

**9-100** A gas-turbine plant operates on the simple Brayton cycle. The net power output, the back work ratio, and the thermal efficiency are to be determined.

**Assumptions** 1 The air-standard assumptions are applicable. 2 Kinetic and potential energy changes are negligible. 3 Air is an ideal gas with variable specific heats.

**Properties** The gas constant of air is  $R = 0.287 \text{ kJ/kg}\cdot\text{K}$  (Table A-1).

**Analysis** (a) For this problem, we use the properties from EES software.

Remember that for an ideal gas, enthalpy is a function of temperature only whereas entropy is functions of both temperature and pressure.

Process 1-2: Compression

$$T_1 = 40^\circ\text{C} \longrightarrow h_1 = 313.6 \text{ kJ/kg}$$

$$\left. \begin{array}{l} T_1 = 40^\circ\text{C} \\ P_1 = 100 \text{ kPa} \end{array} \right\} s_1 = 5.749 \text{ kJ/kg}\cdot\text{K}$$

$$\left. \begin{array}{l} P_2 = 2000 \text{ kPa} \\ s_2 = s_1 = 5.749 \text{ kJ/kg}\cdot\text{K} \end{array} \right\} h_{2s} = 736.7 \text{ kJ/kg}$$

$$\eta_C = \frac{h_{2s} - h_1}{h_2 - h_1} \longrightarrow 0.85 = \frac{736.7 - 313.6}{h_2 - 313.6} \longrightarrow h_2 = 811.4 \text{ kJ/kg}$$

Process 3-4: Expansion

$$T_4 = 650^\circ\text{C} \longrightarrow h_4 = 959.2 \text{ kJ/kg}$$

$$\eta_T = \frac{h_3 - h_4}{h_3 - h_{4s}} \longrightarrow 0.88 = \frac{h_3 - 959.2}{h_3 - h_{4s}}$$

We cannot find the enthalpy at state 3 directly. However, using the following lines in EES together with the isentropic efficiency relation, we find  $h_3 = 1873 \text{ kJ/kg}$ ,  $T_3 = 1421^\circ\text{C}$ ,  $s_3 = 6.736 \text{ kJ/kg}\cdot\text{K}$ . The solution by hand would require a trial-error approach.

$$h_3 = \text{enthalpy}(\text{Air}, T=T_3)$$

$$s_3 = \text{entropy}(\text{Air}, T=T_3, P=P_2)$$

$$h_{4s} = \text{enthalpy}(\text{Air}, P=P_1, s=s_3)$$

The mass flow rate is determined from

$$\dot{m} = \frac{P_1 \dot{V}_1}{RT_1} = \frac{(100 \text{ kPa})(700/60 \text{ m}^3/\text{s})}{(0.287 \text{ kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(40 + 273 \text{ K})} = 12.99 \text{ kg/s}$$

The net power output is

$$\dot{W}_{C,\text{in}} = \dot{m}(h_2 - h_1) = (12.99 \text{ kg/s})(811.4 - 313.6) \text{ kJ/kg} = 6464 \text{ kW}$$

$$\dot{W}_{T,\text{out}} = \dot{m}(h_3 - h_4) = (12.99 \text{ kg/s})(1873 - 959.2) \text{ kJ/kg} = 11,868 \text{ kW}$$

$$\dot{W}_{\text{net}} = \dot{W}_{T,\text{out}} - \dot{W}_{C,\text{in}} = 11,868 - 6464 = \mathbf{5404 \text{ kW}}$$

(b) The back work ratio is

$$r_{\text{bw}} = \frac{\dot{W}_{C,\text{in}}}{\dot{W}_{T,\text{out}}} = \frac{6464 \text{ kW}}{11,868 \text{ kW}} = \mathbf{0.545}$$

(c) The rate of heat input and the thermal efficiency are

$$\dot{Q}_{\text{in}} = \dot{m}(h_3 - h_2) = (12.99 \text{ kg/s})(1873 - 811.4) \text{ kJ/kg} = 13,788 \text{ kW}$$

$$\eta_{\text{th}} = \frac{\dot{W}_{\text{net}}}{\dot{Q}_{\text{in}}} = \frac{5404 \text{ kW}}{13,788 \text{ kW}} = 0.392 = \mathbf{39.2\%}$$

