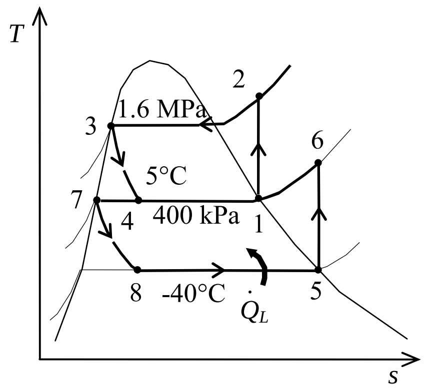
11-64 A two-stage cascade refrigeration system is considered. Each stage operates on the ideal vapor-compression cycle with upper cycle using water and lower cycle using refrigerant-134a as the working fluids. The mass flow rate of R-134a and water in their respective cycles and the overall COP of this system are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Kinetic and potential energy changes are negligible. **3** The heat exchanger is adiabatic.

Analysis From the water and refrigerant tables (Tables A-4, A-5, A-6, A-11, A-12, and A-13),

$$\left.\begin{array}{l} T_1 = 5°\text{C} \\ \text{sat. vapor} \end{array}\right\} \begin{array}{l} h_1 = h_{g\ @\ 5°\text{C}} = 2510.1\ \text{kJ/kg} \\ s_1 = s_{g\ @\ 5°\text{C}} = 9.0249\ \text{kJ/kg}\cdot\text{K} \end{array}$$



$$\left.\begin{array}{l} P_2 = 1.6\ \text{MPa} \\ s_2 = s_1 \end{array}\right\} h_2 = 5083.4\ \text{kJ/kg}$$

$$\left.\begin{array}{l} P_3 = 1.6\ \text{MPa} \\ \text{sat. liquid} \end{array}\right\} h_3 = h_{f\ @\ 1.6\ \text{MPa}} = 858.44\ \text{kJ/kg}$$

$$h_4 \cong h_3 = 858.44\ \text{kJ/kg} \quad \text{(throttling)}$$

$$\left.\begin{array}{l} T_5 = -40°\text{C} \\ \text{sat. vapor} \end{array}\right\} \begin{array}{l} h_5 = h_{g\ @\ -40°\text{C}} = 225.86\ \text{kJ/kg} \\ s_5 = s_{g\ @\ -40°\text{C}} = 0.96866\ \text{kJ/kg}\cdot\text{K} \end{array}$$

$$\left.\begin{array}{l} P_6 = 400\ \text{kPa} \\ s_6 = s_5 \end{array}\right\} h_6 = 267.59\ \text{kJ/kg}$$

$$\left.\begin{array}{l} P_7 = 400\ \text{kPa} \\ \text{sat. liquid} \end{array}\right\} h_7 = h_{f\ @\ 400\ \text{kPa}} = 63.94\ \text{kJ/kg}$$

$$h_8 \cong h_7 = 63.94\ \text{kJ/kg} \quad \text{(throttling)}$$

The mass flow rate of R-134a is determined from

$$\dot{Q}_L = \dot{m}_R(h_5 - h_8) \longrightarrow \dot{m}_R = \frac{\dot{Q}_L}{h_5 - h_8} = \frac{20\ \text{kJ/s}}{(225.86 - 63.94)\ \text{kJ/kg}} = \mathbf{0.1235\ kg/s}$$

An energy balance on the heat exchanger gives the mass flow rate of water

$$\dot{m}_R(h_6 - h_7) = \dot{m}_w(h_1 - h_4)$$

$$\longrightarrow \dot{m}_w = \dot{m}_R \frac{h_6 - h_7}{h_1 - h_4} = (0.1235\ \text{kg/s}) \frac{267.59 - 63.94}{2510.1 - 858.44} = \mathbf{0.01523\ kg/s}$$

The total power input to the compressors is

$$\begin{aligned} \dot{W}_{\text{in}} &= \dot{m}_R(h_6 - h_5) + \dot{m}_w(h_2 - h_1) \\ &= (0.1235\ \text{kg/s})(267.59 - 225.86)\ \text{kJ/kg} + (0.01523\ \text{kg/s})(5083.4 - 2510.1)\ \text{kJ/kg} \\ &= 44.35\ \text{kJ/s} \end{aligned}$$

The COP of this refrigeration system is determined from its definition,

$$\text{COP}_\text{R} = \frac{\dot{Q}_L}{\dot{W}_{\text{in}}} = \frac{20\ \text{kJ/s}}{44.35\ \text{kJ/s}} = \mathbf{0.451}$$