

9-141 A turbopan engine operating on an ideal cycle produces 50,000 N of thrust. The air temperature at the fan outlet needed to produce this thrust is to be determined.

Assumptions 1 Steady operating conditions exist. 2 The air standard assumptions are applicable. 3 Air is an ideal gas with constant specific heats at room temperature. 4 The turbine work output is equal to the compressor work input.

Properties The properties of air at room temperature are $R = 0.287 \text{ kPa} \cdot \text{m}^3/\text{kg} \cdot \text{K}$, $c_p = 1.005 \text{ kJ/kg} \cdot \text{K}$ and $k = 1.4$ (Table A-2a).

Analysis The total mass flow rate is

$$\nu_1 = \frac{RT}{P} = \frac{(0.287 \text{ kPa} \cdot \text{m}^3)(253 \text{ K})}{50 \text{ kPa}} = 1.452 \text{ m}^3/\text{kg}$$

$$\dot{m} = \frac{AV_1}{\nu_1} = \frac{\pi D^2}{4} \frac{V_1}{\nu_1} = \frac{\pi (2.5 \text{ m})^2}{4} \frac{200 \text{ m/s}}{1.452 \text{ m}^3/\text{kg}} = 676.1 \text{ kg/s}$$

Now,

$$\dot{m}_e = \frac{\dot{m}}{8} = \frac{676.1 \text{ kg/s}}{8} = 84.51 \text{ kg/s}$$

The mass flow rate through the fan is

$$\dot{m}_f = \dot{m} - \dot{m}_e = 676.1 - 84.51 = 591.6 \text{ kg/s}$$

In order to produce the specified thrust force, the velocity at the fan exit will be

$$F = \dot{m}_f (V_{\text{exit}} - V_{\text{inlet}})$$

$$V_{\text{exit}} = V_{\text{inlet}} + \frac{F}{\dot{m}_f} = (200 \text{ m/s}) + \frac{50,000 \text{ N}}{591.6 \text{ kg/s}} \left(\frac{1 \text{ kg} \cdot \text{m/s}^2}{1 \text{ N}} \right) = 284.5 \text{ m/s}$$

An energy balance on the stream passing through the fan gives

$$c_p (T_4 - T_5) = \frac{V_{\text{exit}}^2 - V_{\text{inlet}}^2}{2}$$

$$T_5 = T_4 - \frac{V_{\text{exit}}^2 - V_{\text{inlet}}^2}{2c_p}$$

$$= 253 \text{ K} - \frac{(284.5 \text{ m/s})^2 - (200 \text{ m/s})^2}{2(1.005 \text{ kJ/kg} \cdot \text{K})} \left(\frac{1 \text{ kJ/kg}}{1000 \text{ m}^2/\text{s}^2} \right)$$

$$= \mathbf{232.6 \text{ K}}$$

