ME 201 Tutorial #2 Solutions
Vector Calculus:

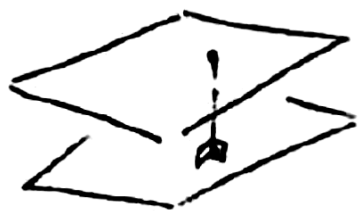
1. Find the distance between two planes $x - y + 2z = 4$ and $3x - 3y + 6z = 10$

Find distance between planes $x-y+2z=4$
$3x-3y+6z=10$

Check if planes intersect
- solve for x

$$x = 4-2z+y = \frac{1}{3}(10-6z+3y) = \frac{10}{3} - 2z + y$$

since $x = 4 \neq \frac{10}{3}$ the planes do not intersection



use distance from plane to point formula

$$d = \frac{|Ax_1 + By_1 + Cz_1 + D|}{\sqrt{A^2+B^2+C^2}}$$

from $x - y + 2z - 4 = 0$ $A=1, B=-1, C=2, D=-4$
from $3x-3y+6z-10=0$ $x_1=1, y_1=1$ then

$$6z = 10-3+3 \quad z = \frac{10}{6}$$

$$d = \frac{|1(1) - 1(1) + 2(\frac{10}{6}) - 4|}{\sqrt{1+1+4}} = \frac{|-\frac{2}{3}|}{\sqrt{6}} \cong 0.272$$

2. If the line $\frac{x+5}{7}=\frac{y-11}{9}=\frac{z}{45}$ is parallel to the plane $9x-2y-z=0$, find the shortest distance between them.

If line $\frac{x+5}{7}=\frac{y-11}{9}=\frac{z}{45}$ is parallel to plane $9x-2y-z=0$, then shortest distance = distance between plane and pt. on line

from $9x-2y-z=0$

$A=9$
$B=-2$
$C=-1$
$D=0$

from $\frac{x+5}{7}=\frac{y-11}{9}$ let $x=1$, $\frac{y-11}{9}=\frac{6}{7}$, $y=\frac{131}{7}$

$\frac{z}{45}=\frac{6}{7}$, $z=\frac{270}{7}$

$$d=\frac{\left|9(1)-2\left(\frac{131}{7}\right)-1\left(\frac{270}{7}\right)+0\right|}{\sqrt{81+4+1}}=\frac{67}{\sqrt{86}}$$

$\cong 7.22$

3. Determine whether the planes x + z = 1 and y + z = 1 are parallel, perpendicular, or neither. If neither, find the angle between them.

The normal vectors of the planes are n1 = < 1, 0, 1 > and n2 = < 0, 1, 1 >. The two vectors are not proportional to each other, thus the planes are not parallel and they intersect each other.

If the dot product of the two normal vectors is 0, then the two planes are perpendicular.

$$\mathbf{n1} \bullet \mathbf{n2} = < 1, 0, 1 > \bullet < 0, 1, 1 > = 1$$

Thus the two planes are neither parallel nor perpendicular. The angle between the planes can be found using the formula:

$$\begin{aligned} \cos\theta &= \frac{\mathbf{n1} \bullet \mathbf{n2}}{|\mathbf{n1}||\mathbf{n2}|} \\ &= \frac{< 1, 0, 1 > \bullet < 0, 1, 1 >}{\sqrt{1^2 + 0^2 + 1^2} * \sqrt{0^2 + 1^2 + 1^2}} \\ &= \frac{1}{2} \end{aligned}$$

$$\theta = \cos^{-1}\left(\frac{1}{2}\right) = 60^\circ$$

Thus the angle between the two planes is 60°