefore\ x-6 &= -3 & x &= 3 \\ y-5 &= -1 & y &= 4 \\ z+2 &= 5 & z &= 3 \end{aligned} \qquad \therefore\ S = (3, 4, 3)$$

4) Section 12.3 (Trim, 2nd edition)

34. Let T_{AC} and T_{BC} be tensions in cables AC and BC. For equilibrium when both cables are taut, x- a y-components of all forces acting at C must sum to zero:

$$0 = T_{BC}\cos 5\pi/12 - T_{AC}\sin 5\pi/12,$$
$$0 = T_{BC}\sin 5\pi/12 - T_{AC}\cos 5\pi/12 - 200g,$$

where $g = 9.81$. When these are solved for T_{AC} and T_{BC}, the result, (to the nearest newton), is $T_{AC} = 586$ N and $T_{BC} = 2188$ N.



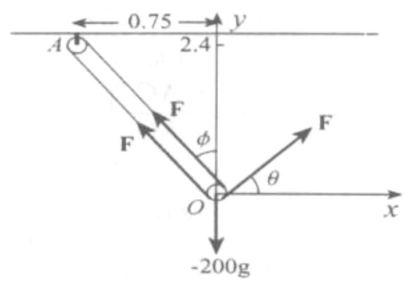
37. Assuming that there is no friction in the pulleys, the tension is the same at all points in the rope. For equilibrium, the sum of the x- and y-components of all forces acting on the pulley at O must be zero. If we assume that the two ropes from O to A are parallel, then

$$0 = |\mathbf{F}|\cos\theta - 2|\mathbf{F}|\sin\phi,$$
$$0 = |\mathbf{F}|\sin\theta + 2|\mathbf{F}|\cos\phi - 200g,$$

where $g = 9.81$. Since $\tan\phi = 0.75/2.4 = 5/16$, it follows that $\sin\phi = 5/\sqrt{281}$ and $\cos\phi = 16/\sqrt{281}$. From the first equation above,

$\cos\theta = 2\sin\phi = 10/\sqrt{281} \Longrightarrow \theta = \pm 0.9316$ radians.

When $\theta = 0.9316$, the second equation gives $|\mathbf{F}| = 200g/(\sin\theta + 2\cos\phi) = 724$ N. When $\theta = -0.9316$, we obtain $|\mathbf{F}| = 1773$ N.



5a) $\vec{v} = \hat{j} - \hat{k}$ $\quad \vec{w} = 6\hat{i} - 2\hat{j} + 3\hat{k}$

$$\vec{v} \cdot \vec{w} = (0)(6) + (1)(-2) + (-1)(3) = -2 - 3 = -5$$

$$(\vec{v} \cdot \vec{w})\vec{u} = -5\left[2\hat{i} - 3\hat{j} + \hat{k}\right] = -10\hat{i} + 15\hat{j} - 5\hat{k}$$

b) $(3\vec{u} - 4\vec{w})$

$$= 3(2\hat{i} - 3\hat{j} + \hat{k}) - 4(6\hat{i} - 2\hat{j} + 3\hat{k})$$
$$= (6-24)\hat{i} + (-9+8)\hat{j} + (3-12)\hat{k}$$
$$= -18\hat{i} - \hat{j} - 9\hat{k}$$

$(2\hat{i} + 3\vec{u} - 2\vec{v})$

$$= 2\hat{i} + 3(2\hat{i} - 3\hat{j} + \hat{k}) - 2(\hat{j} - \hat{k})$$
$$= (2+6)\hat{i} + (-9-2)\hat{j} + (3+2)\hat{k}$$
$$= 8\hat{i} - 11\hat{j} + 5\hat{k}$$

$$(3\vec{u} - 4\vec{w}) \cdot (2\hat{i} + 3\vec{u} - 2\vec{v}) = (-18)(8) + (-1)(-11) + (-9)(5)$$
$$= -144 + 11 - 45 = -178$$

6a) $\vec{v} = (-1, 2, 0)$ $\quad \vec{w} = (-2, -3, 5)$

$$3\vec{v} - \vec{w} = 3(-1, 2, 0) - (-2, -3, 5)$$
$$= (-3, 6, 0) - (-2, -3, 5) = (-1, 9, -5)$$

$$\vec{u} \times (3\vec{v} - \vec{w}) = (3, 1, 4) \times (-1, 9, -5)$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & 4 \\ -1 & 9 & -5 \end{vmatrix} = (-5-36)\hat{i} - (-15-(-4))\hat{j} + (27-(-1))\hat{k}$$
$$= (-41, 11, 28)$$

b) $\vec{v} \times \vec{w} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 2 & 0 \\ -2 & -3 & 5 \end{vmatrix}$

$= (10-0)\hat{i} - (-5-0)\hat{j} + (3-(-4))\hat{k}$

$= (10, 5, 7)$

$\vec{u} \times (\vec{v} \times \vec{w}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & 4 \\ 10 & 5 & 7 \end{vmatrix}$

$= (7-20)\hat{i} - (21-40)\hat{j} + (15-10)\hat{k}$

$= (-13, 19, 5)$

7) PROBLEM 26 – angle between vectors

$\vec{u} = (1, 6)$

$\vec{v} = (-4, 7)$

use dot product $\vec{u} \cdot \vec{v} = |\vec{u}||\vec{v}| \cos\theta$

$\cos\theta = \dfrac{\vec{u} \cdot \vec{v}}{|\vec{u}|\,|\vec{v}|}$ $\quad \vec{u} \cdot \vec{v} = (1)(-4) + (6)(7)$

$= -4 + 42 = 38$

$|\vec{u}| = \sqrt{1 + (6)^2} = \sqrt{37}$

$|\vec{v}| = \sqrt{(-4)^2 + (7)^2} = \sqrt{65}$

$\therefore \theta = \arccos\left[\dfrac{38}{\sqrt{37}\cdot\sqrt{65}}\right]$

$\cong 0.684$ rad.

PROBLEM 30. - angle between $\vec{u} = (1, 3, -2)$
$\vec{v} = (-2, -6, 4)$

$$\theta = \arccos \frac{\vec{u} \cdot \vec{v}}{|\vec{u}||\vec{v}|} \qquad \vec{u} \cdot \vec{v} = (1)(-2) + (3)(-6) + (-2)(4)$$
$$= -2 - 18 - 8 = -28$$

$$\theta = \arccos \frac{-28}{\sqrt{1+9+4} \cdot \sqrt{4+36+16}} = \arccos \left[\frac{-28}{\sqrt{14}\sqrt{56}}\right]$$

$$\theta = \arccos(-1) = \pi \leftarrow \text{angle} = \pi \therefore$$
opposite direction

can verify by inspection
$$\vec{v} = -2\vec{u}$$

PROBLEM 31 find components for vector $\perp$ to
$\vec{u} = (1, 3, 5)$ and $\vec{v} = (-2, 1, 4)$

$$\vec{n} = \vec{u} \times \vec{v} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 3 & 5 \\ -2 & 1 & 4 \end{vmatrix}$$

$$= [(3)(4) - 5]\hat{i} - [4 - (-10)]\hat{j} + [(1 - (-2)(3)]\hat{k}$$
$$= 7\hat{i} - 14\hat{j} + 7\hat{k} = (7, -14, 7)$$

PROBLEM 32 find components for vector $\perp$ to
y axis and vector joining pts $(2, 4 -3)$
and $(1, 5, 6)$

let $\vec{u} = \hat{j} = (0, 1, 0)$
$$\vec{v} = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}$$
$$= (1-2)\hat{i} + (5-4)\hat{j} + (6-(-3))\hat{k}$$
$$= -\hat{i} + \hat{j} + 9\hat{k} = (-1, 1, 9)$$

$\vec{n} = \vec{u} \times \vec{v} = (0,1,0) \times (-1,1,9)$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 1 & 0 \\ -1 & 1 & 9 \end{vmatrix} = 9\hat{i} - 0\hat{j} + \hat{k}$$

$\therefore \vec{n} = (9, 0, 1)$

8a) gvn point $(x_1, y_1, z_1) = (2,1,5)$

and normal vector from $(2,1,5)$ to $(4,2,3)$

vector $(A,B,C) = (4-2,\ 2-1,\ 3-5) = (2,1,-2)$

plane $= (A,B,C) \cdot (x-x_1,\ y-y_1,\ z-z_1) = 0$

$= 2(x-2) + 1(y-1) - 2(z-5) = 0$

$2x + y - 2z = 4 + 1 - 10 = -5$

$\therefore$ equation of plane $2x + y - 2z = -5$

b) gvn points $(1,3,2)$ $(-2,0,-2)$ $(1,4,3)$ on plane.

use cross product to find normal vector

$(A,B,C) = \vec{u} \times \vec{v}$ $\quad \vec{u} = (-2-1,\ 0-3,\ -2-2)$

$= (-3,-3,-4)$

$\vec{v} = (1-1,\ 4-3,\ 3-2)$

$= (0,1,1)$

$$(A,B,C) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -3 & -3 & -4 \\ 0 & 1 & 1 \end{vmatrix} = (-3-(-4))\hat{i} - (-3-0)\hat{j} + (-3-0)\hat{k}$$

$= (1, 3, -3)$

select one point $(x_1, y_1, z_1) = (1,3,2)$

plane $\Rightarrow (A,B,C)(x-x_1, y-y_1, z-z_1) = 0$

$$1(x-1) + 3(y-3) - 3(z-2) = 0$$

$$x + 3y - 3z = 1 + 9 - 6 = 4$$

$\therefore$ plane equation $x + 3y - 3z = 4$.

c) gvn lines $\frac{x-1}{6} = \frac{y}{8} = \frac{z+2}{2} = t$

$\frac{x+1}{3} = \frac{y-2}{4} = z+5 = t$ $\Big\}$ symmetric form.

need to form vectors coincident with each line.

① $t=0 = \frac{x-1}{6}$ $x=1$ $\qquad$ $t=1 = \frac{x-1}{6}$ $x=7$

$0 = \frac{y}{8}$ $y=0$ $\qquad$ $1 = \frac{y}{8}$ $y=8$

$0 = \frac{z+2}{2}$ $z=-2$ $\qquad$ $1 = \frac{z+2}{2}$ $z=0$

$\therefore \vec{u} = (7-1, 8-0, 0-(-2)) = (6,8,2)$

② $t=0 = \frac{x+1}{3}$ $x=-1$ $\qquad$ $t=1 = \frac{x+1}{3}$ $x=2$

$0 = \frac{y-2}{4}$ $y=2$ $\qquad$ $1 = \frac{y-2}{4}$ $y=6$

$0 = z+5$ $z=-5$ $\qquad$ $1 = z+5$ $z=-4$

$\therefore \vec{v} = (2-(-1), 6-2, -4-(-5)) = (3,4,1)$

(A,B,C)

$$= \vec{u}\times\vec{v} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 6 & 8 & 2 \\ 3 & 4 & 1 \end{vmatrix} = (8-8)\hat{i} - (6-6)\hat{j} + (24-24)\hat{k}$$

$$= (0,0,0) = 0$$

<u>because</u> $\vec{u}$ and $\vec{v}$ coincident.

need to find another vector on the plane

use pts for $t=1$ from both lines

$$\vec{v} = (2-7, 6-8, -4-0) = (-5, -2, -4)$$

$$(A,B,C) = \vec{u}\times\vec{v} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 6 & 8 & 2 \\ -5 & -2 & -4 \end{vmatrix} = (-32-(-4))\hat{i} - (-24-(-10))\hat{j} + (-12-(-40))\hat{k}$$

$$= (-28, 14, 28)$$

select point $(x_1, y_1, z_1) = (1, 0, -2)$

equation of plane $= (A,B,C)\cdot(x-x_1, y-y_1, z-z_1)$

$$= -28(x-1) + 14(y-0) + 28(z-(-2)) = 0$$

$$-28x + 14y + 28z = -28 - 56 = -84$$

$$\boxed{-2x + y + 2z = -6}$$

9a) given points $(2,-3,4)$ and $(5,2,-1)$

$$\vec{R} = \vec{R}_0 + t\vec{v}$$

find $\vec{v}$ from points $\vec{v} = (5-2, 2-(-3), -1-4)$

$$= (3, 5, -5)$$

$$\vec{R}_0 = (2, -3, 4)$$

$$\vec{R} = (2,-3,4) + t(3,5,-5)$$

$\vec{R} = (2+3t)\hat{i} + (5t-3)\hat{j} + (4-5t)\hat{k}$ ← vector form

$x = 2+3t \quad y = 5t-3 \quad z = 4-5t$ ← parametric form

$t = \frac{x-2}{3} = y\frac{+3}{5} = \frac{4-z}{5}$ ← symmetric form.

b) gvn. point $(-2,3,1)$ and llel line $x+y=3$
$2x-y+z=-2$

find vectors $\vec{u}, \vec{v} \perp$ to planes

$\vec{u} = (A,B,C)$ $\quad (A,B,C)\cdot(x-x_1, y-y_1, z-z_1) = x+y-3$

$$A(x-x_1) + B(y-y_1) + C(z-z_1) = x+y-3$$

$$\therefore \quad \vec{u} = (1,1,0)$$

$\vec{v} = (A,B,C)$ $\quad A(x-x_1) + B(y-y_1) + C(z-z_1) = 2x-y+z+2$
$= 2(x+1) - y + z$

$$\therefore \vec{v} = (2,-1,1)$$

if $\vec{u}, \vec{v} \perp$ to planes, then $\vec{u} \times \vec{v}$ will be coincident with intersection of planes, ie. line

$$\vec{u}\times\vec{v} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 0 \\ 2 & -1 & 1 \end{vmatrix} = (1-0)\hat{i} - (1-0)\hat{j} + (-1-2)\hat{k}$$
$$= (1,-1,-3)$$

solve $\vec{R} = \vec{R}_0 + t\vec{v}$ $\quad$ where $\vec{v} = (1,-1,-3)$
$R_0 = (-2,3,1)$

$$\vec{R} = (-2,3,1) + t(1,-1,-3)$$
$$= (t-2)\hat{i} + (3-t)\hat{j} + (1-3t)\hat{k}$$ ← vector form

$$\underbrace{\begin{aligned} x &= t-2 \\ y &= 3-t \\ z &= 1-3t \end{aligned}}_{\text{parametric form}} \qquad \underbrace{t = x+2 = 3-y = \frac{1-z}{3}}_{\text{symmetric form}}$$

c) gun surfaces $2x - y = 5$

$$3x + 4y + z = 10$$

assume $x = t$ $\quad 2t - y = 5$ ← first equation

$$y = 2t - 5$$

second equation → $3t + 4(2t-5) + z = 10$

$$z = 10 - 3t - 8t + 20$$
$$= 30 - 11t$$

∴ vector form $\vec{R} = t\,\hat{i} + (2t-5)\,\hat{j} + (30-11t)\,\hat{k}$

parametric form

$$x = t$$
$$y = 2t - 5$$
$$z = 30 - 11t$$

symmetric form $\quad t = x = \dfrac{y+5}{2} = \dfrac{30-z}{11}$