Tutorial #1 Solutions

1) Given Vectors $\vec{u}=(3,1,4),\ \vec{v}=(-1,2,0),\ \vec{w}=(-2,-3,5)$ find $(\vec{u}\,x\,\vec{v})\,x\,\vec{w}$?

Given : $\vec{u} = (3,1,4)$

$\vec{v} = (-1,2,0)$

$\vec{w} = (-2,-3,5)$

Find : $(\vec{u} \times \vec{v}) \times \vec{w}$

Answer: Let $\vec{u} \times \vec{v} = \vec{x}$

$$\vec{x} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & 4 \\ -1 & 2 & 0 \end{vmatrix} = \hat{i}\begin{vmatrix} 1 & 4 \\ 2 & 0 \end{vmatrix} - \hat{j}\begin{vmatrix} 3 & 4 \\ -1 & 0 \end{vmatrix} + \hat{k}\begin{vmatrix} 3 & 1 \\ -1 & 2 \end{vmatrix}$$

$$= \hat{i}(0-8) - \hat{j}(0-(-4)) + \hat{k}(6-(-1))$$

$$\vec{x} = -8\hat{i} - 4\hat{j} + 7\hat{k}$$

$$\vec{x} \times \vec{w} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -8 & -4 & 7 \\ -2 & -3 & 5 \end{vmatrix} = \hat{i}\begin{vmatrix} -4 & 7 \\ -3 & 5 \end{vmatrix} - \hat{j}\begin{vmatrix} -8 & 7 \\ -2 & 5 \end{vmatrix} + \hat{k}\begin{vmatrix} -8 & -4 \\ -2 & -3 \end{vmatrix}$$

$$= \hat{i}(-20-(-21)) - \hat{j}(-40-(-14)) + \hat{k}(24-8)$$

$$\vec{x} \times \vec{w} = \hat{i} + 26\hat{j} + 16\hat{k}$$

$$\therefore \underline{\underline{(\vec{u} \times \vec{v}) \times \vec{w} = \hat{i} + 26\hat{j} + 16\hat{k} = (1, 26, 16)}}$$

2) Find all vectots of magnitude 7 that are perpendicular ro a plane containing the point $(3,0,-4),\ (4,6,-2), and (-1,0,-1)$?

Find $\vec{v} \perp$ to plane $\underset{P}{(3,0,-4)}, \underset{Q}{(4,6,-2)}, \underset{R}{(-1,0,-1)}$

$\vec{PQ} = (1,6,2)$

$\vec{PR} = (-4,0,3)$ $\qquad \vec{PQ} \times \vec{PR} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 6 & 2 \\ -4 & 0 & 3 \end{vmatrix}$

$= (18-0)\hat{i} - (3+8)\hat{j} + (0+24)\hat{k}$

$= (18, -11, 24)$

$\hat{v} = \dfrac{(18, -11, 24)}{\sqrt{18^2+11^2+24^2}} = \dfrac{(18, -11, 24)}{\sqrt{1021}}$ $\qquad$ two vectors

$\vec{v} = 7\hat{v} = \dfrac{(126, -77, 168)}{\sqrt{1021}} = \pm(3.94, -2.41, 5.26)$

3) Find the equation of the plane containing the point (2,-4,3) and the line $\dfrac{x-1}{3}=\dfrac{y+5}{4}=z+2$?

Given: Point : $(2,-4,3) - P_1$
Line : $\dfrac{x-1}{3}=\dfrac{y+5}{4}=z+2$

Find: Equation for the plane containing the point and line provided above.

Answer: From the line equation, get vector and point as follows

$\dfrac{x-1}{3}=t$	$\dfrac{y+5}{4}=t$	$z+2=t$
$x=1+3t=x_0+v_x t$	$y=-5+4t=y_0+v_y t$	$z=-2+t=z_0+v_z t$
$\therefore\ x_0=1,\ v_x=3$	$\therefore\ y_0=-5,\ v_y=4$	$\therefore\ z_0=-2,\ v_z=1$

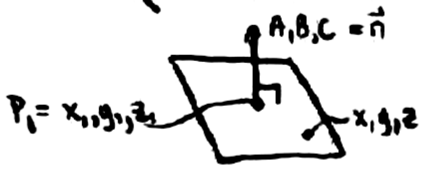
Let $\vec{v}$ be vector from line $\Rightarrow$ $\vec{v}=(v_x,v_y,v_z)=(3,4,1)$
Let P_0 be point on line $\Rightarrow$ $P_0=(1,-5,-2)$

From P_0 and P_1, find vector $\vec{x}=\overrightarrow{P_0P_1}=(2,-4,3)-(1,-5,-2)$
$\vec{x}=(1,1,5)$

Need two vectors ($\vec{x}$ & $\vec{v}$) to find $\perp$ vector needed in order to find equation of plane

$$\vec{n}=\perp \text{ vector} = \vec{x}\times\vec{v}=\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 5 \\ 3 & 4 & 1 \end{vmatrix} = -19\hat{i}+14\hat{j}+\hat{k}=(-19,14,1)=\vec{n}$$

Equation of Plane : $A(x_1-x)+B(y_1-y)+C(z_1-z)=0$
(from class notes)



$-19(2-x)+14(-4-y)+1(3-z)=0$

$\therefore\ \underline{\underline{19x-14y-z-91=0}}$