ASSIGNMENT # 6 SOLUTIONS

1a) $f(x,y) = 3xy - x^3 - y^3$

$$\frac{\partial f}{\partial x} = 3y - 3x^2 = 0$$
$$\frac{\partial f}{\partial y} = 3x - 3y^2 = 0$$

two equations and two unknowns.

(#1) $y = x^2$

(#2) $x = y^2 = x^4$ $x = 0, 1$ $y = 0, 1$

$\therefore$ critical points $(0,0)$ $(1,1)$

$A = f_{xx} = \frac{\partial}{\partial x}(3y - 3x^2) = -6x$ at $(0,0)$ $A = 0$

$B = f_{xy} = \frac{\partial}{\partial y}(3y - 3x^2) = 3$ $B = 3$

$C = f_{yy} = \frac{\partial}{\partial y}(3x - 3y^2) = -6y$ $C = 0$

$\therefore B^2 - AC = 3 > 0$

saddle point at $(0,0)$

at $(1,1)$ $A = -6$

$B = 3$ $B^2 - AC = 9 - (-6)(-6)$ and $A < 0$

$C = -6$ $= 9 - 36 < 0$

relative maximum at $(1,1)$

b) $f(x,y) = x\sin y$

$$\frac{\partial f}{\partial x} = \sin y = 0 \qquad y = n\pi \quad n = 0, 1, 2 \ldots$$
$$\frac{\partial f}{\partial y} = x\cos y = 0 \qquad x = 0$$

$\therefore$ critical points $(0, n\pi)$ $n = 0, 1, 2 \ldots$

$A = f_{xx} = \frac{\partial}{\partial x}(\sin y) = 0$ at $(0, n\pi)$ $A = 0$

$B = f_{xy} = \frac{\partial}{\partial y}(\sin y) = \cos y$ $B = 1$ $n = 0, 2, 4 \ldots$; -1 $n = 1, 3, 5 \ldots$

$C = f_{yy} = \frac{\partial}{\partial y}(x\cos y) = -x\sin y$ $C = 0$

$B^2 - AC = (\cos(n\pi))^2 - 0 > 0$ $\therefore$ all critical points are saddle points

c) $f(x,y) = y^2 - 4x^2y + 3x^4$

$\frac{\partial f}{\partial x} = -8xy + 12x^3 = 0$ (#1)

$\frac{\partial f}{\partial y} = 2y - 4x^2 = 0$ (#2)

$\underbrace{\qquad}_{\text{two equations, two unknowns}}$

from (#2) $y = 2x^2$

substitute into (#1) $-8x(2x^2) + 12x^3 = 0$

$-4x^3 = 0$

$\therefore x = 0$

$y = 0$

critical point $(0,0)$

$A = f_{xx} = \frac{\partial}{\partial x}(-8xy + 12x^3) = -8y + 36x^2$ at $(0,0)$ $A = 0$

$B = f_{xy} = \frac{\partial}{\partial y}(-8xy + 12x^3) = -8x$ $B = 0$

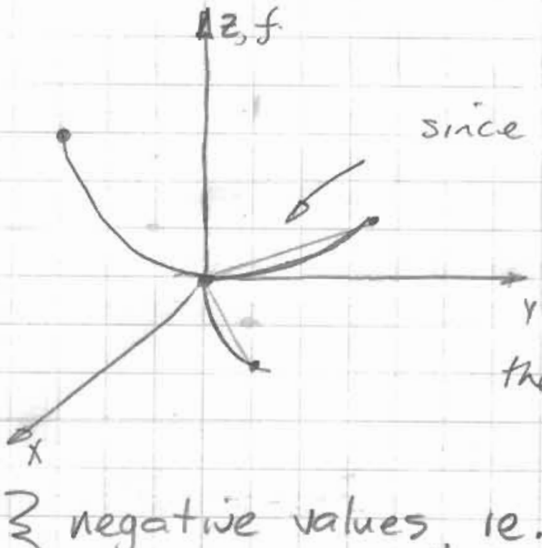
$C = f_{yy} = \frac{\partial}{\partial y}(2y - 4x^2) = 2$ $C = 2$

$\therefore B^2 - AC = 0$ test fails.

use graphical methods. $\rightarrow$ since $(0,0)$ is critical pt, plot region surrounding

x	y	$f = y^2 - 4x^2y + 3x^4$
1	0	3
0	1	1
-1	0	3
0	-1	1
1	1	0
-1	1	0
-1	-1	8
1	-1	8
1/2	1/2	-1/16
-1/2	1/2	-1/16

$\}$ negative values, ie. smaller than $f = z = 0$



since surrounding points have both negative and positive values, then $(0,0)$ is saddle point.

d) $f(x,y,z) = xyz\, e^{x^2+y^2+z^2}$

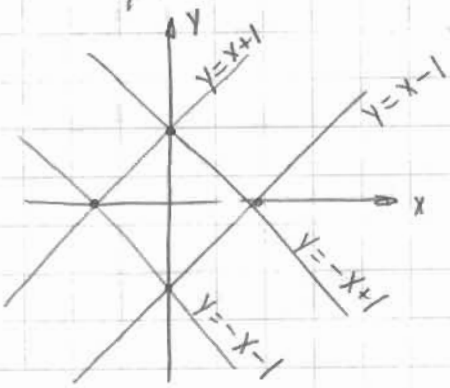
$$\frac{\partial f}{\partial x} = yz e^{x^2+y^2+z^2} + xyz e^{x^2+y^2+z^2} \cdot 2x$$
$$= (2x^2+1)\, yz e^{x^2+y^2+z^2} = 0$$

$$\frac{\partial f}{\partial y} = (2y^2+1)\, xz e^{x^2+y^2+z^2} = 0$$
$$\frac{\partial f}{\partial z} = (2z^2+1)\, yz e^{x^2+y^2+z^2} = 0$$

since $e^{x^2+y^2+z^2} \neq 0$ then
$(2x^2+1)yz = 0 \quad y=0$ or $z=0$
$(2y^2+1)xz = 0 \quad x=0$ or $z=0$
$(2z^2+1)xy = 0 \quad x=0$ or $y=0$

$\therefore$ all points on axes $x=0,\ y=0,\ z=0$ are critical pts.

2a) $f(x,y) = x^2 + x + 3y^2 + y$ on $R =$
$y = x+1$
$y = 1-x$
$y = x-1$
$y = -x-1$



① critical points on R

$$\frac{\partial f}{\partial x} = 2x+1 = 0 \qquad x = -\frac{1}{2}$$
$$\frac{\partial f}{\partial y} = 6y+1 = 0 \qquad y = -\frac{1}{6}$$

② evaluate $f\left(-\frac{1}{2}, -\frac{1}{6}\right) = \frac{1}{4} - \frac{1}{2} + \frac{3}{36} - \frac{1}{6} = \frac{-1}{3}$

③ boundaries - evaluate separately for square shape

a) $y = x+1 \qquad f(x) = x^2 + x + 3(x^2+2x+1) + (x+1)$
$$= 4x^2 + 8x + 4$$
$$\frac{df}{dx} = 8x + 8 = 0 \qquad x = -1$$
$$y = x+1 = 0$$

$f(-1, 0) = 0$

b) $y = x - 1$ $\quad f(x) = x^2 + x + 3(x^2 - 2x + 1) + (x-1)$

$= 4x^2 - 4x + 2$

$\frac{df}{dx} = 8x - 4 = 0 \quad x = \frac{1}{2} \quad y = x - 1 = -\frac{1}{2}$

$f(\frac{1}{2}, -\frac{1}{2}) = \frac{1}{4} + \frac{1}{2} + \frac{3}{4} - \frac{1}{2} = 1$

c) $y = -x + 1$ $\quad f(x) = x^2 + x + 3(x^2 - 2x + 1) - x + 1$

$= 4x^2 - 6x + 4$

$\frac{df}{dx} = 8x - 6 = 0 \quad x = \frac{3}{4} \quad y = -x + 1 = \frac{1}{4}$

$f(\frac{3}{4}, \frac{1}{4}) = \frac{9}{16} + \frac{3}{4} + \frac{3}{16} + \frac{1}{4} = \frac{28}{16} = \frac{7}{4}$

d) $y = -x - 1$ $\quad f(x) = x^2 + x + 3(x^2 + 2x + 1) - x - 1$

$= 4x^2 + 6x + 2$

$\frac{df}{dx} = 8x + 6 = 0 \quad x = \frac{-3}{4} \quad y = -\frac{1}{4}$

$f(-\frac{3}{4}, -\frac{1}{4}) = \frac{9}{16} - \frac{3}{4} + \frac{3}{16} - \frac{1}{4} = -\frac{1}{4}$

4 corners $\quad f(-1, 0) = 0$ ← from $y = x + 1$ boundary.

$f(1, 0) = 1 + 1 + 3(0) + 0 = 2$

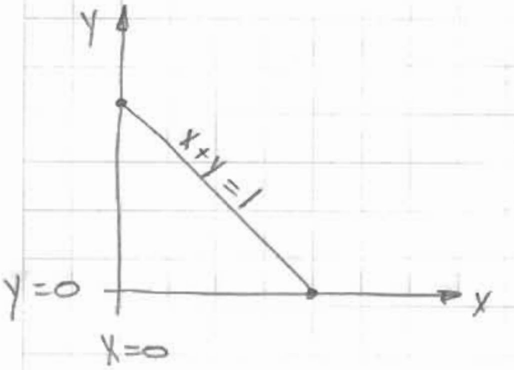
$f(0, -1) = 0 + 3(1) - 1 = 2$

$f(0, 1) = 0 + 3(1) + 1 = 4$

∴ maximum value $f(0, 1) = 4$ (corner)

minimum value $f(-\frac{1}{2}, -\frac{1}{6}) = \frac{-1}{3}$ (critical pt.)

2b) $f(x,y) = x^3 - 3x + y^2 + 2y$ on $R =$ $x=0$, $y=0$, $x+y=1$



① critical points

$$\frac{\partial f}{\partial x} = 3x^2 - 3 = 0 \quad x^2 = 1 \quad x = \pm 1$$

$$\frac{\partial f}{\partial y} = 2y + 2 = 0 \quad y = -1$$

two critical points $(1,-1)$, $(-1,-1)$

both points are outside R

② boundaries - evaluate separately for triangle

a) $x = 0$ $\quad f(y) = y^2 + 2y$

$$\frac{df}{dy} = 2y + 2 = 0 \quad y = -1$$

since $y=-1$ outside R, do not consider

b) $y = 0$ $\quad f(x) = x^3 - 3x$

$$\frac{df}{dx} = 3x^2 - 3 = 0 \quad x = \pm 1$$

since $x = -1$ outside R, consider only $x = 1$

$$f(1,0) = 1 - 3 = -2$$

c) $x + y = 1$ $\quad f(x) = x^3 - 3x + (1 - 2x + x^2) + 2(1-x)$

$y = 1 - x$

$$= x^3 + x^2 - 7x + 3$$

$$\frac{df}{dx} = 3x^2 + 2x - 7 = 0$$

$$x = \frac{-2 \pm \sqrt{4 - 4(3)(-7)}}{6} = \frac{-1 \pm \sqrt{22}}{3} \cong -1.89 \text{ and } 1.23$$

since both x values are outside R, do not consider

③ corners.

$f(0,0) = 0$

$f(1,0) = -2$ from $y=0$ boundary

$f(0,1) = 0+1+2 = 3$

$\therefore$ absolute maximum value $f(0,1) = 3$ (corner)

absolute minimum value $f(1,0) = -2$ (corner)

2c) $f(x,y) = x^3+y^3-3x-3y+2$ for $R = x^2+y^2 \le 1$

① critical points $\frac{\partial f}{\partial x} = 3x^2-3 = 0 \quad x = \pm 1$

$\frac{\partial f}{\partial y} = 3y^2-3 = 0 \quad y = \pm 1$

$\therefore$ 4 critical points $(1,1)\ (1,-1)\ (-1,-1)\ (-1,1)$

all 4 critical points are outside of R
$\therefore$ do not consider

② boundary - since $x^2+y^2=1$ circle, define curve parametrically

$\left.\begin{array}{l} x = \cos t \\ y = \sin t \end{array}\right\}$ substitute into $f(x,y)$ and solve for critical points

$f(t) = \cos^3 t + \sin^3 t - 3\cos t - 3\sin t + 2$

$\frac{df}{dt} = (3\cos^2 t)(-\sin t) + (3\sin^2 t)(\cos t) + 3\sin t - 3\cos t = 0$

$3\sin t\cos t(\sin t - \cos t) + 3(\sin t - \cos t) = 0$

$3(\sin t\cos t + 1)(\sin t - \cos t) = 0$

evaluate each part separately

$(\sin t \cos t + 1) = 0 \qquad \sin t \cos t = -1$

from Schaums (pg. 45) $\sin 2A = 2\sin A \cos A$

$\therefore \ \sin 2t = -2$

for all t values $-1 \le \sin 2t \le 1$

$\therefore$ cannot equal zero

S A / T C

$(\sin t - \cos t) = 0 \qquad \sin t = \cos t$

$t = \frac{\pi}{4}, \ \frac{5\pi}{4}$

$$f\left(\tfrac{\pi}{4}\right) = \cos^3\left(\tfrac{\pi}{4}\right) + \sin^3\left(\tfrac{\pi}{4}\right) - 3\cos\left(\tfrac{\pi}{4}\right) - 3\sin\left(\tfrac{\pi}{4}\right) + 2$$

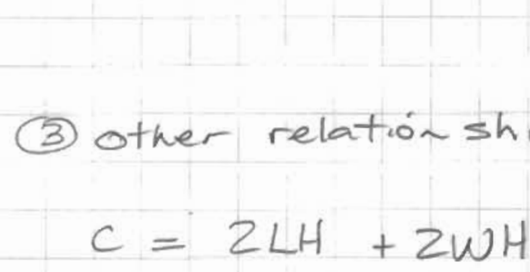
$$= \left(\tfrac{1}{\sqrt{2}}\right)^3 + \left(\tfrac{1}{\sqrt{2}}\right)^3 - \tfrac{3}{\sqrt{2}} - \tfrac{3}{\sqrt{2}} + 2$$

$$= 2 - \tfrac{5}{\sqrt{2}} \cong -1.536$$ ← absolute minimum

$$f\left(\tfrac{5\pi}{4}\right) = \left(-\tfrac{1}{\sqrt{2}}\right)^3 + \left(-\tfrac{1}{\sqrt{2}}\right)^3 + \tfrac{3}{\sqrt{2}} + \tfrac{3}{\sqrt{2}} + 2$$

$$= 2 + \tfrac{5}{\sqrt{2}} \cong 5.536$$ ← absolute maximum

3)



① function to be minimized:

C = cost per unit area.

② constraints:

$V = LWH = 1000$ litres $= 1 m^3$

③ other relationships $S = 2LH + 2WH + LW$

$C = 2LH + 2WH + 3LW$ ← base costs 3X side cost per unit area.

④ substitute constraint

$$LWH = 1 \qquad H = \frac{1}{LW}$$

$$C = 2L\left(\frac{1}{LW}\right) + 2W\left(\frac{1}{LW}\right) + 3LW$$

$$= \frac{2}{W} + \frac{2}{L} + 3LW$$

⑤ find critical point

$$\frac{\partial C}{\partial L} = -\frac{2}{L^2} + 3W = 0$$

$$\frac{\partial C}{\partial W} = -\frac{2}{W^2} + 3L = 0$$

solve 2 equations, 2 unknowns

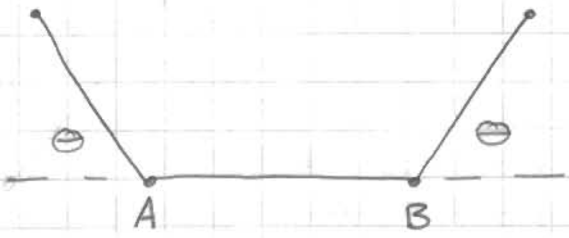
$$W = \frac{2}{3} \cdot \frac{1}{L^2} \qquad \frac{-2}{\left(\frac{4}{9} \cdot \frac{1}{L^4}\right)} + 3L = -\frac{9}{2}L^4 + 3L$$

$$L^3 = \frac{2}{3} \qquad \boxed{L \cong 0.874}$$

$$W = \frac{2}{3} \cdot \frac{1}{L^2} = \frac{2}{3} \cdot \frac{1}{(0.874)^2} \qquad \boxed{W \cong 0.874}$$

$$H = \frac{1}{LW} = \frac{1}{(0.874)^2} \qquad \boxed{H \cong 1.310}$$

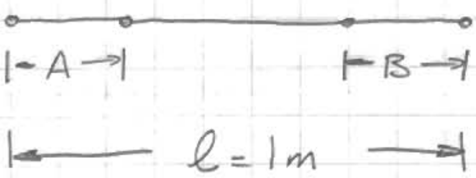
4)



① select quantity to be maximized

flow in channel ∝ cross sectional area

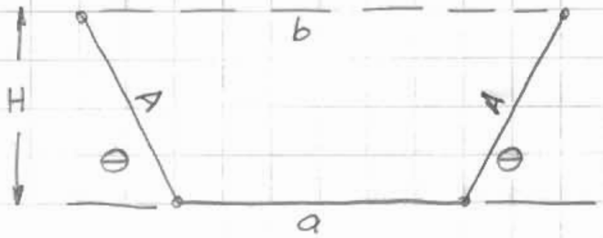
∴ S = area.



② constraints $A = B$

$l = 1m$

(3) other relationships



Schaums (pg. 13)

$$S = \frac{1}{2} H (a+b)$$

$$H = A\sin\theta$$

$$a = 1 - 2A$$

$$b = a + 2A\cos\theta = (1 - 2A + 2A\cos\theta)$$

$$\therefore S = \frac{1}{2}(A\sin\theta)\left(\frac{1}{2} - 2A + \frac{1}{2} - 2A + 2A\cos\theta\right)$$

$$S = A\sin\theta\,(1 - 2A + A\cos\theta)$$

$$= A\sin\theta - 2A^2\sin\theta + A^2\cos\theta\sin\theta$$

critical points

$$\frac{\partial S}{\partial A} = \sin\theta - 4A\sin\theta + 2A\sin\theta\cos\theta = 0$$

$$\sin\theta\,(1 - 4A + 2A\cos\theta) = 0 \qquad (\#1)$$

$$\frac{\partial S}{\partial \theta} = A\cos\theta - 2A^2\cos\theta + A^2\cos^2\theta - A^2\sin^2\theta = 0$$

$$= A\cos\theta\,(1-2A) + 2A^2\cos^2\theta - A^2 \qquad (\#2)$$

from (#1) since $\sin\theta = 0$ for $\theta = 0, \pi$ – not physical

$$\therefore 1 - 4A + 2A\cos\theta = 0$$

$$\cos\theta = \frac{4A-1}{2A}$$

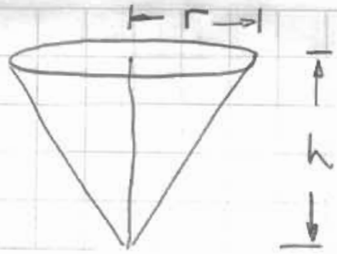
substitute into (#2) $A\left(\frac{4A-1}{2A}\right)(1-2A) + 2A^2\left(\frac{4A-1}{2A}\right)^2 - A^2 = 0$

$$\frac{1}{2}(4A - 8A^2 - 1 + 2A) + \frac{2}{4}(16A^2 - 8A + 1) - A^2 = 0$$

$$3A^2 - A = 0$$

$$3A = 1 \qquad \boxed{A = 1/3}$$

$$\theta = \cos^{-1}\left(\frac{4A-1}{2A}\right) = \cos^{-1}\left(\frac{1}{2}\right) \qquad \boxed{\theta = \frac{\pi}{3}}$$

5) 

① minimize surface area S

② constraints $V = \frac{\pi}{3} r^2 h = 100\text{ cm}^3$

③ other relationships $S = \pi r \sqrt{r^2 + h^2}$ (Schaums pg. 17)

lateral surface area of cone

④ substitute constraint

$$h = \frac{300}{\pi} \cdot \frac{1}{r^2} \qquad S = \pi r \sqrt{r^2 + \frac{(300)^2}{\pi^2} \cdot \frac{1}{r^4}}$$

⑤ find critical points – S is function of 1 independent variable only (r)

$\therefore$ $\frac{dS}{dr}$ = messy – better to optimize on S^2

$$S^2 = \pi^2 r^2 \left(r^2 + \frac{(300)^2}{\pi^2} \cdot \frac{1}{r^4}\right) = \pi^2 r^4 + \frac{(300)^2}{r^2}$$

$$\frac{dS^2}{dr} = 4\pi^2 r^3 - \frac{2(300)^2}{r^3} = 0$$

$$4\pi^2 r^3 = \frac{2(300)^2}{r^3} \qquad r^6 = \frac{(300)^2}{2\pi^2} \quad \therefore r = \left(\frac{45000}{\pi^2}\right)^{\frac{1}{6}}$$

$$\boxed{r \cong 4.0721\text{ cm}}$$

$$h = \frac{300}{\pi} \cdot \frac{1}{r^2} = \frac{300}{\pi} \frac{1}{(4.072)^2}$$

$$\boxed{h \cong 5.759}$$

$$\boxed{\frac{h}{r} = 1.41426 = \sqrt{2}}$$

6 fit data using straight line

$$P_i = aA_i + b$$

$$S = \sum_{i=1}^{N} [P_i - \overline{P_i}]^2 = \sum_{i=1}^{N} [aA_i + b - \overline{P_i}]^2$$

$$\frac{\partial S}{a} = \sum_{i=1}^{N} 2[aA_i + b - \overline{P_i}](A_i) = 0$$

$$\frac{\partial S}{\partial b} = \sum_{i=1}^{N} 2[aA_i + b - \overline{P_i}] = 0$$

$$a(\sum A_i^2) + b(\sum A_i) = \sum \overline{P_i} A_i$$

$$a(\sum A_i) + bN = \sum \overline{P_i}$$

$$\sum A_i = 4+5+6+7+8+9+10+11+12+13+14+15+16 = 130$$

$$\sum A_i^2 = 16+25+36+49+64+81+100+121+144+169+196+225+256 = 1482$$

$$\sum \overline{P_i} = 85+87+90+92+95+98+100+105+108+110+112+115+118 = 1315$$

$$\sum \overline{P_i} A_i = (4)(85)+5(87)+6(90)+7(92)+8(95)+9(98)+10(100)+11(105)+12(108)+13(110)+14(112)+15(115)+16(118) = 13663$$

$$1482a + 130b = 13663$$
$$(130a + 13b = 1315) \times 10$$

$$1482a + 130b = 13663$$
$$1300a + 130b = 13150$$
$$182a = 513$$

$$\boxed{a = \frac{513}{182} = 2.8187}$$

$$b = \frac{1315 - 130(2.8187)}{13}$$

$$\boxed{b = 72.967}$$

$$\boxed{\therefore\ P = 2.8187\,A\,[yr] + 72.967}$$

7

$$y = ax^3 + bx^2 + cx + d$$

$$S = \sum_{i=1}^{N} [y_i - \bar{y}_i]^2 = \sum_{i=1}^{N} [ax_i^3 + bx_i^2 + cx_i + d - \bar{y}_i]^2$$

$$\frac{\partial S}{\partial a} = \sum_{i=1}^{N} 2[ax_i^3 + bx_i^2 + cx_i + d - \bar{y}_i]x_i^3 = 0$$

$$\frac{\partial S}{\partial b} = \sum_{i=1}^{N} 2[ax_i^3 + bx_i^2 + cx_i + d - \bar{y}_i]x_i^2 = 0$$

$$\frac{\partial S}{\partial c} = \sum_{i=1}^{N} 2[ax_i^3 + bx_i^2 + cx_i + d - \bar{y}_i]x_i = 0$$

$$\frac{\partial S}{\partial d} = \sum_{i=1}^{N} 2[ax_i^3 + bx_i^2 + cx_i + d - \bar{y}_i] = 0$$

$$(\Sigma x_i^6)a + (\Sigma x_i^5)b + (\Sigma x_i^4)c + (\Sigma x_i^3)d = \Sigma \bar{y}_i x_i^3$$
$$(\Sigma x_i^5)a + (\Sigma x_i^4)b + (\Sigma x_i^3)c + (\Sigma x_i^2)d = \Sigma \bar{y}_i x_i^2$$
$$(\Sigma x_i^4)a + (\Sigma x_i^3)b + (\Sigma x_i^2)c + (\Sigma x_i)d = \Sigma \bar{y}_i x_i$$
$$(\Sigma x_i^3)a + (\Sigma x_i^2)b + (\Sigma x_i)c + Nd = \Sigma \bar{y}_i$$

SOLVE SUMMATIONS IN EXCEL.

$\sum x_i^6 = 19280 \quad \sum x_i^3 = 410.69 \quad \sum \bar{y}_i x_i^3 = 20660$

$\sum x_i^5 = 5214.9 \quad \sum x_i^2 = 121.04 \quad \sum \bar{y}_i x_i^2 = 5613.1$

$\sum x_i^4 = 1442.9 \quad \sum x_i = 37.2 \quad \sum \bar{y}_i x_i = 1559.9$

$\sum \bar{y}_i = 445.69$

SET OF EQUATIONS

$$\begin{aligned}
19280a + 5214.9b + 1442.9c + 410.69d &= 20660 \\
5214.9a + 1442.9b + 410.69c + 121.04d &= 5613.1 \\
1442.9a + 410.69b + 121.04c + 37.2d &= 1559.9 \\
410.69a + 121.04b + 37.2c + 12d &= 445.69
\end{aligned}$$

SOLVE for a, b, c, d using Maple (or Mathcad)

$a = 0.1193 \qquad c = -2.2145$

$b = 4.8113 \qquad d = -8.6081$

$y = 0.1193x^3 + 4.8113x^2 - 2.2145x - 8.6081$

x_i	2	2.2	2.4	2.6	2.8	3	3.2	3.4	3.6	3.8	4	4.2
y_i	7.163	11.078	15.439	20.255	25.531	31.271	37.482	44.17	51.34	59.0	67.15	75.8
$\bar{y}_i$	7.06	11.34	15.62	19.5	25.62	31.94	37.02	44.32	51.56	58.72	67.08	75.91

$$\% \text{ diff} = \frac{(y_i - \bar{y}_i)}{\bar{y}_i} \times 100$$

%	1.45	-2.32	-1.16	3.87	-0.35	-2.09	1.25	-0.34	-0.43	0.47	0.10	-0.14
$\%^2$	2.11	5.38	1.34	15.01	0.12	4.38	1.56	0.11	0.18	0.22	0.01	0.02

$$\text{RMS \% difference} = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(\%^2)} = \sqrt{\frac{30.45}{12}} = 1.59\%$$

function form $\quad t_i = a F_i^{\,b}$

$$T_i = \log(t_i) = \log(a) + b\log(F_i)$$

$$T_i = A + b f_i \quad \leftarrow \text{linear fit}$$

$\therefore$ least squares for linear function

$$(\Sigma f_i^2)b + (\Sigma f_i)A = (\Sigma T_i f_i)$$

$$(\Sigma f_i)b + NA = \Sigma T_i$$

F_i	$f_i = \log(F_i)$	t_i	$T_i = \log(t_i)$
70	1.845	470	2.672
100	2	288	2.459
200	2.301	84	1.924
300	2.477	52	1.716
400	2.602	32	1.505

$\Sigma f_i = 11.225 \qquad \Sigma T_i f_i = 22.442$

$\Sigma f_i^2 = 25.605 \qquad \Sigma T_i = 10.276$

$$(25.605b + 11.225A = 22.442) \times 5$$

$$(11.225b + 5A = 10.276) \times 11.225$$

$$2.0244b = -3.1381$$

$$b = -1.5501$$

$$A = \frac{10.276 + (11.225)(1.5501)}{5}$$

$$= 5.5352$$

$\log(a) = A$

$a = 10^A = 10^{5.5352}$

$= 342926$

$$\boxed{\therefore\ t_i = 342926\ F_i^{-1.5501}}$$

9) function form $y = \dfrac{1}{ax+b}$

let $Y_i = \dfrac{1}{y_i} = ax_i + b$

least squares - linear equation

$(\Sigma x_i^2)a + (\Sigma x_i)b = (\Sigma \bar{Y}_i x_i)$

$(\Sigma x_i)a + Nb = \Sigma \bar{Y}_i$

x	$\bar{y}_i$	$\bar{Y}_i$
5	1.335	0.749
6	1.431	0.699
7	1.247	0.802
8	1.197	0.835
9	1.118	0.894

$\Sigma x_i = 35$

$\Sigma x_i^2 = 255$

$\Sigma x_i \bar{Y}_i = 28.28$

$\Sigma \bar{Y}_i = 3.979$

$255a + 35b = 28.28$

$(35a + 5b = 3.979) \times 7$

$10a = 0.427 \quad a = 0.0427$

$b = \dfrac{3.979 - 35(0.0427)}{5}$

$b = 0.4969$

$$\boxed{\therefore\ y = \dfrac{1}{0.0427x + 0.4969}}$$