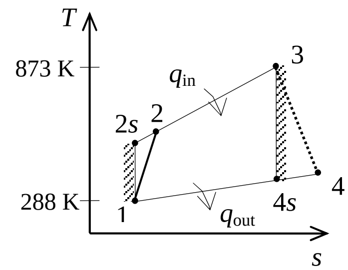
9-94 A simple Brayton cycle with air as the working fluid operates between the specified temperature and pressure limits. The effects of non-isentropic compressor and turbine on the back-work ratio is to be compared.

Assumptions **1** Steady operating conditions exist. **2** The air-standard assumptions are applicable. **3** Kinetic and potential energy changes are negligible. **4** Air is an ideal gas with constant specific heats.

Properties The properties of air at room temperature are c_p = 1.005 kJ/kg·K and k = 1.4 (Table A-2a).

Analysis For the compression process,



$$T_{2s} = T_1\left(\frac{P_2}{P_1}\right)^{(k-1)/k} = (288\text{ K})(12)^{0.4/1.4} = 585.8\text{ K}$$

$$\eta_C = \frac{h_{2s}-h_1}{h_2-h_1} = \frac{c_p(T_{2s}-T_1)}{c_p(T_2-T_1)} \longrightarrow T_2 = T_1 + \frac{T_{2s}-T_1}{\eta_C}$$

$$= 288 + \frac{585.8-288}{0.90} = 618.9\text{ K}$$

For the expansion process,

$$T_{4s} = T_3\left(\frac{P_4}{P_3}\right)^{(k-1)/k} = (873\text{ K})\left(\frac{1}{12}\right)^{0.4/1.4} = 429.2\text{ K}$$

$$\eta_T = \frac{h_3-h_4}{h_3-h_{4s}} = \frac{c_p(T_3-T_4)}{c_p(T_3-T_{4s})} \longrightarrow T_4 = T_3 - \eta_T(T_3-T_{4s})$$

$$= 873-(0.90)(873-429.2)$$

$$= 473.6\text{ K}$$

The isentropic and actual work of compressor and turbine are

$$W_{\text{Comp},s} = c_p(T_{2s}-T_1) = (1.005\text{ kJ/kg}\cdot\text{K})(585.8-288)\text{K} = 299.3\text{ kJ/kg}$$

$$W_{\text{Comp}} = c_p(T_2-T_1) = (1.005\text{ kJ/kg}\cdot\text{K})(618.9-288)\text{K} = 332.6\text{ kJ/kg}$$

$$W_{\text{Turb},s} = c_p(T_3-T_{4s}) = (1.005\text{ kJ/kg}\cdot\text{K})(873-429.2)\text{K} = 446.0\text{ kJ/kg}$$

$$W_{\text{Turb}} = c_p(T_3-T_4) = (1.005\text{ kJ/kg}\cdot\text{K})(873-473.6)\text{K} = 401.4\text{ kJ/kg}$$

The back work ratio for 90% efficient compressor and isentropic turbine case is

$$r_{\text{bw}} = \frac{W_{\text{Comp}}}{W_{\text{Turb},s}} = \frac{332.6\text{ kJ/kg}}{446.0\text{ kJ/kg}} = \mathbf{0.7457}$$

The back work ratio for 90% efficient turbine and isentropic compressor case is

$$r_{\text{bw}} = \frac{W_{\text{Comp},s}}{W_{\text{Turb}}} = \frac{299.3\text{ kJ/kg}}{401.4\text{ kJ/kg}} = \mathbf{0.7456}$$

The two results are almost identical.