13-55 The volume fractions of components of a gas mixture are given. This mixture is expanded isentropically to a specified pressure. The work produced per unit mass of the mixture is to be determined.

Assumptions All gases will be modeled as ideal gases with constant specific heats.

Properties The molar masses of H_2, He, and N_2 are 2.0, 4.0, and 28.0 kg/kmol, respectively (Table A-1). The constant-pressure specific heats of these gases at room temperature are 14.307, 5.1926, and 1.039 kJ/kg·K, respectively (Table A-2a).

Analysis We consider 100 kmol of this mixture. Noting that volume fractions are equal to the mole fractions, mass of each component are

$$m_{\text{H2}} = N_{\text{H2}} M_{\text{H2}} = (30\text{ kmol})(2\text{ kg/kmol}) = 60\text{ kg}$$
$$m_{\text{He}} = N_{\text{He}} M_{\text{He}} = (40\text{ kmol})(4\text{ kg/kmol}) = 160\text{ kg}$$
$$m_{\text{N2}} = N_{\text{N2}} M_{\text{N2}} = (30\text{ kmol})(28\text{ kg/kmol}) = 840\text{ kg}$$

30% H_2
40% He
30% N_2
(by volume)
5 MPa, 600°C

The total mass is

$$m_m = m_{\text{H2}} + m_{\text{He}} + m_{\text{N2}} = 60 + 160 + 840 = 1060\text{ kg}$$

Then the mass fractions are

$$\text{mf}_{\text{H2}} = \frac{m_{\text{H2}}}{m_m} = \frac{60\text{ kg}}{1060\text{ kg}} = 0.05660$$
$$\text{mf}_{\text{He}} = \frac{m_{\text{He}}}{m_m} = \frac{160\text{ kg}}{1060\text{ kg}} = 0.1509$$
$$\text{mf}_{\text{N2}} = \frac{m_{\text{N2}}}{m_m} = \frac{840\text{ kg}}{1060\text{ kg}} = 0.7925$$

The apparent molecular weight of the mixture is

$$M_m = \frac{m_m}{N_m} = \frac{1060\text{ kg}}{100\text{ kmol}} = 10.60\text{ kg/kmol}$$

The constant-pressure specific heat of the mixture is determined from

$$\begin{aligned} c_p &= \text{mf}_{\text{H2}} c_{p,\text{H2}} + \text{mf}_{\text{He}} c_{p,\text{He}} + \text{mf}_{\text{N2}} c_{p,\text{N2}} \\ &= 0.05660 \times 14.307 + 0.1509 \times 5.1926 + 0.7925 \times 1.039 \\ &= 2.417\text{ kJ/kg}\cdot\text{K} \end{aligned}$$

The apparent gas constant of the mixture is

$$R = \frac{R_u}{M_m} = \frac{8.314\text{ kJ/kmol}\cdot\text{K}}{10.60\text{ kg/kmol}} = 0.7843\text{ kJ/kg}\cdot\text{K}$$

Then the constant-volume specific heat is

$$c_v = c_p - R = 2.417 - 0.7843 = 1.633\text{ kJ/kg}\cdot\text{K}$$

The specific heat ratio is

$$k = \frac{c_p}{c_v} = \frac{2.417}{1.633} = 1.480$$

The temperature at the end of the expansion is

$$T_2 = T_1 \left(\frac{P_2}{P_1}\right)^{(k-1)/k} = (873\text{ K})\left(\frac{200\text{ kPa}}{5000\text{ kPa}}\right)^{0.48/1.48} = 307\text{ K}$$

An energy balance on the adiabatic expansion process gives

$$w_{\text{out}} = c_p (T_1 - T_2) = (2.417\text{ kJ/kg}\cdot\text{K})(873 - 307)\text{ K} = \mathbf{1368\text{ kJ/kg}}$$