1a $$\nabla f = \frac{\partial f}{\partial x}\hat{i} + \frac{\partial f}{\partial y}\hat{j} + \frac{\partial f}{\partial z}\hat{k}$$

$$\frac{\partial f}{\partial x} = \frac{\partial}{\partial x}\left[\frac{1}{\sqrt{x^2+y^2+z^2}}\right] = \left[\frac{-2x}{2(x^2+y^2+z^2)^{3/2}}\right] = \frac{-x}{(x^2+y^2+z^2)^{3/2}}$$

$$\frac{\partial f}{\partial y} = \frac{-y}{(x^2+y^2+z^2)^{3/2}} \qquad \frac{\partial f}{\partial z} = \frac{-z}{(x^2+y^2+z^2)^{3/2}}$$

$$\nabla f = -\frac{(x\hat{i} + y\hat{j} + z\hat{k})}{(x^2+y^2+z^2)^{3/2}}$$

b) $$\nabla\cdot\vec{F} = \frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} + \frac{\partial R}{\partial z}$$

$$\frac{\partial P}{\partial x} = \frac{\partial}{\partial x}(2xe^y) = 2e^y \qquad \frac{\partial Q}{\partial y} = \frac{\partial}{\partial y}(3x^2z) = 0$$

$$\frac{\partial R}{\partial z} = \frac{\partial}{\partial z}(-2x^2yz) = -2x^2y$$

$$\nabla\cdot\vec{F} = 2(e^y - x^2y)$$

c) $$\nabla \cdot \vec{F} = \frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} + \frac{\partial R}{\partial z}$$

$$\frac{\partial P}{\partial x} = \frac{\partial}{\partial x}\left(\frac{x}{\sqrt{x^2+y^2+z^2}}\right) = \frac{1}{\sqrt{x^2+y^2+z^2}} - \frac{x^2}{(x^2+y^2+z^2)^{3/2}}$$

$$\frac{\partial Q}{\partial y} = \frac{\partial}{\partial y}\left(\frac{y}{\sqrt{x^2+y^2+z^2}}\right) = \frac{1}{\sqrt{x^2+y^2+z^2}} - \frac{y^2}{(x^2+y^2+z^2)^{3/2}}$$

$$\frac{\partial R}{\partial z} = \frac{1}{\sqrt{x^2+y^2+z^2}} - \frac{z^2}{(x^2+y^2+z^2)^{3/2}}$$

$$\nabla \cdot \vec{F} = \frac{3}{\sqrt{x^2+y^2+z^2}} - \frac{(x^2+y^2+z^2)}{(x^2+y^2+z^2)^{3/2}} = \frac{2}{\sqrt{x^2+y^2+z^2}}$$

d) $$\nabla \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ P & Q & R \end{vmatrix} = \left(\frac{\partial R}{\partial y} - \frac{\partial Q}{\partial z}\right)\hat{i} - \left(\frac{\partial R}{\partial x} - \frac{\partial P}{\partial z}\right)\hat{j} + \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right)\hat{k}$$

$P = x^2 z \qquad \partial P/\partial y = 0 \qquad \partial P/\partial z = x^2$

$Q = 12xyz \qquad \partial Q/\partial x = 12yz \qquad \partial Q/\partial z = 12xy$

$R = 32y^2z^4 \qquad \partial R/\partial x = 0 \qquad \partial R/\partial y = 64yz^4$

$$\nabla \times \vec{F} = \left(64yz^4 - 12xy\right)\hat{i} - \left(-x^2\right)\hat{j} + \left(12yz\right)\hat{k}$$

e) $$\nabla \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ y & -x & 0 \end{vmatrix}$$

$$= (0)\hat{i} - (0)\hat{j} + (-1-1)\hat{k} = -2\hat{k}$$

$$\nabla \times \vec{F}\,\big|_{(2,0)} = -2\hat{k}$$

2 $$\phi(x,y) = -k \ln\sqrt{x^2+y^2} \qquad x>0,\ y>0$$

$$\nabla\phi = \frac{\partial\phi}{\partial x}\hat{i} + \frac{\partial\phi}{\partial y}\hat{j} \qquad \frac{\partial\phi}{\partial x} = \left(\frac{-k}{\sqrt{x^2+y^2}}\right)\left(\frac{2x}{2\sqrt{x^2+y^2}}\right)$$

$$= \frac{-kx}{x^2+y^2}$$

$$\frac{\partial\phi}{\partial y} = \frac{-ky}{x^2+y^2} \qquad \nabla\phi = \frac{-k(x\hat{i}+y\hat{j})}{x^2+y^2}$$

$$\nabla\cdot\nabla\phi = \left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j}\right)\cdot\left(\frac{-k(x\hat{i}+y\hat{j})}{x^2+y^2}\right)$$

$$\frac{\partial}{\partial x}\left(\frac{-kx}{x^2+y^2}\right) = \frac{-k}{(x^2+y^2)} - kx\left(\frac{-2x}{(x^2+y^2)^2}\right)$$

$$= \frac{-k}{(x^2+y^2)} + \frac{2kx^2}{(x^2+y^2)^2}$$

$$\frac{\partial}{\partial y}\left(\frac{-ky}{x^2+y^2}\right) = \frac{-k}{(x^2+y^2)} + \frac{2ky^2}{(x^2+y^2)^2}$$

$$\nabla\cdot\nabla\phi = \frac{-2k}{(x^2+y^2)} + \frac{2k(x^2+y^2)}{(x^2+y^2)^2} = 0$$