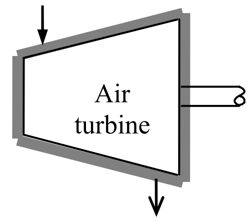
7-209 Air is expanded by an adiabatic turbine with an isentropic efficiency of 85%. The outlet temperature, the work produced, and the entropy generation are to be determined.

Assumptions **1** This is a steady-flow process since there is no change with time. **2** Kinetic and potential energy changes are negligible. **3** The device is adiabatic and thus heat transfer is negligible. **4** Air is an ideal gas with constant specific heats.

Properties The properties of air at the anticipated average temperature of 400 K are c_p = 1.013 kJ/kg·°C and k = 1.395 (Table A-2b). Also, R = 0.287 kJ/kg·K (Table A-2a).



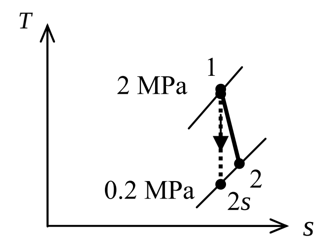
Analysis We take the turbine as the system, which is a control volume since mass crosses the boundary. Noting that one fluid stream enters and leaves the turbine, the energy balance for this steady-flow system can be expressed in the rate form as

$$\underbrace{\dot{E}_{\text{in}} - \dot{E}_{\text{out}}}_{\substack{\text{Rate of net energy transfer}\\ \text{by heat, work, and mass}}} = \underbrace{\Delta \dot{E}_{\text{system}}^{\nearrow 0 \text{ (steady)}}}_{\substack{\text{Rate of change in internal, kinetic,}\\ \text{potential, etc. energies}}} = 0$$

$$\dot{E}_{\text{in}} = \dot{E}_{\text{out}}$$

$$\dot{m}h_1 = \dot{W}_{a,\text{out}} + \dot{m}h_2 \quad (\text{since } \dot{Q} \cong \Delta\text{ke} \cong \Delta\text{pe} \cong 0)$$

$$\dot{W}_{a,\text{out}} = \dot{m}(h_1 - h_2) = \dot{m}c_p(T_1 - T_2)$$



The isentropic exit temperature is

$$T_{2s} = T_1\left(\frac{P_{2s}}{P_1}\right)^{(k-1)/k} = (300 + 273\text{ K})\left(\frac{200\text{ kPa}}{2200\text{ kPa}}\right)^{0.395/1.395} = 290.6\text{ K}$$

From the definition of the isentropic efficiency,

$$w_{a,\text{out}} = \eta_T w_{s,\text{out}} = \eta_T c_p(T_1 - T_{2s}) = (0.90)(1.013\text{ kJ/kg}\cdot\text{K})(573 - 290.6)\text{K} = \mathbf{257.5\ kJ/kg}$$

The actual exit temperature is then

$$w_{a,\text{out}} = c_p(T_1 - T_{2a}) \longrightarrow T_{2a} = T_1 - \frac{w_{a,\text{out}}}{c_p} = T_1 - \frac{w_{a,\text{out}}}{c_p} = 573\text{ K} - \frac{257.5\text{ kJ/kg}}{1.013\text{ kJ/kg}\cdot\text{K}} = \mathbf{318.8\ K}$$

The rate of entropy generation in the turbine is determined by applying the rate form of the entropy balance on the turbine:

$$\underbrace{\dot{S}_{\text{in}} - \dot{S}_{\text{out}}}_{\substack{\text{Rate of net entropy transfer}\\ \text{by heat and mass}}} + \underbrace{\dot{S}_{\text{gen}}}_{\substack{\text{Rate of entropy}\\ \text{generation}}} = \underbrace{\Delta \dot{S}_{\text{system}}^{\nearrow 0 \text{ (steady)}}}_{\substack{\text{Rate of change}\\ \text{of entropy}}}$$

$$\dot{m}_1 s_1 - \dot{m}_2 s_2 + \dot{S}_{\text{gen}} = 0 \quad (\text{since } \dot{Q} = 0)$$

$$\dot{S}_{\text{gen}} = \dot{m}(s_2 - s_1)$$

$$s_{\text{gen}} = s_2 - s_1$$

Then, from the entropy change relation of an ideal gas,

$$\begin{aligned} s_{\text{gen}} = s_2 - s_1 &= c_p \ln\frac{T_2}{T_1} - R\ln\frac{P_2}{P_1} \\ &= (1.013\text{ kJ/kg}\cdot\text{K})\ln\frac{318.8\text{ K}}{573\text{ K}} - (0.287\text{ kJ/kg}\cdot\text{K})\ln\frac{200\text{ kPa}}{2200\text{ kPa}} \\ &= \mathbf{0.0944\ kJ/kg\cdot K} \end{aligned}$$