13-56 The mass fractions of components of a gas mixture are given. This mixture is enclosed in a rigid, well-insulated vessel, and a paddle wheel in the vessel is turned until specified amount of work have been done on the mixture. The mixture's final pressure and temperature are to be determined.

Assumptions All gases will be modeled as ideal gases with constant specific heats.

Properties The molar masses of N_2 , He, CH_4 , and C_2H_6 are 28.0, 4.0, 16.0, and 30.0 kg/kmol, respectively (Table A-1). The constant-pressure specific heats of these gases at room temperature are 1.039, 5.1926, 2.2537, and 1.7662 kJ/kg·K, respectively (Table A-2a).

Analysis We consider 100 kg of this mixture. The mole numbers of each component are

$$N_{\text{N2}} = \frac{m_{\text{N2}}}{M_{\text{N2}}} = \frac{15 \text{ kg}}{28 \text{ kg/kmol}} = 0.5357 \text{ kmol}$$

$$N_{\text{He}} = \frac{m_{\text{He}}}{M_{\text{He}}} = \frac{5 \text{ kg}}{4 \text{ kg/kmol}} = 1.25 \text{ kmol}$$

$$N_{\text{CH4}} = \frac{m_{\text{CH4}}}{M_{\text{CH4}}} = \frac{60 \text{ kg}}{16 \text{ kg/kmol}} = 3.75 \text{ kmol}$$

$$N_{\text{C2H6}} = \frac{m_{\text{C2H6}}}{M_{\text{C2H6}}} = \frac{20 \text{ kg}}{30 \text{ kg/kmol}} = 0.6667 \text{ kmol}$$

$$15\% \text{ N}_2$$

$$5\% \text{ He}$$

$$60\% \text{ CH}_4$$

$$20\% \text{ C}_2\text{H}_6$$
(by mass)
$$10 \text{ m}^3$$

$$200 \text{ kPa}$$

$$20\% \text{ C}_2\text{ C}_2\text{$$

The mole number of the mixture is

$$N_m = N_{N2} + N_{He} + N_{CH4} + N_{C2H6} = 0.5357 + 1.25 + 3.75 + 0.6667 = 6.2024 \text{ kmol}$$

The apparent molecular weight of the mixture is

$$M_m = \frac{m_m}{N_m} = \frac{100 \text{ kg}}{6.2024 \text{ kmol}} = 16.12 \text{ kg/kmol}$$

The constant-pressure specific heat of the mixture is determined from

$$\begin{split} c_p &= \mathrm{mf_{N2}} c_{p,\mathrm{N2}} + \mathrm{mf_{He}} c_{p,\mathrm{He}} + \mathrm{mf_{CH4}} c_{p,\mathrm{CH4}} + \mathrm{mf_{C2H6}} c_{p,\mathrm{C2H6}} \\ &= 0.15 \times 1.039 + 0.05 \times 5.1926 + 0.60 \times 2.2537 + 0.20 \times 1.7662 \\ &= 2.121 \, \mathrm{kJ/kg \cdot K} \end{split}$$

The apparent gas constant of the mixture is

$$R = \frac{R_u}{M_m} = \frac{8.134 \text{ kJ/kmol} \cdot \text{K}}{16.12 \text{ kg/kmol}} = 0.5158 \text{ kJ/kg} \cdot \text{K}$$

Then the constant-volume specific heat is

$$c_v = c_p - R = 2.121 - 0.5158 = 1.605 \,\text{kJ/kg} \cdot \text{K}$$

The mass in the container is

$$m_m = \frac{P_1 V_m}{RT_1} = \frac{(200 \text{ kPa})(10 \text{ m}^3)}{(0.5158 \text{ kPa} \cdot \text{m}^3/\text{kg} \cdot \text{K})(293 \text{ K})} = 13.23 \text{ kg}$$

An energy balance on the system gives

$$W_{\rm sh,in} = m_m c_v (T_2 - T_1) \longrightarrow T_2 = T_1 + \frac{W_{\rm sh,in}}{m_m c_v} = (293 \text{ K}) + \frac{100 \text{ kJ}}{(13.23 \text{ kg})(1.605 \text{ kJ/kg} \cdot \text{K})} = 297.7 \text{ K}$$

Since the volume remains constant and this is an ideal gas,

$$P_2 = P_1 \frac{T_2}{T_1} = (200 \text{ kPa}) \frac{297.7 \text{ K}}{293 \text{ K}} = 203.2 \text{ kPa}$$