Week 1: Lecture 1

Review

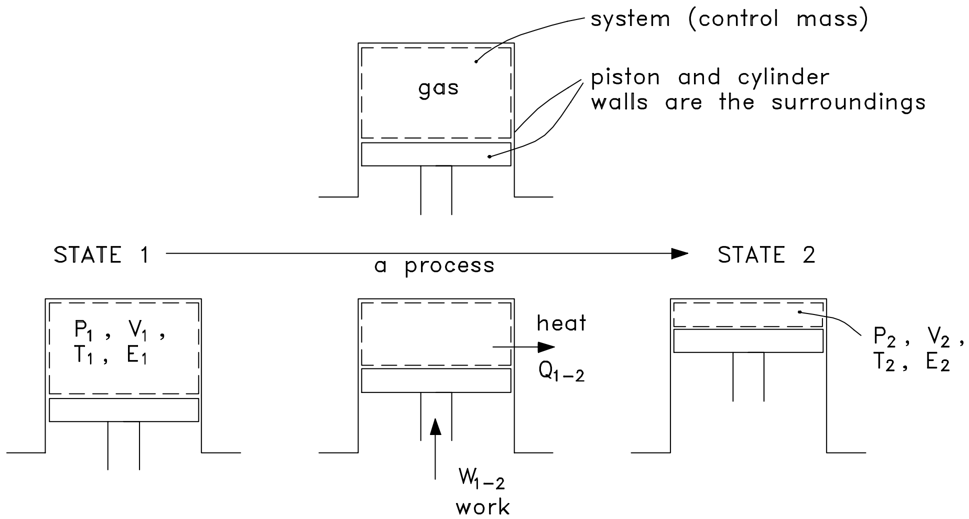
- any specified collection of matter under study is called a **system**
- all systems posses properties like mass, energy, entropy, volume, pressure, temperature, etc
- thermodynamics deals with these properties of matter as a system interacts with its surroundings through work and heat transfer
- work and heat transfer are not properties, they are the forms energy takes to cross the system boundary

First Law of Thermodynamics

Conservation of Energy: the energy content of an isolated system is constant

$$energy\ entering \ - \ energy\ leaving \ = \ change\ of\ energy\ within\ the\ system$$

Control Mass Analysis



first law energy balance

$$\left\{\begin{array}{c}\text{Initial}\\ \text{Energy}\\ E_1\end{array}\right\} \begin{array}{c}+\\ \\ -\end{array} \left\{\begin{array}{c}\text{Energy gain } W_{1-2}\\ \\ \text{Energy loss } Q_{1-2}\end{array}\right\} = \left\{\begin{array}{c}\text{Final}\\ \text{Energy}\\ E_2\end{array}\right\}$$

$$\boxed{E_1 + W_{1-2} - Q_{1-2} = E_2}$$

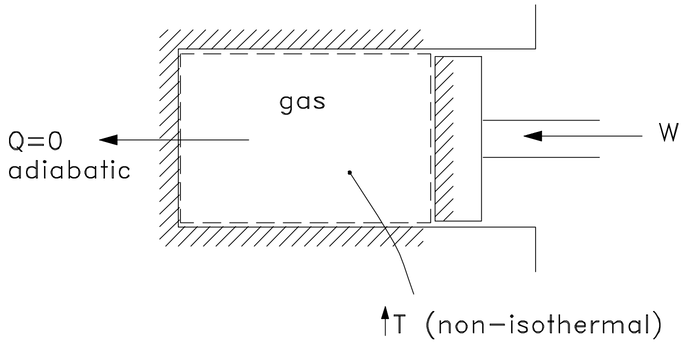
Week 1: Lecture 1

Some Interesting Points

- the internal energy of a system will depend to a large extent but not exclusively on temperature

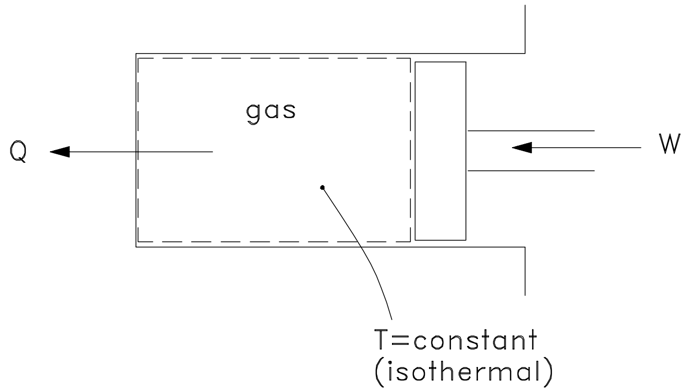
$$u = u(T) \quad (only)$$

- a change in the system temperature does not necessarily imply that heat transfer is taking place



- no change in the system temperature does not necessarily imply "no energy transfer as heat" (isothermal compression of a gas)

Isothermal Compression of a Gas

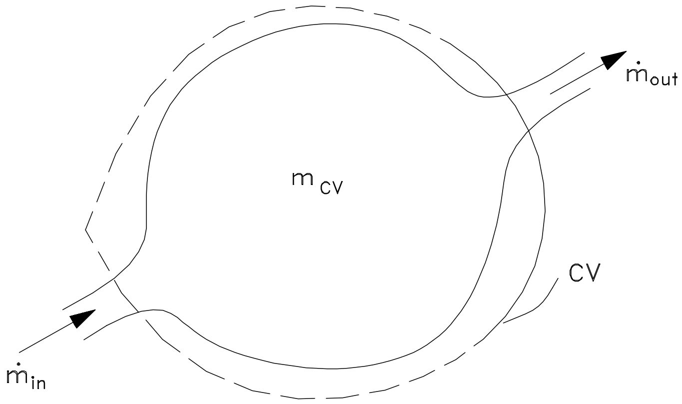


- Adiabatic does not always mean isothermal

Week 1: Lecture 2

Conservation of Mass

Control Volume Analysis



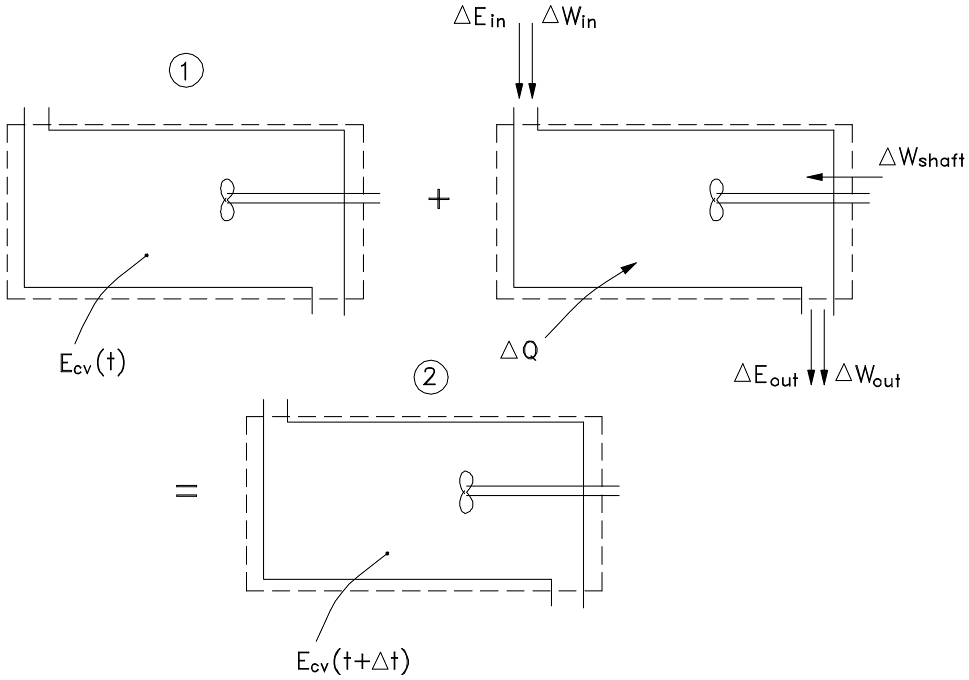
$$\left\{ \begin{array}{c} \text{Rate of Increase} \\ \text{of mass within} \\ \text{the CV} \end{array} \right\} = \left\{ \begin{array}{l} \text{Net rate of} \\ \text{mass flow} \\ \text{IN} \end{array} \right\} - \left\{ \begin{array}{c} \text{Net rate of} \\ \text{mass flow} \\ \text{OUT} \end{array} \right\}$$

$$\frac{d}{dt}(m_{CV}) = \dot{m}_{in} - \dot{m}_{out}$$

Week 1: Lecture 2

First Law of Thermodynamics

Control Volume Analysis



$$E_{CV}(t + \Delta t) - E_{CV}(t) = \Delta Q + \Delta W_{shaft} + \Delta m_{in}(e + Pv)_{in} - \Delta m_{out}(e + Pv)_{out}$$

The 1st law for a control volume can also be written as a rate equation by dividing the above equation by Δt and taking the limit as $\Delta t \to 0$

$$\frac{dE_{CV}}{dt} = \dot{Q} + \dot{W}_{shaft} + [\dot{m}(e + Pv)]_{in} - [\dot{m}(e + Pv)]_{out}$$

where $\dot{m} = \rho v^* A$

Note that

$$\begin{aligned} e + Pv &= \underbrace{u + Pv}_{} + \frac{(v^*)^2}{2} + gz \\ &= h(enthalpy) + KE + PE \end{aligned}$$

Week 1: Lecture 3

Second Law of Thermodynamics

Fundamentals

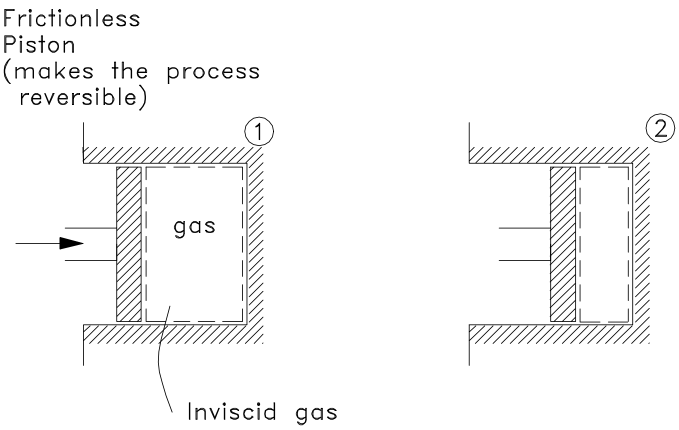
1. like mass and energy, every system has entropy. Entropy is a measure of the degree of microscopic disorder and represents our uncertainty about the microscopic state.
2. Unlike mass and energy, entropy can be produced but it can never be destroyed. That is, the entropy of a system plus its surroundings (i.e. and isolated system) can never decrease (2nd law).

$$(\Delta S)_{system} + (\Delta S)_{surr.} \geq 0$$

3. In a perfect crystal of a pure substance at $T = 0\ K$, the molecules are completely motionless and are stacked precisely in accordance with the crystal structure. Since entropy is a measure of microscopic disorder, then in this case $S = 0$. That is there is no uncertainty about the microscopic state.
4. For a given system, an increase in microscopic disorder results in a loss of ability to do useful work.
5. Energy transfer as heat takes place as work at the microscopic level but in a random, disorganized way. This type of energy transfer carries with it some chaos and thus results in entropy flow in or out of the system.

   Energy transfer by work is microscopically organized and therefore entropy-free.

Slow Adiabatic Compression of a Gas



A process $1 \rightarrow 2$ is said to be reversible if the reverse process $2 \rightarrow 1$ restores the system to its original state without leaving any change in either the system or its surroundings

$\rightarrow$ idealization where $S_2 = S_1 \Rightarrow \mathcal{P}_S = 0$

Week 1: Lecture 3

Analysis

$T_2 > T_1 \Rightarrow$ increased microscopic disorder

$V_2 < V_1 \Rightarrow$ reduced uncertainty about the whereabouts of molecules

The net effect is $S_2 = S_1$ and $\mathcal{P}_S = 0$. Therefore

$$Reversible + Adiabatic\ Process \Rightarrow Isentropic\ Process$$

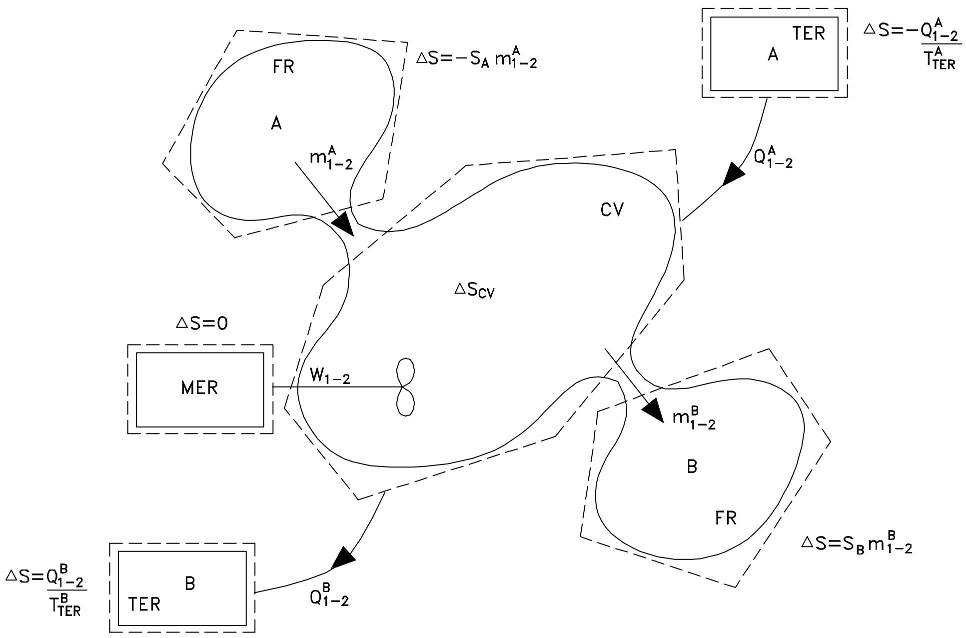
But does

$$Isentropic\ Process = Reversible + Adiabatic$$

NOT ALWAYS

Week 1: Lecture 3

Second Law Analysis for a Control Volume



$$\left(\frac{dS}{dt}\right)_{CV} = \left(s\dot{m} + \frac{\dot{Q}}{T}\right)_{in} - \left(s\dot{m} + \frac{\dot{Q}}{T}\right)_{out} + \dot{\mathcal{P}}_S$$

$$accumulation = in - out + generation$$

Week 2: Lecture 1

Equations of State

Perfect Gas

The defining equation for a perfect gas is

$$\frac{Pv}{T} = \frac{PV}{Tm} \quad = \quad \text{constant} \; = \; R$$

where R is a constant for a particular gas, defined as

$$R = \mathcal{R}/\hat{M}$$

and $\mathcal{R}$ the universal gas constant is 8.314 $kJ/(kmole \cdot K)$, and $\hat{M}$ is the molal mass defined as:

$$\hat{M} = \frac{m}{n} = \frac{mass}{amount\ of\ substance\ (mole)} = (kg/kmole)$$

In practice, no gases obeys this law rigidly but most gases tend towards it. An imaginary ideal gas which obeys this law is called a perfect gas.

The perfect gas assumption is viable when

- temperatures are considerably in excess of the critical temperature of the fluid
- at very low pressures

Gibb's Equation

By combining a first law energy balance and the differential form of the entropy equation we can obtain Gibb's equation

$$dQ \quad = \quad dW + dU \quad (energy\ equation)$$

$$dS \quad = \quad dQ/T$$

$$\boxed{ds = \frac{du}{T} + \frac{Pdv}{T}}$$

Week 2: Lecture 2

Isentropic Processes for Ideal Gases

Assume:

- constant specific heats over a wide range of temperature
- $ds = 0$
- $du = c_v dT \quad \equiv c_v = \left(\frac{\partial u}{\partial T}\right)_v$
- $dh = c_p dT \quad \equiv c_p = \left(\frac{\partial h}{\partial T}\right)_P$

For isentropic processes

$$\frac{T_2}{T_1} = \left(\frac{v_1}{v_2}\right)^{k-1} = \left(\frac{P_2}{P_1}\right)^{(k-1)/k}$$

where

$$k = \frac{c_p}{c_v}$$

Polytropic Processes for Ideal Gases

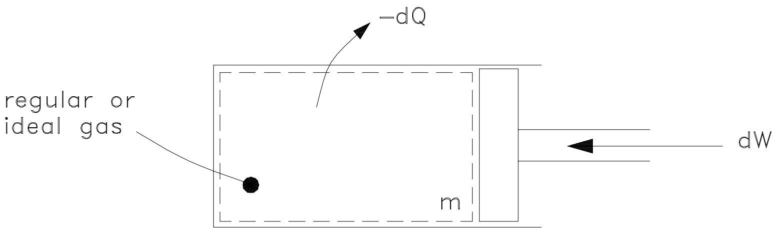
The isentropic process is a special case of a more general process known as a polytropic process, where

$$Pv^n = constant$$

isothermal process	$n = 1$	$Pv = RT = constant$
isobaric process	$n = 0$	$Pv^0 = constant$
isentropic process	$n = k$	$k = c_p/c_v$
constant volume process	$n \to \infty$	$Pv^\infty = constant$

Week 2: Lecture 2

Reversible Compression/Expansion for a Fixed Mass:



From the first law

$$mdu = dW - dQ \quad (1)$$

From the second law

$$mds = -\frac{dQ}{T} \quad (2)$$

Combining (1) and (2) through the dQ term

$$mdu = dW + mTds \quad (3)$$

From Gibb's equation

$$mdu = mTds - mPdv \quad (4)$$

Combining (3) and (4)

$$\frac{W_{1-2}}{m} = \int_1^2 -Pdv$$

This is suitable for reversible processes with any gas, real or ideal.

Week 2: Lecture 2

Summary

For a perfect gas with constant specific heat

$$
\begin{aligned}
Pv &= RT \\
u_2 - u_1 &= c_v(T_2 - T_1) \\
h_2 - h_1 &= c_p(T_2 - T_1) \\
s_2 - s_1 &= c_v \ln \frac{T_2}{T_1} + R \ln \frac{v_2}{v_1} \\
&= c_p \ln \frac{T_2}{T_1} + R \ln \frac{P_2}{P_1} \\
&= c_p \ln \frac{v_2}{v_1} + R \ln \frac{P_2}{P_1} \\
R &= c_p - c_v
\end{aligned}
$$

For a reversible, isothermal process for an ideal gas

$$
\begin{aligned}
W_{1-2} &= mRT \ln \frac{V_1}{V_2} \\
&= mRT \ln \frac{P_1}{P_2}
\end{aligned}
$$

Week 2: Lecture 3

Incompressible Liquids

- a substance whose volume cannot be changed
- no substance is truly incompressible, but this model is good for most liquids and solids

A simple compressible substance has only one work mode and that work mode is Pdv work.

If a substance is assumed to be incompressible then its internal energy, for example, cannot be varied independently by work transfer but it can be varied by heat transfer at constant volume.

For a incompressible substance

$$c_p = c_v = c$$

From Gibb's equation we can see that

$$s_2 - s_1 = c \ln \left(\frac{T_2}{T_1} \right)$$

For an isothermal process ($T_2 = T_1$) we see that the process becomes isentropic ($S_2 - S_1$).

In summary

$$\begin{aligned} u_2 - u_1 &= c(T_2 - T_1) \\ h_2 - h_1 &= (u_2 - u_1) + v(P_2 - P_1) \\ s_2 - s_1 &= c \ln(T_2/T_1) \end{aligned}$$

Week 2: Lecture 3

Specific Heat of an Incompressible Substance

In a general sense, $u = u(T, v)$, which when differentiated will give:

$$du = \left.\frac{\partial u}{\partial T}\right|_v dT + \left.\frac{\partial u}{\partial v}\right|_T dv \tag{1}$$

where the specific heat at constant volume is defined as:

$$c_v = \left(\frac{\partial u}{\partial T}\right)_v \tag{2}$$

In a similar manner, $h = h(T, P)$ and

$$dh = \left.\frac{\partial h}{\partial T}\right|_P dT + \left.\frac{\partial h}{\partial P}\right|_T dP \tag{3}$$

where the specific heat at constant pressure is defined as:

$$c_P = \left(\frac{\partial h}{\partial T}\right)_P \tag{4}$$

Combining Eqs. 1-4

$$du = c_v dT + \left.\frac{\partial u}{\partial v}\right|_T dv \tag{5}$$

$$dh = c_P dT + \left.\frac{\partial h}{\partial P}\right|_T dP \tag{6}$$

But we also know that $h = u + Pv$, which when differentiated gives

$$dh = du + d(Pv) = du + Pdv + vdP \tag{7}$$

but $dv \to 0$ for an incompressible substance.

Substituting Eq. 5 and 6 into Eq. 7, we obtain

$$c_P dT + \left.\frac{\partial h}{\partial P}\right|_T dP = c_v dT + \left.\frac{\partial u}{\partial v}\right|_T dv + vdP \tag{8}$$

Week 2: Lecture 3

But if we take the derivative of h with respect to P at constant temperature we obtain

$$\begin{aligned} \left.\frac{\partial h}{\partial P}\right|_T &= \left.\frac{\partial (u(T,v))}{\partial P}\right|_T + P\left.\frac{\partial v}{\partial P}\right|_T + v\left.\frac{\partial P}{\partial P}\right|_T \\ &= v \end{aligned} \tag{9}$$

Therefore Eq. 8 can be written as

$$c_P dT + v dP = c_v dT + v dP \tag{10}$$

Therefore

$$c_p = c_v = c \tag{11}$$

Week 3: Lecture 1

Second Law Analysis of Systems

Availability: The theoretical maximum amount of work that can be obtained from a system at a given state P_1 and T_1 when interacting with a reference atmosphere at the constant pressure and temperature P_0 and T_0.

Availability describes the work potential of a given system. It is also refered to as "exergy".

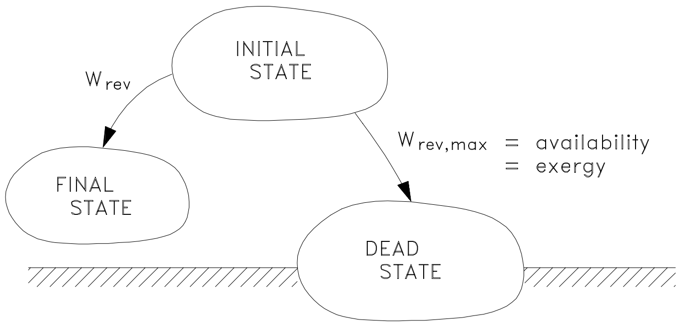
The following observations can be made about availability:

1. Availability is a property - since any quantity that is fixed when the state is fixed is a property. For a system at state 1 and specified values of the atmosphere of T_0 and P_0, the maximum useful work that can be produced is fixed.
2. Availability is an external property - since its value depends upon an external datum - the temperature and pressure of the dead state.
3. Availability is never negative - as long as a system is not at its dead state, it will always be possible to produce some useful work by interacting with the reference atmosphere.
4. Availability of a system is 0 at its dead state when $T = T_0$ and $P = P_0$. It is not possible for the system to interact with the reference atmosphere at the dead state. The system is said to be in thermodynamic equilibrium with its surroundings.
5. Unless otherwise stated, assume the dead state to be:

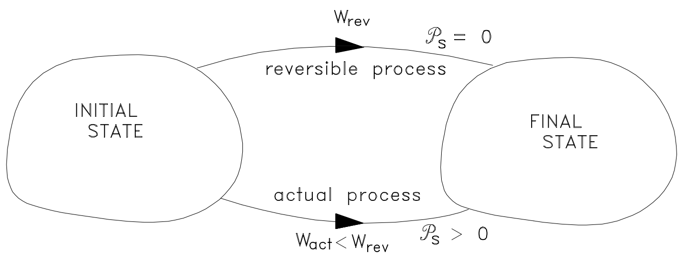
$$P_0 = 1\ atm$$

$$T_0 = 25°C$$

The maximum work is obtained through a reversible process to the dead state.

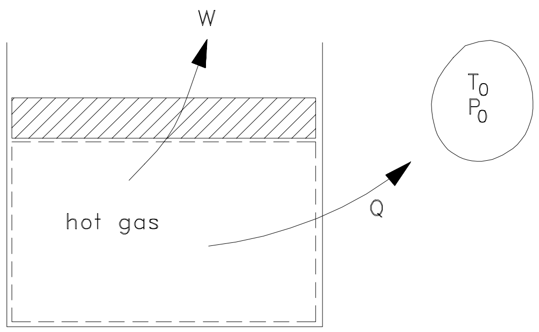


Week 3: Lecture 1



$$\underbrace{IRREVERSIBILITY}_{I} = \underbrace{REVERSIBLE\ WORK}_{W_{rev}} - \underbrace{ACTUAL\ WORK}_{W_{act}}$$

Control Mass Analysis



- part of the work done is against the surroundings

$$W_{sur} = P_0(V_2 - V_1)$$

- this is unavoidable $\rightarrow$ this is not useful work. Nothing is gained by pushing the atmosphere away.
- note the direction of W
-

$$W_{useful} = W_{actual} - W_{sur}$$

To find W_{actual}:

From the 1st law

$$E_1 - Q - W_a = E_2 \quad \rightarrow \quad Q = E_1 - E_2 - W_a$$

From the 2nd law

$$\begin{aligned} \mathcal{P}_s &= \Delta S_{system} + \Delta S_{sur} \geq 0 \\ &= S_2 - S_1 + \frac{Q}{T_0} \end{aligned}$$

Week 3: Lecture 1

Rearranging the above equation gives

$$
\begin{aligned}
W_a &= (E_1 - E_2) - T_0(S_1 - S_2) - T_0\mathcal{P}_s \\
W_u &= (E_1 - E_2) - T_0(S_1 - S_2) + P_0(V_1 - V_2) - T_0\mathcal{P}_s \\
W_{rev} &= (E_1 - E_2) - T_0(S_1 - S_2) + P_0(V_1 - V_2)
\end{aligned}
$$

Define

$$
\begin{aligned}
\Phi &= CONTROL\ MASS\ AVAILABILITY \\
&= W_{rev}\ (in\ going\ to\ the\ dead\ state) \\
&= (E_1 - E_0) - T_0(S_1 - S_0) + P_0(V_1 - V_0) \\
&= (h_1 - h_0) - T_0(S_1 - S_0)
\end{aligned}
$$

where the specific availability is defined as

$$\phi = \frac{\Phi}{m}$$

In going from one state to another the reversible work is

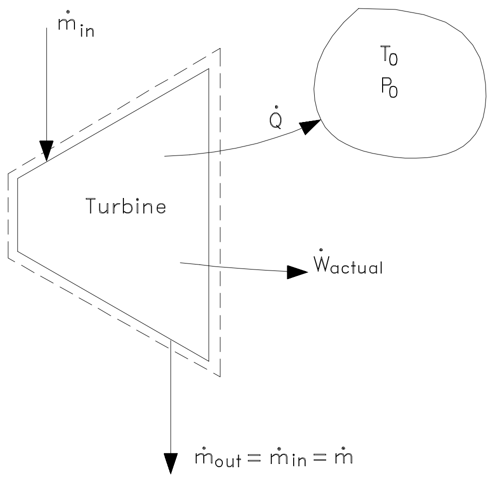
$$W_{rev} = \Phi_1 - \Phi_2$$

The availability destroyed is

$$I = W_{rev} - W_{useful} = T_0\mathcal{P}_s$$

Week 3: Lecture 2

Control Volume Analysis



From the 1st law

$$\cancelto{0}{\frac{dE_{cv}}{dt}} = -\dot{W}_a - \dot{Q} + [\dot{m}(h + \frac{(v^*)^2}{2} + gz)]_{in} - [\dot{m}(h + \frac{(v^*)^2}{2} + gz)]_{out}$$

From the 2nd law

$$\cancelto{0}{\frac{dS_{cv}}{dt}} = \left(\dot{m}s + \frac{\cancelto{0}{\dot{Q}}}{T_{TER}}\right)_{in} - \left(\dot{m}s + \frac{\dot{Q}}{T_0}\right)_{out} + \dot{\mathcal{P}}_s$$

Which leads to the actual work output of the turbine, given as

$$\dot{W}_a = \dot{m}\left[-T_0\Delta s + \Delta h + \Delta KE + \Delta PE\right] - (T_0\dot{\mathcal{P}}_s)$$

The specific flow availability is given as

$$\psi = -T_0(s - s_0) + (h - h_0) + \left(\frac{(v^*)^2}{2} - \frac{(\cancelto{0}{v_0^*})^2}{2}\right) + g(z - \cancelto{0}{z_0})$$

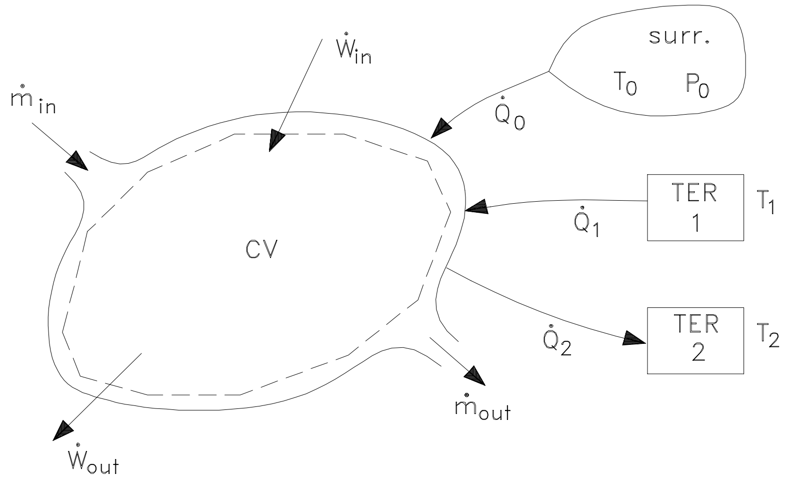
Week 3: Lecture 2

For a steady state, steady flow process

$$\begin{aligned} \dot{W}_{rev} &= (\dot{m}\psi)_{in} - (\dot{m}\psi)_{out} \\ \dot{I} &= \dot{W}_{rev} - \dot{W}_a = T_0 \dot{\mathcal{P}}_s \end{aligned}$$

Week 3: Lecture 3

The General Exergy Equation



From the 1st law

$$\frac{dE_{cv}}{dt} = \dot{W}_{in} - \dot{W}_{out} + \dot{Q}_0 + \dot{Q}_1 - \dot{Q}_2 + [\dot{m}(e + Pv)]_{in} - [\dot{m}(e + Pv)]_{out}$$

From the 2nd law

$$\frac{dS_{cv}}{dt} = \left(\dot{m}s + \frac{\dot{Q}_0}{T_0} + \frac{\dot{Q}_1}{T_1}\right)_{in} - \left(\dot{m}s + \frac{\dot{Q}_2}{T_2}\right)_{out} + \dot{\mathcal{P}}_s$$

Which leads to the generalized exergy equation

$$\frac{d\Phi_{CV}}{dt} = P_0\frac{dV_{CV}}{dt} + \left[\dot{W} + \dot{m}\psi + \dot{Q}\left(1 - \frac{T_0}{T_{TER}}\right)\right]_{in} - \left[\dot{W} + \dot{m}\psi + \dot{Q}\left(1 - \frac{T_0}{T_{TER}}\right)\right]_{out} - \dot{I}$$

Week 3: Lecture 3

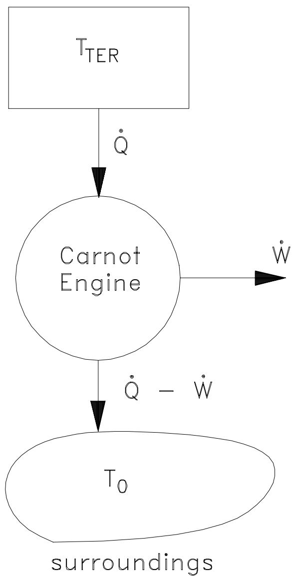
where

$$\dot{I} = T_0\dot{\mathcal{P}}_s = exergy\ destruction\ rate$$

$$\Phi_{CV} = [(E - E_0) + P_0(V - V_0) - T_0(S - S_0)]_{CV} = nonflow\ exergy$$

$$\psi = (h - h_0) - T_0(s - s_0) + \frac{1}{2}[(v^*)^2 - (v_0^*)^2] + g(z - z_0) = flow\ exergy$$

$$\dot{W}_{useful} = (\underbrace{\dot{W}_{in} - \dot{W}_{out}}_{\dot{W}_{actual}}) - \left(P_0\frac{dV_{CV}}{dt}\right)$$



Notice that:

$$\eta = \frac{\dot{W}}{\dot{Q}} = 1 - \frac{T_0}{T_{TER}}$$

Therefore

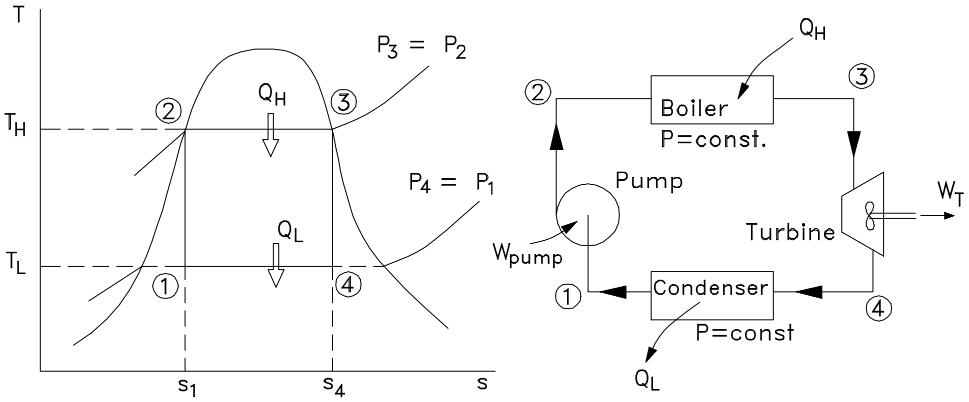
$$\dot{W} = \underbrace{\dot{Q}\left(1 - \frac{T_0}{T_{TER}}\right)}_{\substack{\text{appears in the}\\ \text{general exergy equation}}}$$

This term represents the work potential (exergy) of a given TER with respect to the surroundings (dead state) at T_0.

Week 4: Lecture 1

Carnot Cycle

- an ideal theoretical cycle that is the most efficient conceivable
- based on a fully reversible heat engine
- in practice the thermal efficiency of real world heat engines are about half that of the ideal Carnot cycle



Process	State Points	Description
Pump	$1 \rightarrow 2$	isentropic compression from $T_L \rightarrow T_H$ to return vapour to a liquid state
Heat Supply	$2 \rightarrow 3$	heat is supplied at constant temperature and pressure
Work Output	$3 \rightarrow 4$	the vapour expands isentropically from the high pressure and temperature to the low pressure
Condenser	$4 \rightarrow 1$	the vapour which is wet at 4 has to be cooled to state point 1

Week 4: Lecture 1

Cycle Efficiency

- defined as the net work output divided by the gross heat supplied

$$\begin{aligned}\eta &= \frac{W_{net}}{Q_H} \\ &= \frac{Q_H - Q_L}{Q_H} \\ &= 1 - \frac{T_L}{T_H}\end{aligned}$$

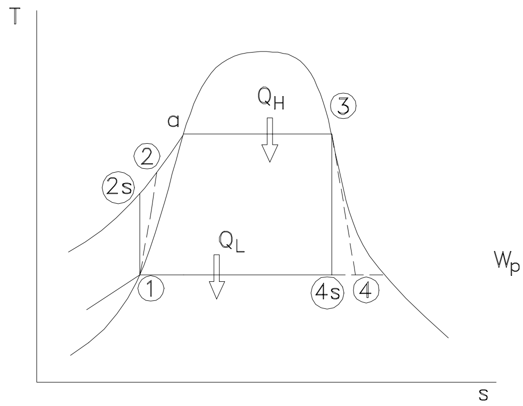
Practical Problems

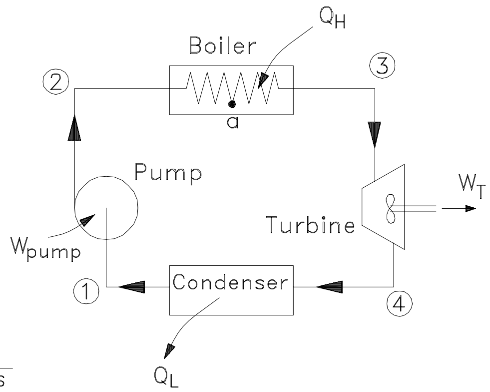
- at state point 1 the steam is wet at T_L and it is difficult to pump water/steam (two phase) to state point 2
- the pump can be sized smaller if the fluid is 100% liquid water
- the pump is smaller, cheaper and more efficient
- can we devise a Carnot cycle to operate outside the wet vapour region
  - between state points 2 and 3 the vapour must be isothermal an at different pressures - this is difficult to achieve
  - the high temperature and pressure at 2 and 3 present metallurgical limitations

The net effect is that the Carnot cycle is <u>not</u> feasible for steam power plants.

Week 4: Lecture 2

Rankine Cycle





- working fluid is alternately vaporized and condensed as it recirculates is a closed cycle
- water is typically used as the working fluid because of its low cost and relatively large value of enthalpy of vaporization
- the condensation process is allowed to proceed to completion → state point 1 - saturated liquid
- the water at state point 1 can be conveniently pumped to the boiler pressure at state point 2
- but the water is <u>not</u> at the saturation temperature corresponding to the boiler pressure
- heat must be added to change the water at 2 to saturated water at 'a'

Rankine Efficiency

$$\eta_R = \frac{\textit{net work output}}{\textit{heat supplied in the boiler}}$$

$$= \frac{(h_3 - h_4) + (h_1 - h_2)}{(h_3 - h_2)}$$

$$= \frac{(h_3 - h_4) + (h_1 - h_2)}{(h_3 - h_1) + (h_1 - h_2)}$$

If the pump work is considered negligible in relation to the turbine work i.e. $(h_1 - h_2) \rightarrow 0$

Therefore

$$\eta_R = \frac{(h_3 - h_4)}{(h_3 - h_1)}$$

If the pump work is included and we consider the fluid to be incompressible

$$(h_1 - h_2) = v(P_1 - P_2)$$

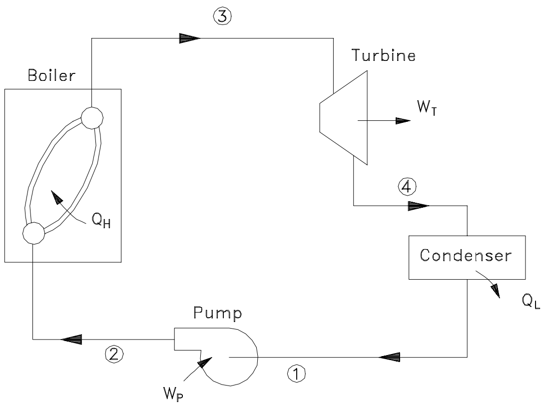
Since the actual process is irreversible, an isentropic efficiency can be defined such that

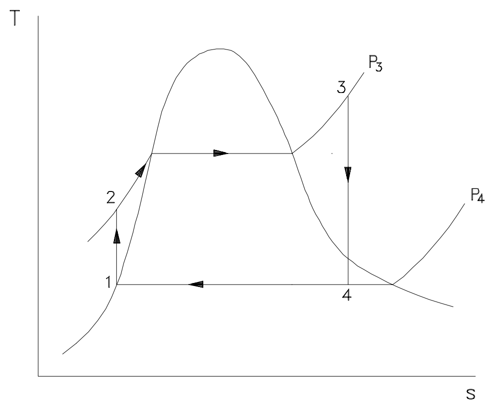
$$\text{Expansion process} \quad \Rightarrow \quad \text{Isentropic efficiency} \quad = \quad \frac{\text{actual work}}{\text{isentropic work}}$$

$$\text{Compression process} \quad \Rightarrow \quad \text{Isentropic efficiency} \quad = \quad \frac{\text{isentropic work}}{\text{actual work}}$$

Rankine Cycle with Superheat

- the average temperature at which heat is supplied in the boiler can be increased by superheating the steam
- the dry saturated steam from the boiler is passed through a second bank of smaller bore tubes within the boiler until the steam reaches the required temperature

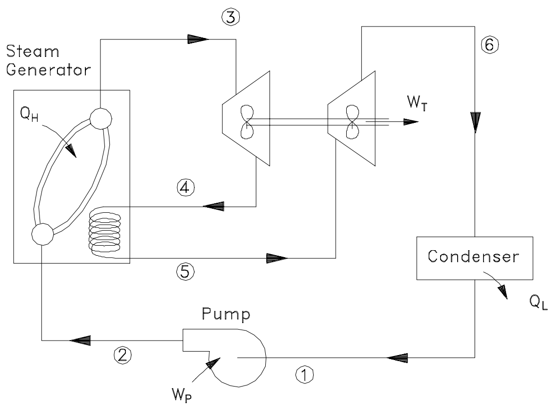


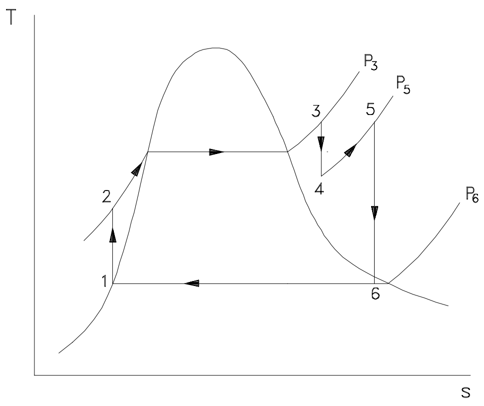


Week 4: Lecture 2

Rankine Cycle with Reheat

- the wetness at the exhaust of the turbine should be no greater that 10% - this can result in physical erosion of the turbine blades
- but high boiler pressures are required for high efficiency - tends to lead to a high wetness ratio
- to improve the exhaust steam conditions, the steam can be reheated with the expansion carried out in two steps

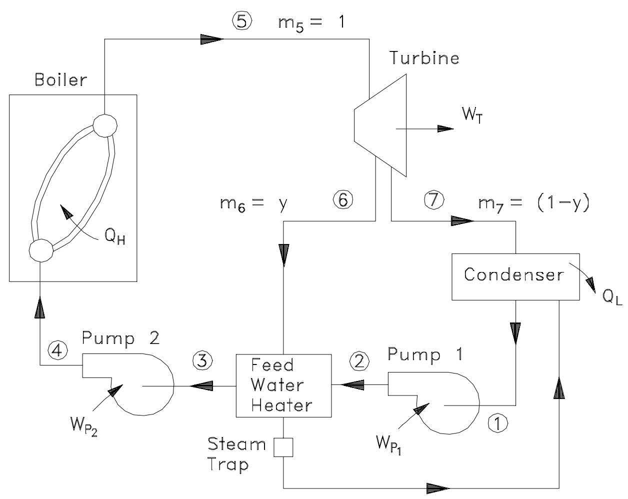


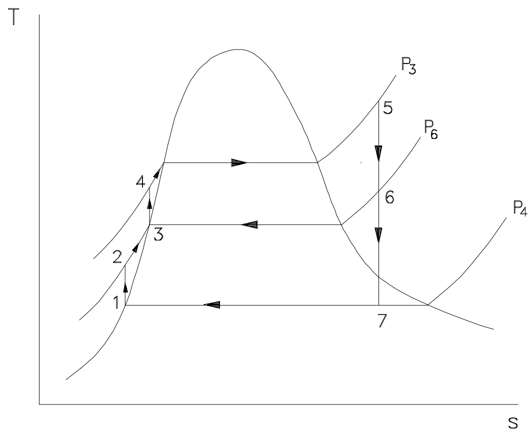


Week 4: Lecture 3

Rankine Cycle with Regeneration

- Carnot cycle has efficiency - $\eta = 1 - T_L/T_H$
  - add Q_H at as high a T_H as possible
  - reject Q_L at as low a T_L as possible
- the Rankine cycle can be used with a “Feedwater Heater” to heat the high pressure sub-cooled water at the pump exit to the saturation temperature
  - most of the heat addition (Q_H) is done at high temperature





Week 4: Lecture 3

Feedwater Heaters

1. **OPEN FWH** - streams mix $\rightarrow$ high temperature steam with low temperature water at constant pressure)
   - working fluid passes isentropically through the turbine stages and pumps
   - steam enters the first state turbine at state 1 and expands to state 2 - where a fraction of the total flow is bled off into an open feedwater heater at P_2
   - the rest of the steam expands into the second stage turbine at state point 3 - this portion of the fluid is condensed and pumped as a saturated liquid to the FWH at P_2
   - a single mixed stream exists the FWH at state point 6
2. **CLOSED FWH** - heat exchangers, streams do not mix (different pressures)
   - the incoming feedwater does not mix with the extracted steam
   - both streams flow separately through the heater
   - this allows the two streams to be at different pressures
   - a steam trap which allows only liquid to pass, is inserted in the condensed steam line, leaving the heater to insure all the extracted steam condenses before it leaves the heater
   - the condensate is then passed to the low pressure line of the condenser

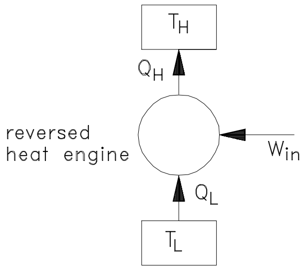
Week 5: Lecture 1

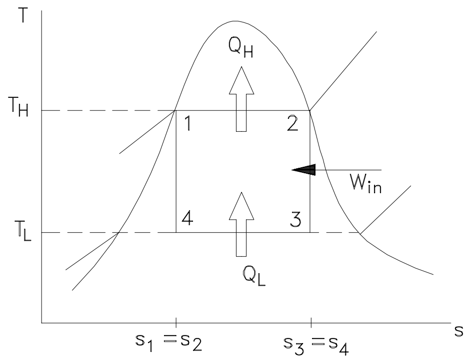
Refrigeration Cycle

A refrigeration system removes thermal energy from a low-temperature region and transfers heat to a high-temperature region.

The Carnot cycle can serve as the initial model of the ideal refrigeration cycle.

- operates as a reversed heat cycle engine
- transfer a quantity of heat, Q_L from a cold source at temperature, T_L





where:

$$
\begin{aligned}
Q_L &= T_L(s_3 - s_2) \\
Q_H &= T_H(s_4 - s_1) \\
W_{in} &= Q_{net} \\
&= (T_H - T_L)(s_3 - s_2)
\end{aligned}
$$

Coefficient of Performance

$$COP = \frac{benefit}{cost}$$

$$COP_{refrig} = \frac{Q_L}{W_{in}} = \frac{1}{\dfrac{T_H}{T_L} - 1} = \frac{T_L}{T_H - T_L}$$

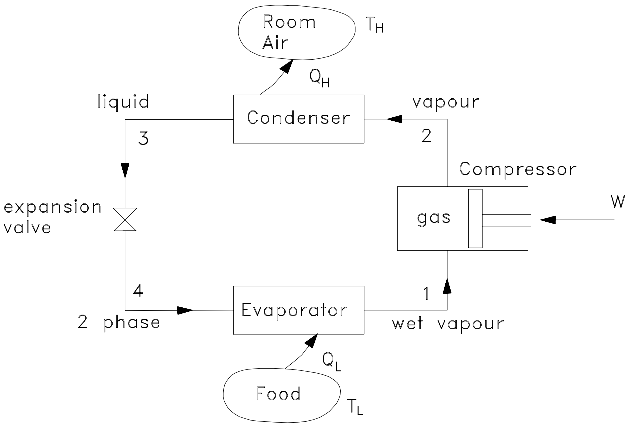
$$COP_{heat\ pump} = \frac{Q_H}{W_{in}} = \frac{1}{1 - \dfrac{T_L}{T_H}} = \frac{T_H}{T_H - T_L}$$

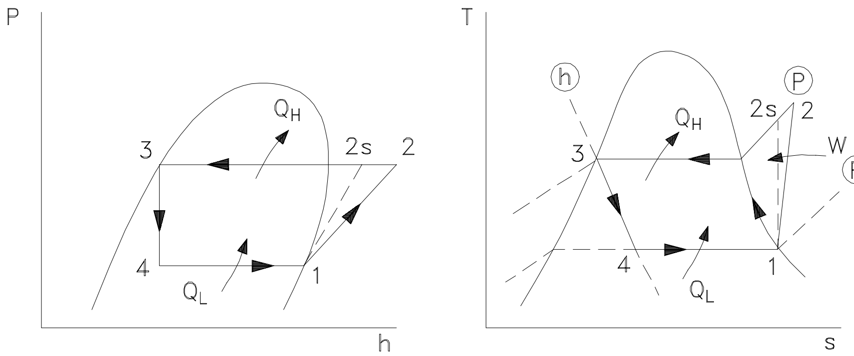
Week 5: Lecture 1

Vapour Compression Refrigeration Cycle

Assumptions for Ideal VCRC

- irreversibilities within the evaporator, condenser and compressor are ignored
- no frictional pressure drops
- refrigerant flows at constant pressure through the two heat exchangers
- stray heat losses to the surroundings are ignored
- compression process is isentropic





1-2s:	isentropic compression of the refrigerant
2s-3:	heat transfer from the refrigerant as it flows at constant pressure through the condenser
3-4	throttling process from state 3 to a two phase liquid-vapour mixture at state 4
4-1	heat transfer to the refrigerant as it flows at constant pressure through the evaporator

Week 5: Lecture 2

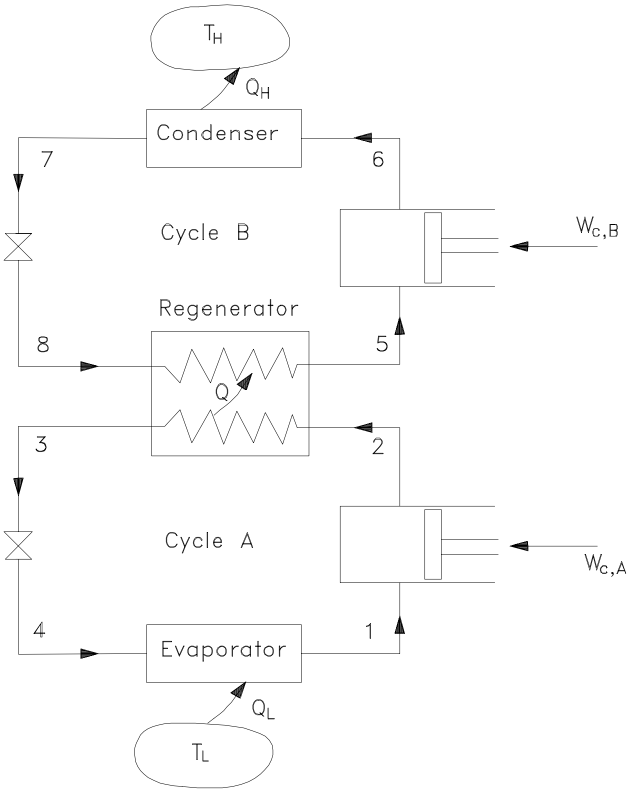
Common Refrigerants

R12	- CCl_2F_2 - dichlorofluoromethane
R22	- $CHClF_2$ - has less chlorine, a little better for the environment
R134a	- $C_2H_2F_4$ - tetrafluorethane - no chlorine
Ammonia	- NH_3 - corrosive and toxic

How to Choose a Refrigerant

- the heat of vaporization of the refrigerant should be high. The higher h_{fg}, the greater the refrigerating effect per kg of fluid circulated
- the specific heat of the refrigerant should be low. The lower the specific heat, the less heat it will pick up for a given change in temperature during the throttling or in flow through the piping, and consequently the greater the refrigerating effect per kg of refrigerant
- the specific volume of the refrigerant should be low to minimize the work required per kg of refrigerant circulated
- the cirtical temperature of the refrigerant should be higher than the condesning pressure to prevent excessive power comsumption
- since evaporation and condenser temperatures are fixed by the temperatures of the surroundings - selection is based on operating pressures in the evaporator and the condenser
- selection is based on the suitability of the pressure-temperature relationship of the refrigerant
- other factors include:
  - chemical stability
  - toxicity
  - cost
  - environmental friendliness
  - does not result in very low pressures in the evaporator (air leakage)
  - does not result in very high pressures in the condenser (refrigerant leakage)

Week 5: Lecture 2

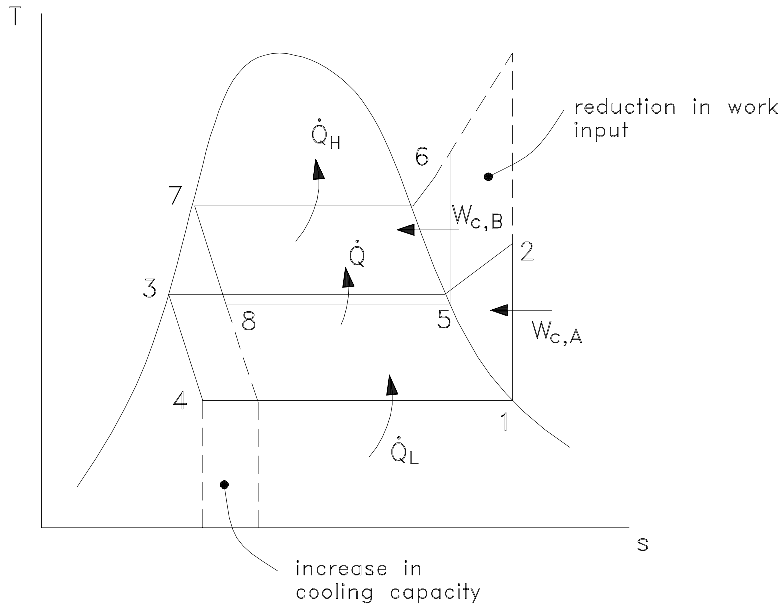


Cascade Refrigeration System

- combined cycle arrangements
- two or more vapour vapour compression refrigeration cycles are combined

Advantages

- the refrigerants can be selected to have reasonable evaporator and condenser pressures in the two or more temperature ranges



$$COP = \frac{Q_L(\uparrow)}{W_{net}(\downarrow)} \quad overall(\uparrow)$$

Week 5: Lecture 3

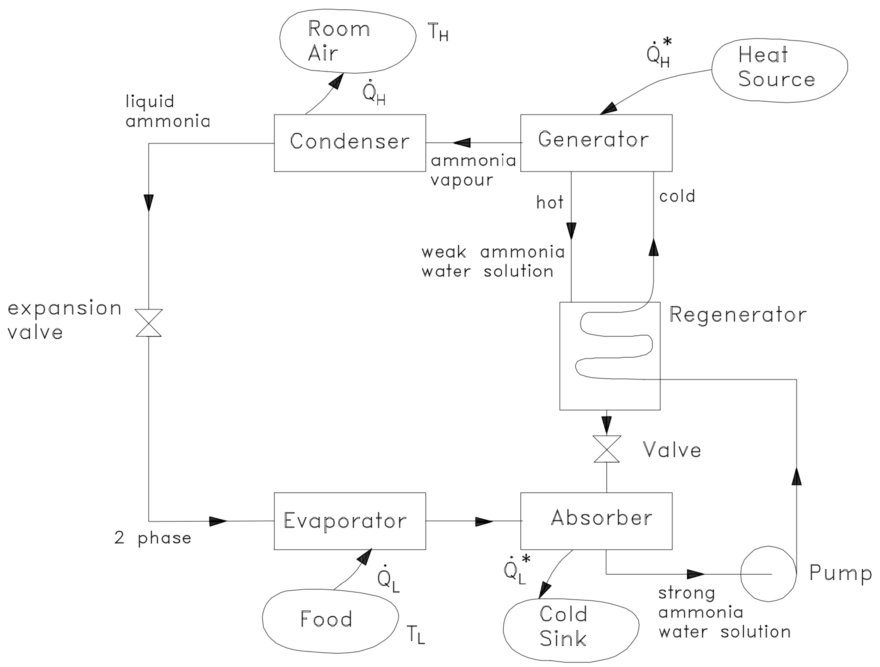
Absorption Refrigeration System

Differences with VCRC

VCRC	Absorption RS
• vapour is compressed between the evaporator and the condenser	• the refrigerant is absorbed by an absorbent material to form a liquid solution
• process is driven by work	• heat is added to the process to retrieve the refrigerant vapour from the liquid solution
	• process is driven by heat

Common Refrigerant/Absorber Combinations

Refrigerant	Absorber
ammonia	water
water	lithium bromide



Week 5: Lecture 3

- absorption is proportional to $1/T \quad \Rightarrow$ the cooler the better
- in the generator, ammonia is driven out of the solution by Q_H^*
- in the absorber, ammonia vapour is absorbed into the water at low T and P maintained by means of Q_L^*
- big savings in using a liquid pump as opposed to the gas compressor in a conventional refrigerator, but it must be weighed against the cost of extra components

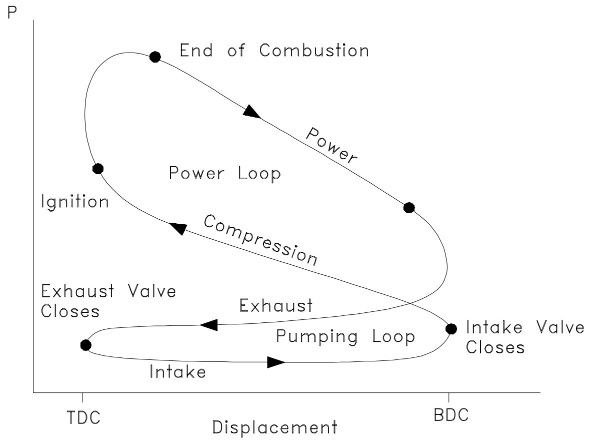
Process

- ammonia circulates through the condenser, expansion valve and evaporator (as in the VCRC)
- compressor is replaced by an absorber, pump, generator and a valve
- in the absorber, ammonia vapour is absorbed by liquid
- the process is exothermic
- the solution is cooled - more ammonia can be dissolved in water
- pump raises solution to the pressure of the generator
- generator drives ammonia vapour out of solution (endothermic reaction)
- vapour passes to the condenser
- a weak ammonia, water solution passes back to the absorber through a valve

Week 7: Lecture 1

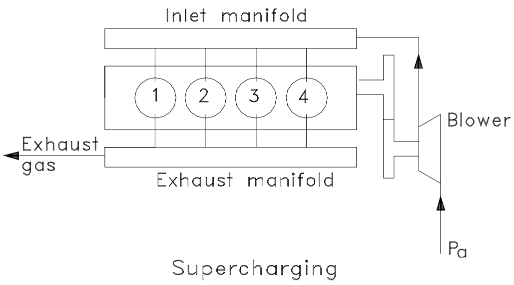
IC Engines

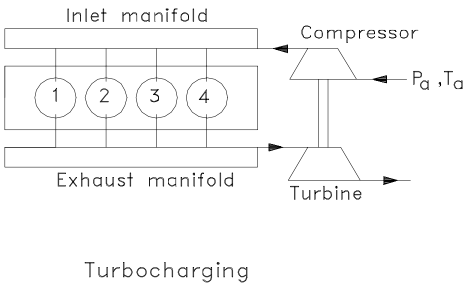
1. **spark ignition**:
   - a mixture of fuel and air is ignited by a spark plug
   - applications requiring power to about 225 kW (300 HP)
   - relatively light and low in cost



2. **compression ignition engine**:
   - air is compressed to a high enough pressure and temperature that combustion occurs when the fuel is injected
   - applications where fuel economy and relatively large amounts of power are required

Power Enhancements:





1. **Supercharging**:
   - compressor is run directly from the engine shaft
2. **Turbocharging**:
   - compressor is run by the exhaust gases
   - more efficient than a supercharger

Week 7: Lecture 2

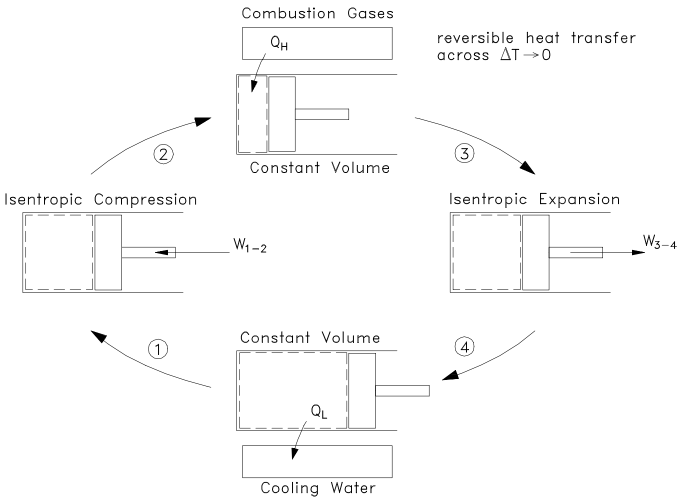
Air Standard Cycle

Assumptions

- air is an ideal gas with constant c_p and c_v
- no intake or exhaust processes
- the cycle is completed by heat transfer to the surroundings
- the internal combustion process is replaced by a heat transfer process from a TER
- all processes are reversible
- heat addition occurs instantaneously while the piston is at TDC

Otto Cycle

- consists of four internally reversible processes
- heat is transferred to the working fluid at constant volume



1 → 2 isentropic compression or air

2 → 3 constant volume heat addition at TDC (represents the ignition process)

3 → 4 isentropic expansion (power stroke)

4 → 1 constant volume heat rejection at BDC

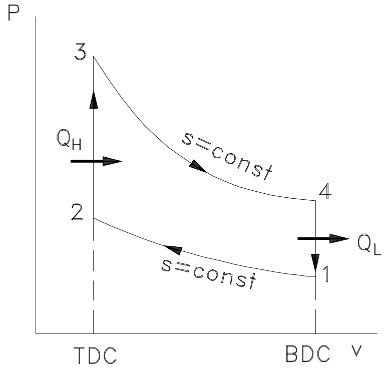
$$\eta_{Otto} = 1 - r^{1-k} \qquad \textbf{where :} \qquad r = \frac{V_1}{V_2} = \frac{V_4}{V_3} \quad \textbf{(compression ratio)}$$

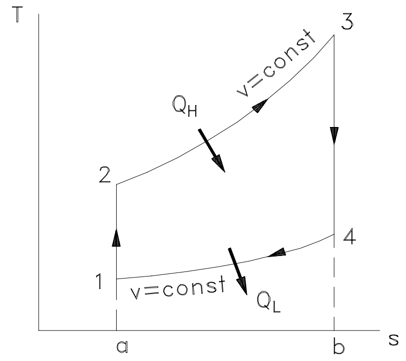
Otto cycle limitations on compression ratio

- increased tendency for the fuel to detonate as the compression ratio increases
- the pressure wave gives rise to engine knock
- can be reduced by adding tetraethyl lead to the fuel
- not good for the environment

Week 7: Lecture 2

Otto Cycle Process Diagrams





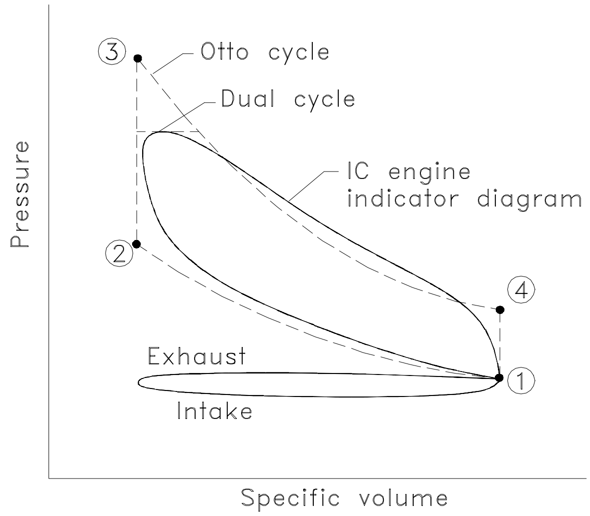
The Diesel Cycle

- heat is transferred to the working fluid at constant pressure

$$\eta_{Diesel} = 1 - \frac{1}{r^{k-1}}\left(\frac{1}{k}\right)\left(\frac{r_v^k - 1}{r_v - 1}\right) \qquad \textbf{where :} \qquad r_v = \frac{V_3}{V_2} = \quad \textbf{(volume ratio)}$$

Comparison of the Otto and the Diesel Cycle

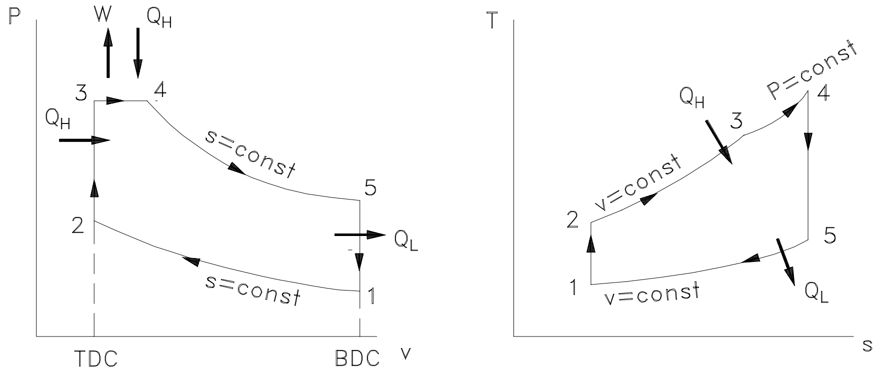
- $\eta_{Otto} > \eta_{Diesel}$ for the same compression ratio
- <u>but</u> a diesel engine can tolerate a higher ratio since only air is compressed in a diesel cycle and spark knock is not an issue
- direct comparisons are difficult



Week 7: Lecture 3

Dual Cycle

- this is a better representation of the combustion process in both the gasoline and the diesel engines
- in a compression ignition engine, combustion occurs at TDC and the piston moves down to maintain a constant pressure



1 - 2	isentropic compression
2 - 3	constant volume fuel injection and initial combustion
	→ modelled as a reversible, constant volume heat addition
3 - 4	isobaric expansion as the fuel burns
	→ modelled as a reversible, isobaric heat addition
4 - 5	isentropic expansion
5 - 1	exhausting of spent gases
	→ modelled as a constant volume heat rejection process

- $\boldsymbol{\eta_{dual} = 1 - \dfrac{r_p r_v^k - 1}{[(r_p - 1) + k r_p (r_v - 1)]\, r^{k-1}}}$

  where

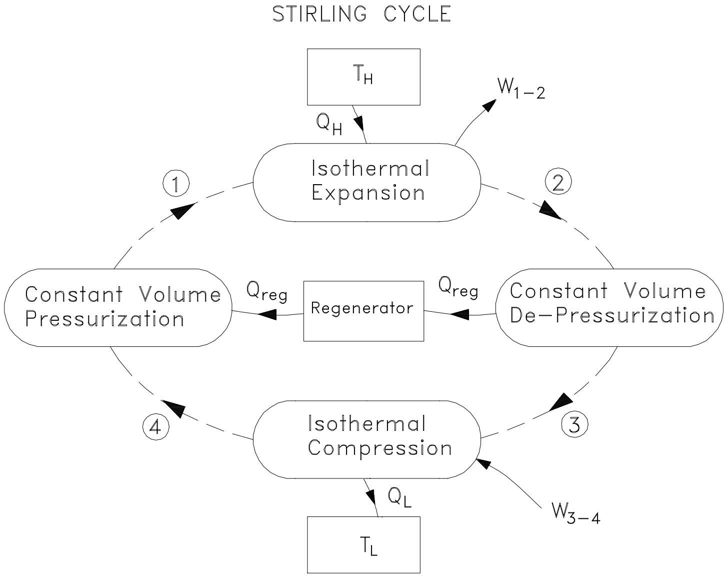
$$\boldsymbol{r = \frac{V_1}{V_2}} \quad \textbf{(compression ratio)}$$

$$\boldsymbol{r_v = \frac{V_4}{V_3}} \quad \textbf{(volume ratio)}$$

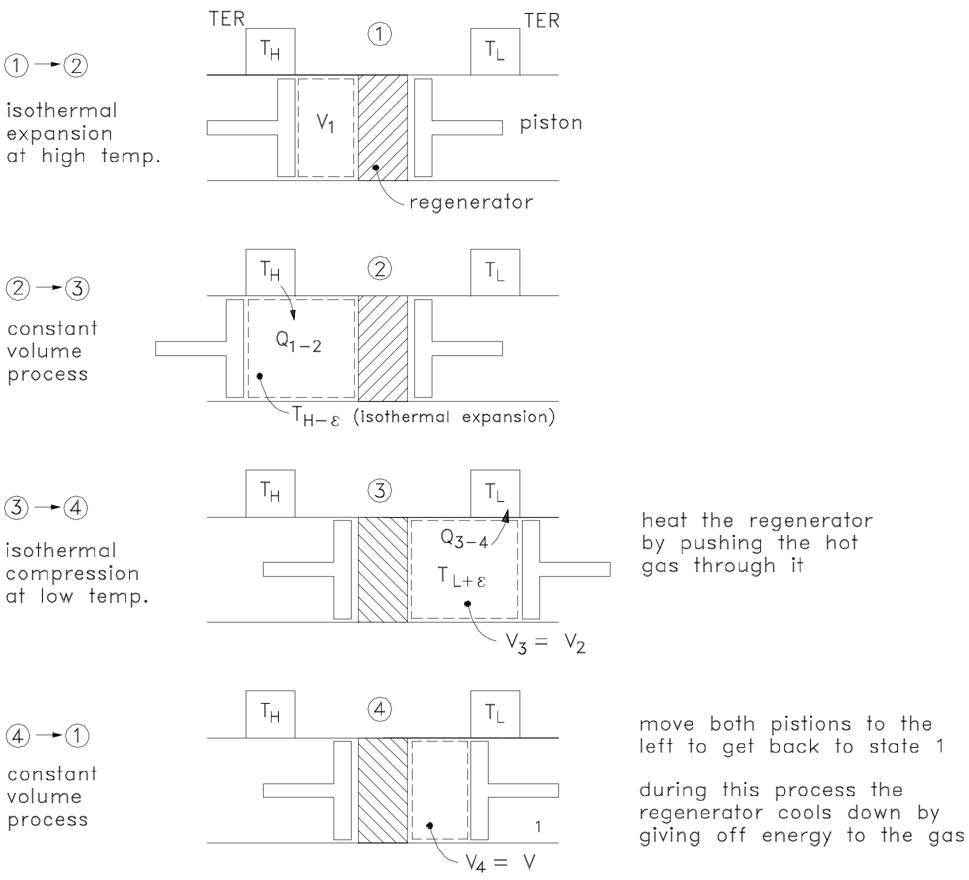
$$\boldsymbol{r_p = \frac{P_3}{P_2}} \quad \textbf{(pressure ratio)}$$

Week 7: Lecture 3

Stirling Cycle

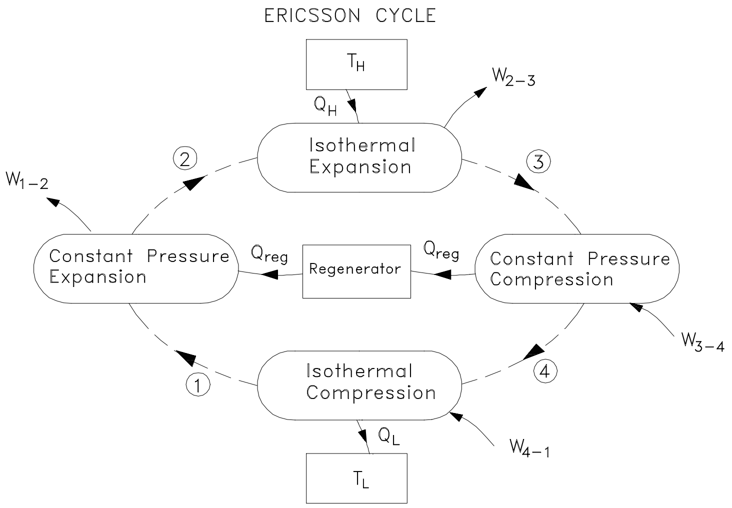


- reversible regenerator used as an energy storage device
- possible to recover all heat given up by the working fluid in the constant volume cooling process
- all the heat received by the cycle is at T_H and all heat rejected at T_L
- $\eta_{Stirling} = 1 - T_L/T_H$ (Carnot efficiency)

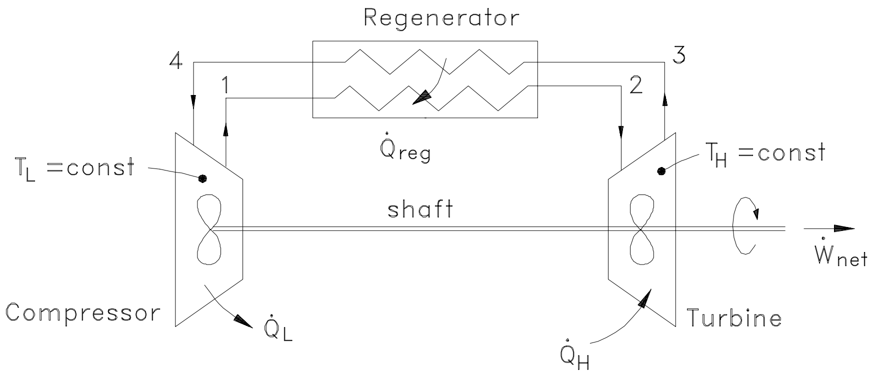


Week 7: Lecture 3

Ericsson Cycle



- hardware is not complicated
- but it is very difficult to operate turbines and compressors isothermally
- can approach isothermal conditions by repeated intercooling (compressor) and repeated reheating (turbines)
- $\eta_{Stirling} = 1 - T_L/T_H$ (Carnot efficiency)



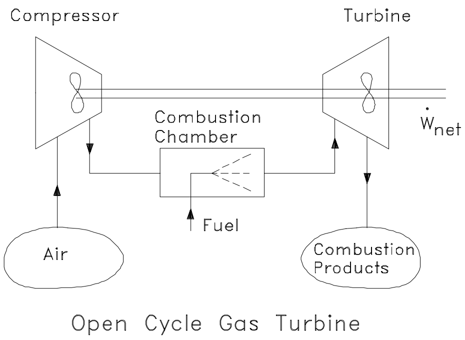
Week 8: Lecture 1

Open Cycle Gas Turbine Engines

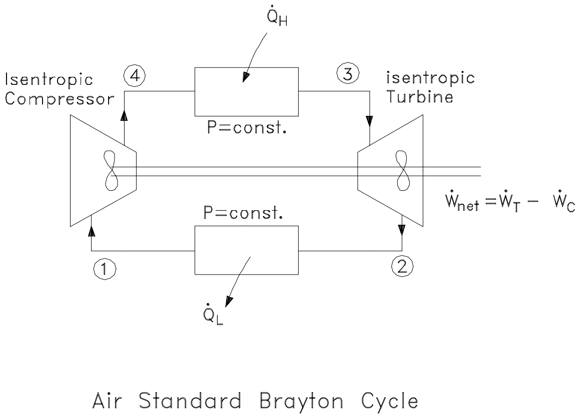
- after compression, air enters a combustion chamber into which fuel is injected
- the resulting products of combustion expand and drive the turbine
- combustion products are discharged to the atmosphere
- compressor power requirements vary from 40-80% of the power output of the turbine (remainder is net power output)
- high power requirement is typical when gas is compressed because of the large specific volume of gases in comparison to that of liquids

Advantages

- can use different fuels
- simple in construction, easy to maintain
- small weight-to-power ratio



Open Cycle Gas Turbine

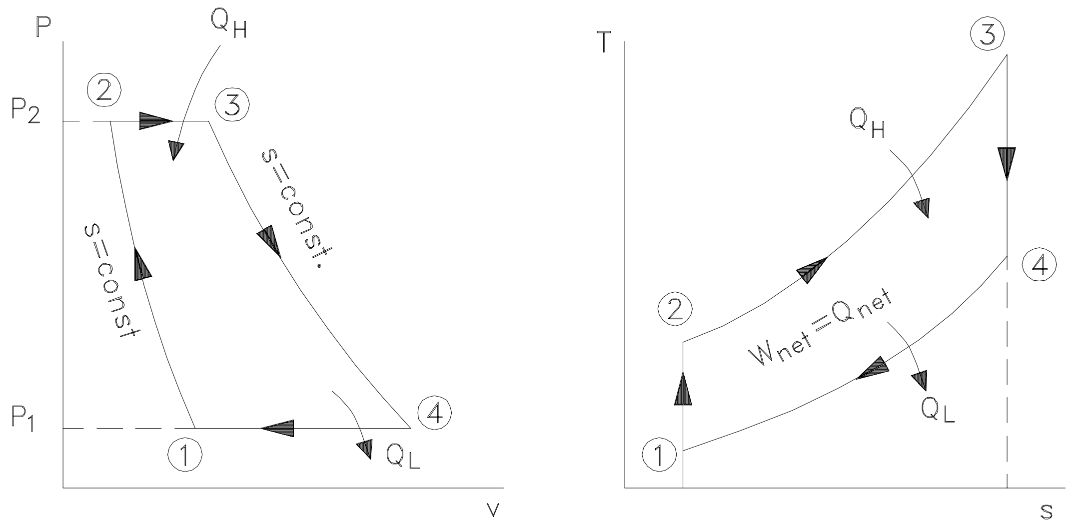


Air Standard Brayton Cycle

Idealized Air-standard Brayton Cycle

- closed loop
- constant pressure heat addition and rejection
- ideal gas with constant specific heats

Week 8: Lecture 1



Brayton cycle efficiency $\Rightarrow$ $\boxed{\eta = 1 - (r_p)^{\frac{1-k}{k}}}$

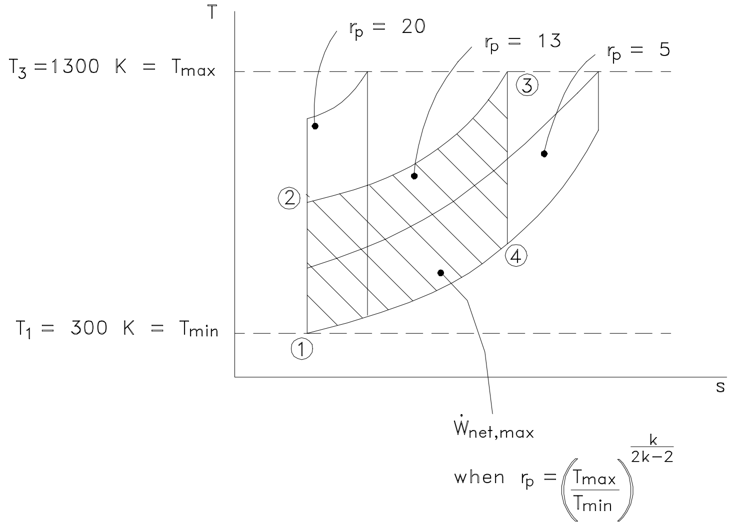
where $r_p = \dfrac{P_2}{P_1} = \dfrac{P_3}{P_4}$

The maximum temperature in the cycle (T_3) is limited by metallurgical conditions because the turbine blades cannot sustain temperatures above 1300 K. Higher temperatures (up to 1600 K can be obtained with ceramic turbine blades).

Optimum Work Output

The maximum work output can be achieved when the pressure ratio is:

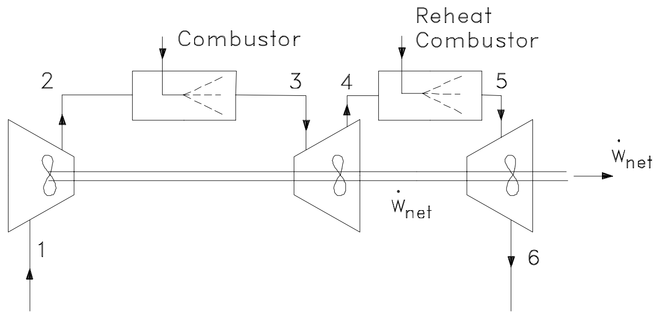
$$r_p = \left(\frac{T_{max}}{T_{min}}\right)^{\dfrac{k}{2k-2}}$$

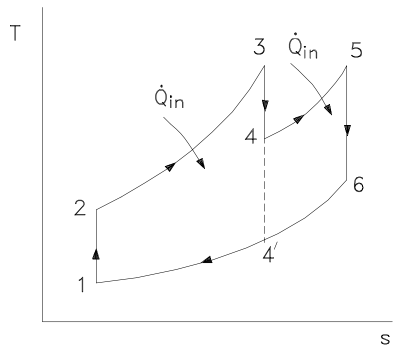


Week 8: Lecture 2

Brayton Cycle with Reheat

- the maximum temperature at T_3 entering the turbine is limited due to metallurgical constraints
- excess air is extracted and fed into a second stage combustor and turbine
- total work is increased
- but additional heat input is required
- net efficiency may or may not increase
- turbine outlet temperature is increased with reheat, therefore potential for regeneration is enhanced

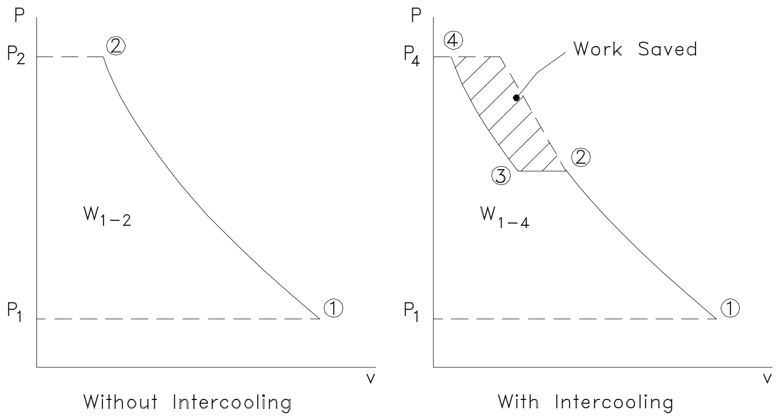




Week 8: Lecture 2

Compression with Intercooling

- the work required to compress in a steady flow device can be reduced by compressing in stages
- cooling the gas reduces the specific volume and in turn the work required for compression
- by itself compression with intercooling does not provide a significant increase in efficiency
- the lower temperature at the compressor exit enhances the potential for regeneration



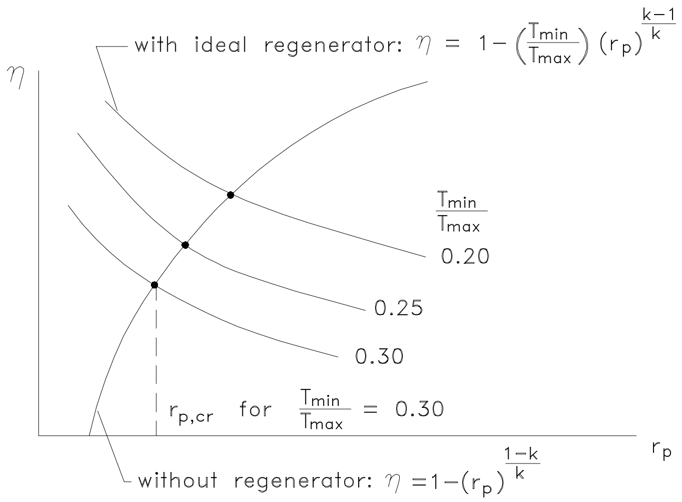
Week 8: Lecture 3

Brayton Cycle with Regeneration

- a regenerator (heat exchanger) is used to reduce the fuel consumption to provide the required $\dot{Q}_H$
- the efficiency with a regenerator is given by

$$\eta = 1 - \left(\frac{T_{min}}{T_{max}}\right)(r_p)^{\frac{k-1}{k}}$$

- for a given T_{min}/T_{max}, the use of a regenerator above a certain r_p will result in a reduction of η



The regenerator effectiveness can be defined as

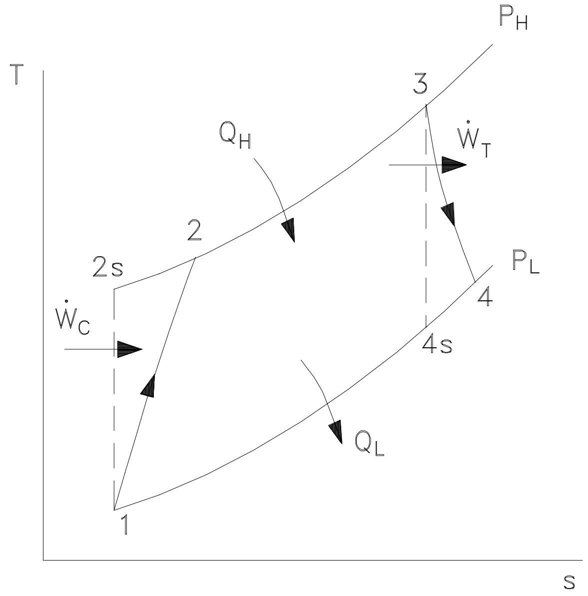
$$\epsilon = \frac{\dot{Q}_{reg,actual}}{\dot{Q}_{reg,ideal}}$$

Typical values of effectivenes are ≤ 0.7

Repeated intercooling, reheating and regeneration will provide a system that approximates the Ericsson Cycle which has Carnot efficiency ($\eta = 1 - \frac{T_L}{T_H}$).

Week 8: Lecture 3

Compressor and Turbine Efficiencies



Isentropic Efficiencies

(1) $$\eta_{comp} = \frac{h_{2,s} - h_1}{h_2 - h_1} = \frac{\cancel{c}_p\,(T_2, s - T_1)}{\cancel{c}_p\,(T_2 - T_1)}$$

(2) $$\eta_{turb} = \frac{h_3 - h_4}{h_3 - h_{4,s}} = \frac{\cancel{c}_p\,(T_3 - T_4)}{\cancel{c}_p\,(T_3 - T_{4,s})}$$

(3) $$\eta_{cycle} = \frac{W_{net}}{Q_H} = \frac{Q_H - Q_L}{Q_H} = 1 - \frac{Q_L}{Q_H} = 1 - \frac{\cancel{c}_p\,(T_4 - T_1)}{\cancel{c}_p\,(T_3 - T_2)}$$

Given the turbine and compressor efficiencies and the maximum (T_3) and the minimum (T_1) temperatures in the process, find the cycle efficiency (η_{cycle}).

(4) Calculate T_{2s} from the isentropic relationship,
$$\frac{T_{2,s}}{T_1} = \left(\frac{P_2}{P_1}\right)^{(k-1)/k}.$$
Get T_2 from (1).

(5) Do the same for T_4 using (2) and the isentropic relationship.

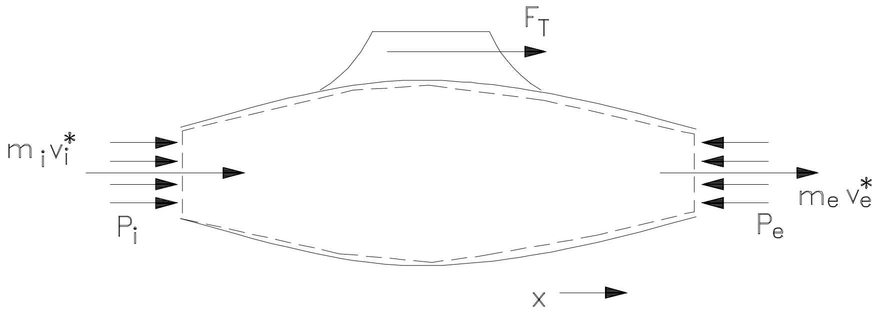
(6) substitute T_2 and T_4 in (3) to find the cycle efficiency.

Week 9: Lecture 1

Gas Turbines for Aircraft Propulsion

- gas turbines are well suited to aircraft propulsion because of their favorable power-to-weight ratio
- the exhaust pressure of the turbine will be greater than that of the surroundings
- since the gases leave at a high velocity, the change in momentum that the gas undergoes gives a thrust to the aircraft
- gases are expanded in the turbine to a pressure where the turbine work is just equal to the compressor work plus some auxiliary power for pumps and generators i.e. the net work output is zero
- typically operate at higher pressure ratios, often in the range of 10 to 25

Conservation of Momentum



$$\frac{d(Mom)_{x,cv}}{dt} = (Mom)_{x,in} - (Mom)_{x,out} + \sum F_x$$

for steady flow $\Rightarrow \frac{d}{dt} = 0$ and

$$\dot{m}_{fuel} << \dot{m}_i \Rightarrow \dot{m}_i \approx \dot{m}_e$$

Therefore

$$\begin{aligned} F_T &= \dot{m}_e v_e^* - \dot{m}_i v_i^* - \underbrace{P_{atm}(A_i - A_e)}_{negligible} \\ &= \dot{m}_i (v_e^* - v_i^*) \end{aligned}$$

Week 9: Lecture 1

Specific Impulse: $\boldsymbol{I = \frac{F_T}{\dot{m}_i} = v_e^* - v_i^*}$

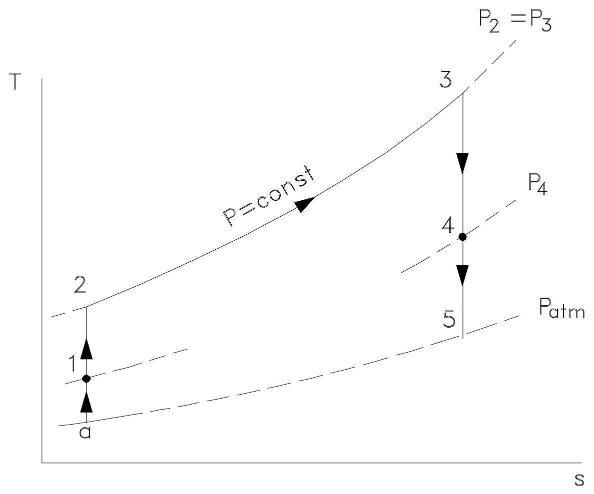
Propulsive Power: $\boldsymbol{\dot{W}_T = F_T v_i^* \approx \dot{m}_i (v_e^* - v_i^*) v_i^*}$

Propulsive Efficiency: $\boldsymbol{\eta = \frac{\dot{W}_T}{\dot{Q}_{in}}}$

Since the net work output is zero, we must define the propulsive efficiency as propulsive power over the heat flow rate in the combustion process. This then becomes a measure of how efficiently the energy released during the combustion process is converted to propulsive energy.

Week 9: Lecture 2

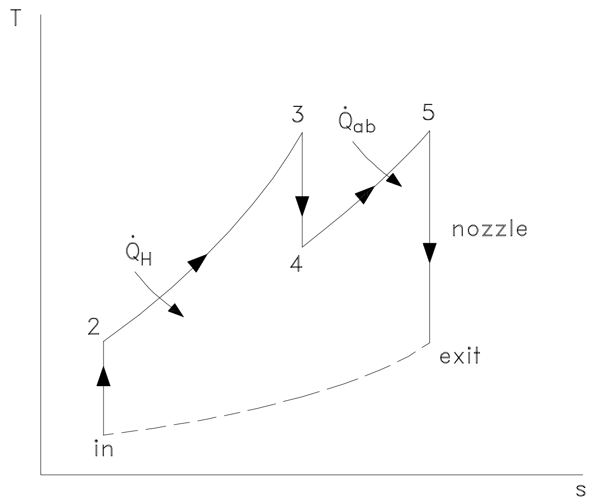
Turbojet



- **a-1**: diffuser
  - decelerates the incoming flow relative to the engine
  - a pressure rise known as a ram effect occurs
- **1-4**: gas generator
  - compressor, combustor and turbine
  - turbine power just enough to drive the compressor
  - air and fuel are mixed and burned in the combustion chamber at constant pressure
  - $\boldsymbol{P_T >> P_{atm}}$
- **4-5**: nozzle
  - gases leave the turbine significantly higher in pressure than atmospheric pressure
  - gases are expanded to produce a high velocity
  - $\boldsymbol{v_e^* >> v_i^*}$
  - results in a thrust

Week 9: Lecture 2

Afterburner



By performing a 1st law energy over the nozzle we can obtain an expression for the exit velocity in terms of the entrance temperature to the nozzle.

$$\frac{d\cancel{E}^{\,0}}{dt} = \cancel{\dot{Q}}^{\,0} + \cancel{\dot{W}}^{\,0} \left\{ \dot{m} \left[h_4 + \frac{(v_4^*)^2}{2} \right] \right\} - \left\{ \dot{m} \left[h_e + \frac{(v_e^*)^2}{2} \right] \right\}$$

If we assume that the air velocity leaving the turbine is relatively small, the kinetic energy term at 4 can be assumed to go to zero and we get

$$\begin{aligned} v_e^* &= \sqrt{2(h_4 - h_e} \\ &= \sqrt{2c_p(T_4 - T_e)} \end{aligned}$$

- exit velocity proportional to $v_e^* \propto \sqrt{2c_p(T_4 - T_e)}$
- afterburner is used to increase T_4 to T_5
- similar to a reheat device
- produces a higher temperature at the nozzle inlet

Week 9: Lecture 3

Other Types of Engines

1. Turbo-prop Engine
   - gas turbine drives the compressor and the propeller
   - propellers are best suited for low speed ($<$ 300 mph) flight
   - by-pass ratio defined as

$$\boldsymbol{by - pass\ ratio = \frac{mass\ flow\ by - passing\ the\ combustion\ chamber}{mass\ flow\ through\ the\ combustion\ chamber}}$$

   - by-pass ratio of 100:1 or more
2. Turbo-fan Engine (Ducted Turbo-prop Engine)
   - best choice for fuel economy and speed
   - high speed exhaust gases are mixed with the lower speed air in the by-pass resulting in a considerable noise reduction
   - by-pass ratio can be adjusted
   - by-pass provides thrust for takeoff
   - the core provides thrust for cruising
   - used for speeds up to 600 mph
   - increasing the by-pass ration results in increased thrust
   - typical by-pass ratios are 5-6
3. Ramjet
   - no moving parts
   - compression is achieved by decelerating the high-speed incoming air in the diffuser
   - aircraft must already be in flight at a high speed

Week 10: Lecture 1

Non-Reacting Mixtures

Definitions

- the total mass of a mixture

$$m = m_1 + m_2 + \cdots + m_j = \sum_{i=1}^{j} m_i$$

- the relative amount of components specified in terms of mass fractions

$$Y_i = \frac{m_i}{m} \quad \Rightarrow \quad \sum_{i=1}^{j} Y_i = 1$$

- the total number of moles of a mixture

$$n = n_1 + n_2 + \cdots + n_j = \sum_{i=1}^{j} n_i$$

- the relative amount of components specified in terms of mole fractions

$$X_i = \frac{n_i}{n} \quad \Rightarrow \quad \sum_{i=1}^{j} X_i = 1$$

- m_i and n_i are related by the molecular weight $\tilde{M}_i$

$$m_i = n_i \cdot \tilde{M}_i$$

therefore

$$m = \sum_{i=1}^{j} n_i \cdot \tilde{M}_i$$

Week 10: Lecture 1

- the mixture molecular weight can be calculated as a mole-fraction average of the component molecular weights

$$\tilde{M} = \frac{m}{n} = \frac{\sum_{i=1}^{j} n_i \tilde{M}_i}{n} = \sum_{i=1}^{j} X_i \tilde{M}_i$$

- X_i and Y_i are also related by the molecular weight

$$\frac{Y_i}{X_i} = \frac{m_i/m}{n_i/n} = \left(\frac{m_i}{n_i}\right)\left(\frac{m}{n}\right) = \left(\tilde{M}_i\right)\left(\frac{1}{\tilde{M}}\right)$$

therefore

$$Y_i = X_i \left[\frac{\tilde{M}_i}{\sum_{i=1}^{j} X_i \tilde{M}_i}\right]$$

Week 10: Lecture 2

P-V-T Relationships for Ideal Gas Mixtures

Amagat Model (law of additive volumes)

- each component mixture behaves as an ideal gas as if it existed separately at the pressure, $\boldsymbol{P}$ and the temperature, $\boldsymbol{T}$, of the mixture

$$
\begin{array}{|c|c|}
\hline
T, P & T, P \\
m_A & m_B \\
n_A & n_B \\
V_A & V_B \\
\hline
\end{array}
\quad
\begin{array}{c}
\text{remove} \\
\rightarrow \\
\text{partition}
\end{array}
\quad
\begin{array}{|c|}
\hline
T, P \\
m_C = m_A + m_B \\
n_C = n_A + n_B \\
V_C = V_A + V_B \\
\hline
\end{array}
$$

$$\frac{V_i}{V} = \frac{n_i}{n} = X_i$$

which leads to

$$V = \sum_{i=i}^{j} V_i$$

Dalton Model (law of additive pressures)

- each mixture component behaves as an ideal gas as if it were alone at the temperature, $\boldsymbol{T}$ and the volume, $\boldsymbol{V}$ of the mixture

$$
\begin{array}{|c|}
\hline
V, T \\
m_A, n_A, P_A \\
\hline
\end{array}
+
\begin{array}{|c|}
\hline
V, T \\
m_B, n_B, P_B \\
\hline
\end{array}
=
\begin{array}{|c|}
\hline
V, T \\
m_C = m_A + m_B \\
n_C = n_A + n_B \\
P_C = P_A + P_B \\
\hline
\end{array}
$$

Week 10: Lecture 2

for a mixture of ideal gases the pressure is the sum of the partial pressures of the individual components

$$P = \sum_{i=i}^{j} P_i$$

By combining the results of the Amagat and Dalton models, we obtain

$$\frac{P_i}{P} = \frac{V_i}{V} = \frac{n_i}{n}$$

Ideal Gas Law for a Mixture

- the gas constant can be expressed as

$$R = \sum_{i=1}^{j} Y_i R_i$$

- the relative mass fractions and mole fractions can be related in terms of the gas constant as

$$X_i = Y_1 \left[\frac{R_i}{\sum_{i=1}^{j} Y_i R_i} \right]$$

Week 10: Lecture 2

Non-Reacting Mixtures

Changes in Internal Energy, Enthalpy and Entropy of Mixtures

$$u_2 - u_1 = \sum Y_i (u_2 - u_1)_i = \int_{T_1}^{T_2} c_v dT$$

$$h_2 - h_1 = \sum Y_i (h_2 - h_1)_i = \int_{T_1}^{T_2} c_p dT$$

$$s_2 - s_1 = \sum Y_i (s_2 - s_1)_i = \int_{T_1}^{T_2} c_p \frac{dT}{T} - R \ln \frac{P_2}{P_1}$$

The above equations can also be expressed on a per mole basis.

Entropy Change Due to Mixing of Ideal Gases

- when ideal gases are mixed, a change in entropy occurs as a result of the increase in disorder in the system
- if the initial temperatures of all constituents are the same and the mixing process is adiabatic

$$\mathcal{P}_s = \Delta S = -\left(m_A R_A \ln \frac{P_A}{P} + m_B R_B \ln \frac{P_B}{P} + \ldots\right)$$

$$= -\sum m_i R_i \ln \frac{P_i}{P}$$

$$= -\mathcal{R} \sum n_i \ln X_i$$

Week 10: Lecture 3

Psychrometric Applications

- studies involving mixtures of dry air and water vapour
- used in the design of air-conditioning systems, cooling towers and most processes involving the control of vapour content in air
- for $T \leq 50°C \quad (P_{sat} \leq 13\ kPa) \quad \Rightarrow \quad h \approx h(T)$
- water vapour can be treated as an ideal gas

Moist Air

$$\begin{aligned} P &= P_a + P_w \\ P_a &= \frac{m_a R_a T}{V} \\ P_w &= \frac{m_w R_w T}{V} \end{aligned}$$

where

P_a is the partial pressure of air and P_w is the partial pressure of water vapour.

Relative Humidity

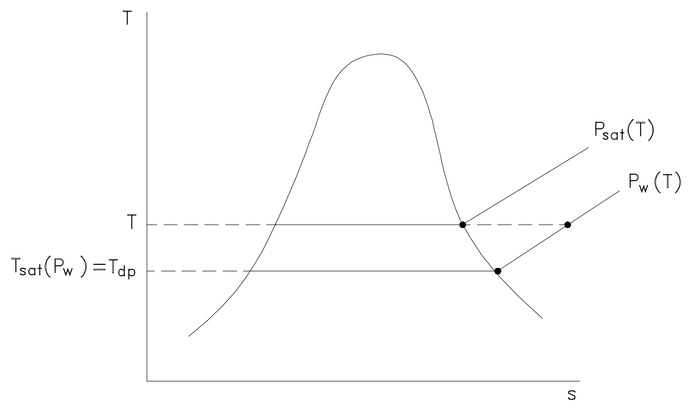
$$\phi = \frac{P_w(T)}{P_{sat}(T)} = \frac{\textbf{vapour pressure at the prevailing T}}{\textbf{saturation pressure at the prevailing T}}$$

The relative humidity can also be written as

$$\phi = \frac{P_w}{P_{sat}} = \frac{\rho_w}{\rho_s at} = \frac{v_g}{v_w}$$

where v_g is the mixture specific volume and v_w is water specific volume.

Week 10: Lecture 3



Specific Humidity (Humidity ratio)

$$\gamma = \omega = \frac{m_w}{m_a} = \frac{\textbf{mass of water vapour}}{\textbf{mass or air}}$$

$$= \left(\frac{\tilde{M}_w}{\tilde{M}_a}\right)\left(\frac{P_w}{P_a}\right)$$

$$= 0.622 \left(\frac{P_w}{P_a}\right)$$

In addition γ can be written as

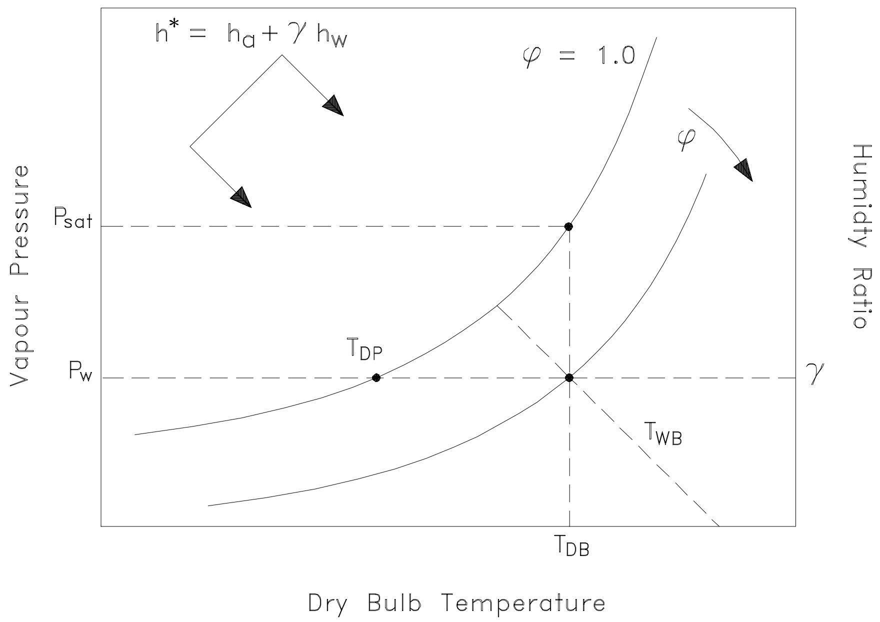
$$\gamma = 0.622 \left(\frac{P_w}{P_a}\right) = 0.622 \left(\frac{P_w}{P - P_w}\right) = 0.622 \left(\frac{\phi P_{sat}}{P - \phi P_{sat}}\right)$$

which can be rearranged in terms of relative humidity

$$\phi = \frac{P\gamma}{P_{sat}\left(\gamma + \frac{\tilde{M}_w}{\tilde{M}_a}\right)} = \frac{P\gamma}{P_{sat}(\gamma + 0.622)}$$

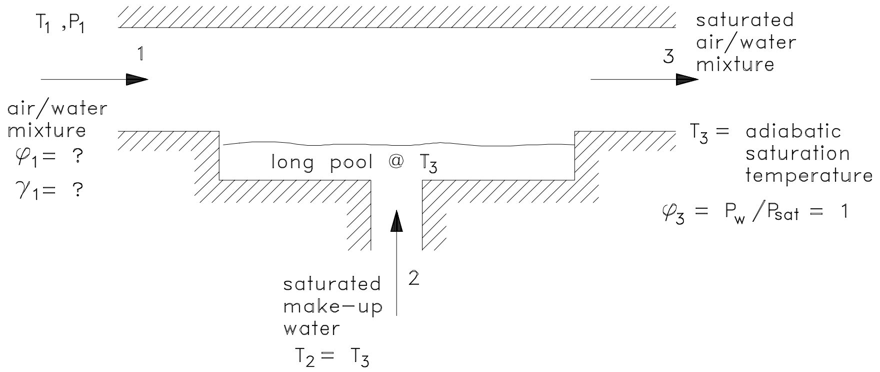
Week 10: Lecture 3

Psychrometric Chart



Week 11: Lecture 1

An Adiabatic Saturator



- used to measure humidity
- two inlet, single exit device through which moist air passes
- air-water mixture of unknown humidity enters at a known pressure and temperature
- if air/water mixture is not saturated, some water from the pool will evaporate
- the energy required to evaporate the water comes from the moist air $\rightarrow$ mixture temperature decreases
- for a sufficient long duct, the moisture exits with $\boldsymbol{\phi = 1}$
- the temperature of the exiting mixture is called the adiabatic saturation temperature

Mass and energy balances lead to:

$$\gamma_1 = \frac{c_{p_a}(T_3 - T_1) + \gamma_3(h_{fg}(T_3))}{(h_{w_1} - h_{w_2})}$$

If we assume that h_{w_1} is very close to the enthalpy for the saturated vapour at T_1 and h_{w_2} is close to the enthalpy of the saturated liquid at T_1 (this assumption can be made with confidence since $h_f << h_g$), we can write γ_1 as

$$\gamma_1 = \frac{c_{p_a}(T_3 - T_1) + \gamma_3(h_{fg}(T_3))}{(h_{fg}(T_1))}$$

Week 11: Lecture 1

Dry Bulb Temperature - the temperature measured by a thermometer placed in a mixture of air and water vapour

Wet Bulb Temperature
- thermometer surrounded by a saturated wick
- if air/water vapour mixture is not saturated, some water in the wick evaporates and diffuses into the air $\rightarrow$ cooling the water in the wick
- at the temperature of the water drops, heat is transferred to the water from both the air and the thermometer
- the steady state temperature is the wet-bulb temperature

Sling Thermometer - a rotating set of thermometers one of which measures wet bulb temperature and the other dry bulb temperature. $\boldsymbol{T_{DB}}$ and $\boldsymbol{T_{WB}}$ are sufficient to fix the state of the mixture.

Week 11: Lecture 2

Applications

1. Simple Heating and Cooling

$$\frac{\dot{Q}}{\dot{m}_a} = (h_2^* - h_1^*) \quad (heating)$$

where

$$h_1^* = h_{a,1} + \gamma_1 h_{w_1}$$

$$h_2^* = h_{a,2} + \gamma_2 h_{w_2}$$

2. Heating with Humidification

$$\dot{Q} = \dot{m}_a \; (h_2^* - h_1^*) \quad (heating)$$

where

$$\dot{m}_a = \frac{\dot{V}}{v_{a_a}}$$

$$v_{a_a} = \frac{R_a T_1}{P_{a_1}} = \frac{R_a T_1}{(P - P_{w_1})} = \frac{R_a T_1}{(P - \phi_1 P_{sat_1})}$$

and

$$\dot{m}_{steam} = \dot{m}(\gamma_3 - \gamma_2)$$

3. Evaporative Cooling

$$h_1^* + \underbrace{(\gamma_2 - \gamma_1) h_\ell}_{\text{negligible}} = h_2^*$$

Therefore

$$h_1^* = h_2^*$$

Week 11: Lecture 2

4. Mixing of Two Moist Air Streams

$$\frac{\dot{m}_{a,1}}{\dot{m}_{a,2}} = \frac{h_2^* - h_3^*}{h_3^* - h_1^*} = \frac{\gamma_2 - \gamma_3}{\gamma_3 - \gamma_1}$$

where

$$\dot{m}_{a,1} = \rho_{a_1} \dot{V}_1$$

$$\dot{m}_{a,2} = \rho_{a_2} \dot{V}_2$$

Week 11: Lecture 3

Reacting Mixtures and Combustion

Definitions

Combustion Process

- a fuel made up of hydrocarbons is said to have burned completely if:
  - all the carbon present in the fuel is burned to carbon dioxide
  - all the hydrogen is burned to water
- if the conditions are not fulfilled the combustion is incomplete

Combustion Reactions

$$\textbf{reactants} \rightarrow \textbf{products}$$

or

$$\textbf{fuel} + \textbf{oxidizer} \rightarrow \textbf{products}$$

- in all cases the mass is conserved

  mass of products = mass of reactants

Fuels

- fuel is simply a combustible substance
- hydrocarbon fuels exist as liquids, gases and solids

Combustion Air

- oxygen is required in every combustion reaction
- in most combustion reactions air provides the needed oxygen

Week 11: Lecture 3

Air-Fuel Ratio

$$\frac{mass\ of\ air}{mass\ of\ fuel} = \frac{moles\ of\ air \times \tilde{M}_{air}}{moles\ of\ fuel \times \tilde{M}_{fuel}}$$

$$AF = \bar{AF}\left(\frac{\tilde{M}_{air}}{\tilde{M}_{fuel}}\right)$$

Theoretical or Stoichiometric Air

- the minimum amount of air that supplies sufficient oxygen for complete combustion of all carbon and hydrogen in the fuel
- no free oxygen would appear in the products
- normally the amount of air supplied is given as a percentage of the theoretical value i.e. 150% = 1.5 × the theoretical air
- the equivalence ratio is defined as

$$equivalence\ ratio = \frac{AF_{actual}}{AF_{theoretical}}$$

- if the equivalence ratio is:
  - $> \mathbf{1}$ $\rightarrow$ lean mixture
  - $< \mathbf{1}$ $\rightarrow$ rich mixture

Week 12: Lecture 1

Conservation of Energy for Reacting Systems

Enthalpy of Formation

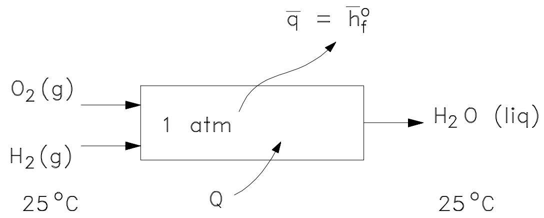
- previous calculations involving enthalpy were all based on differences and the reference used to determine enthalpy did not matter
- when chemical reactions occur, reactants disappear and products are formed → differences cannot be calculated for all substances involved
- it is necessary to establish a common base to account for differences in composition
- the enthalpy datum for reacting systems is set to zero at standard temperature and pressure
  - $T_{ref} = 25°C \rightarrow 298\ K$
  - $P_{ref} = 1\ atm$
- $h = 0$ assigned to elements in their most stable form i.e. $O_2, N_2, C, etc.$
- the enthalpy of formation is defined as:
  - the energy released or absorbed when a compound is formed from its stable elements at STP

$$\delta \overline{h}_f^o = \overline{h}_{comp}^o - \sum \overline{n_i} \overline{h}_i^o \quad (kJ/kmole)$$

where

$\overline{n}_i$ = # of moles of i'th elemental substance in forming a single mole of compound (unitless)

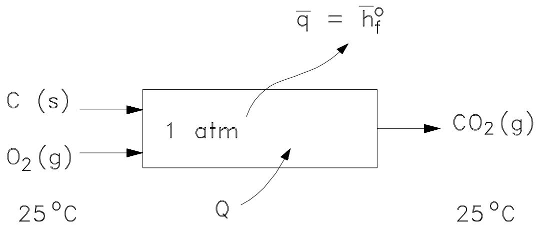
Consider the following reactions taking place at atmospheric pressure, such that $T_{prod} = T_{react} = 298\ K$



$$\frac{1}{2}O_2 + H_2 \rightarrow H_2O$$

where $\overline{h}_f^o = -286\ MJ/kmole\ of H_2O$ (heat transferred from the combustion chamber)

Week 12: Lecture 1



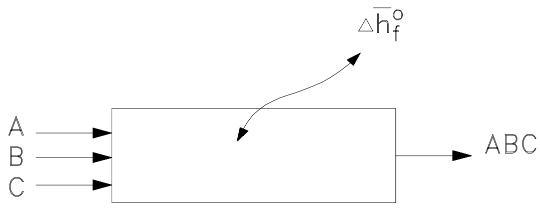
$$C + O_2 \rightarrow CO_2$$

where $\overline{h}_f^o = -393\ MJ/kmole\ of CO_2$ (heat transferred from the combustion chamber)

These reactions are called formation reactions because a substance is formed from its elements in their respective natural states at STP

$$\overline{q} = standard\ heats\ of\ formation$$

$$\overline{h}_f^o = enthalpy\ of\ formation$$



$$\delta\overline{h}_f^o = \overline{h}_{comp}^o - \sum \overline{n}_i \overline{h}_i^o \quad (kJ/kmole)$$

An energy balance gives

$$\underbrace{a\overline{h}_A^o + b\overline{h}_B^o + c\overline{h}_C^o}_{generally\ 0} + \delta\overline{h}_f^o \rightarrow \overline{h}_{ABC}^o$$

If

$\overline{h}_f^o > 0$ - endothermic
$\overline{h}_f^o < 0$ - exothermic

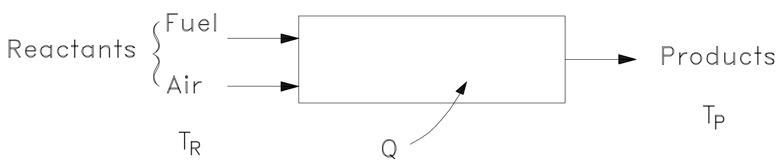
Week 12: Lecture 2

Enthalpy of Combustion

- for various types of fuels the enthalpy of formation is not always known
  - the chemical composition of most fuels may vary from batch to batch
  - generally not tabulated
- enthalpy of combustion is an experimental alternative to enthalpy of formation
  - defined as the difference between the enthalpy of the products and the enthalpy of the reactants where complete combustion occurs at a given temperature and pressure

$$\boldsymbol{Q = \sum (mh)_P - \sum (mh)_R = H_P(T_P) - H_R(T_R)}$$

$$\boldsymbol{Q = \sum (n\overline{h})_P - \sum (n\overline{h})_R = \underbrace{\overline{H}_P(T_P) - \overline{H}_R(T_R)}_{H_{RP}}}$$



- when enthalpy of formation data is available for all products and reactants the above equation can be used
- otherwise a calorimeter must be used to measure the enthalpy of combustion
- $\boldsymbol{+ve\ Q}$ $\Rightarrow$ endothermic reaction
- $\boldsymbol{-ve\ Q}$ $\Rightarrow$ exothermic reaction

Heating Value

- the heating value of a fuel is a positive value equal to the magnitude of the enthalpy of combustion

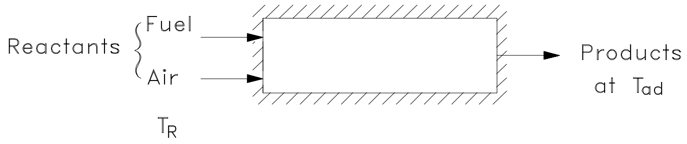
Week 12: Lecture 2

- two values are used
  - HHV: higher heating value - obtained when all the water formed by combustion is a liquid
  - LHV: lower heating value - obtained when all the water formed by combustion is a vapour
- the HHV exceeds the LHV by the energy required to vaporize the liquid formed

$$HHV = LHV + (mh_{fg})_{H_2O}$$

Adiabatic Flame Temperature

- under adiabatic conditions the maximum temperature attained by the products when combustion is complete is called the adiabatic flame or adiabatic combustion temperature



$$H_P(T_{ad}) = H_R(T_R)$$

$$\sum_P n_e(\overline{h}_f^o + \Delta\overline{h})_e = \sum_R n_i(\overline{h}_f^o + \Delta\overline{h})_i$$

$$\sum_P n_e(\Delta\overline{h})_e = \sum_R n_i(\Delta\overline{h})_i + \sum_R n_i\overline{h}_{f_i}^o - \sum_P n_e\overline{h}_{f_e}^o$$

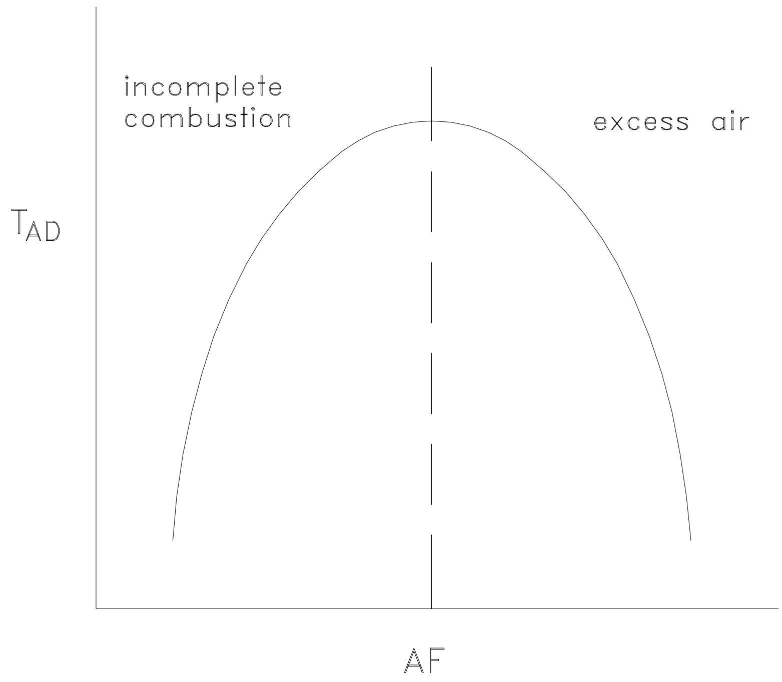
where

n	-	obtained from reaction equation
$\overline{h}_{f_i}^o$	-	from tabulated data
$\Delta\overline{h}$	-	from tables or calculated knowing T_R

Therefore all terms on the right hand side can be evaluated. While the left hand side accounts for the change in enthalpy from T_{ref} to T_{ad}, which is unknown. The procedure requires iteration.

The temperature for the products is assumed. We then keep trying until we get agreement with the RHS. A typical value of $T_{ad} \approx 2300\ K$ for most hydro carbon fuels

Week 12: Lecture 2



Week 12: Lecture 3

Evaluation of Entropy for Reacting Systems

- a common datum must be used to assign entropy values for each substance involved in the reaction
- an entropy of 0 for pure crystalline substances is obtained at absolute zero
- the entropy relative to this datum is called absolute entropy
- absolute entropy at 1 atm and temperature T is denoted as $s^o(T)$ or $\overline{s}^o(T)$ for a per unit mass or per mole basis (note: referred to as $\hat{\phi}$ in Reynolds and Perkins)
- the entropy at any value of T and P can be calculated as

$$\overline{s}(T,P) = \overline{s}^o(T) - \mathcal{R}\ln\left(\frac{P_i}{P_{ref}}\right)$$

- the entropy of the i'th component of an ideal gas mixture is evaluated at temperature T and the partial pressure P_i where the partial pressure is given by

$$P_i = X_i P$$

and

$$\overline{s}(T,P_i) = \overline{s}_i^o(T) - \mathcal{R}\ln\left(\frac{X_i P}{P_{ref}}\right)$$

Incomplete Combustion

- when the amount of air supplied is less than the theoretical air required the combustion is incomplete
- the usual result is carbon unites with oxygen to form carbon monoxide (CO) instead of CO_2

Week 12: Lecture 3

Dew Point

- since water is formed when hydrocarbon fuels are burned, the mole fraction of water vapour in the form of gaseous products can be significant
- if the gaseous products of combustion are cooled at constant mixture pressure the dew point temperature is reached when water vapour begins to condense
- since corrosion of duct work, mufflers etc. can occur, the knowledge of dew point temperature is important